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Endogenous Leverage and Asset Pricing in Double Auctions

Thomas Breuer, Hans-Joachim Vollbrecht, Martin Summer

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Editorial

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Thomas Breuer* Hans-Joachim Vollbrecht† Martin Summer‡

July 19, 2013

We study the trading of real assets financed by collateralized loans in an agent based model of a continuous double auction. This approach provides a complementary perspective on recent advances in the general equilibrium theory of endogenous leverage by studying a model that simultaneously describes dynamic and equilibrium properties of the market. Rather than taking prices as parametric there is an explicit price formation process which can be simulated or studied empirically. This is important because the economics of leverage is key to the understanding of financial crisis. We find that simulated double auctions converge to stable final states close to the theoretical equilibrium state. Consistent with equilibrium theory, real assets are traded at a price above fundamental value in the double auction. The equilibrium level of leverage also emerges in the simulations of the double auction.

Keywords: Leverage, Asset Pricing, Double Auction, Agent Based Modeling

JEL-Classification Numbers: D53, G12, G14, C63

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1. Introduction

In this paper we study the trading of collateralized loans in continuous double auctions. A typical real world example of such markets would be repo markets, where short term debt is issued against full recourse collateral with a specific haircut determined in the market. In finance jargon this haircut is often referred to as margin. The margin can be directly translated into the leverage that can be achieved by borrowing. For example, if the purchase of assets worth 100 is financed by a repo using the assets as collateral and the margin is 20 we say that the leverage in this particular financial arrangement is 5. Understanding how such markets work is key to the understanding of financial crisis more generally because leverage makes agents more sensitive to changes in asset prices. Leverage is also closely related to asset price bubbles. The recent literature on the financial crisis identifies leverage as one of the key drivers of systemic risk (see Adrian and Shin [2010], Shin [2010] and Gorton and Metrick [2012]) among others.

Perhaps somewhat surprisingly economic theory has found it hard to explain how leverage is determined endogenously. The traditional literature on collateral, most importantly Kiyotaki and Moore [1997] and Bernanke et al. [1996] emphasized how borrowing capacity varies with the price of collateral holding leverage constant. Only recently Geanakoplos and Zame [2010] and Geanakoplos [2010][1] came up with a theory of endogenous leverage that relies on a pure mechanism of supply and demand.

The theory rationalizes asset prices and leverage as an equilibrium phenomenon. It does not explain how collateral equilibrium comes about. We are thus interested in a complementary perspective on endogenous leverage where the market mechanism and the price and allocation dynamics can be explicitly described and the relations between equilibrium behavior and out of equilibrium dynamics can be explicitly studied. In our paper this complementary perspective is provided by the continuous double auction. The continuous double auction has been extensively studied in experimental economics. The experimental economics literature has repeatedly found that the double auction converges to outcomes predicted by an abstract general equilibrium model. A synthetic overview of these insights can be found for instance in Smith [2008]. An overview more specifically related to asset markets is Sunder [1995]. It is thus an interesting candidate for providing a complementary perspective on a competitive equilibrium theory of collateralized lending and leverage.

We are going to explain how a double auction works in the case of collateralized lending. Unlike double auctions that have been studied in the literature we have to allow simultaneous markets for the collateral and the collateralized loan, and specify the collateral constraint that couples the two markets. The continuous double auction has been inherently difficult to study theoretically (see Wilson [1987]). In this paper we therefore analyze the double auction in collateralized debt markets by means of a *simulation* of an agent based model (Tesfastion [2006]) with so called zero intelligence agents (see Gode and Sunder [1993]).

We believe that a complementary perspective on the theory of endogenous leverage is

[1] These papers are based on previous work by Geanakoplos [1997] and Geanakoplos [2003]

not only inherently interesting but it is indeed necessary if we want to understand the main mechanisms behind the theory. In contrast to the general equilibrium theory of leverage, our process based analysis can give insights into the potential mechanisms through which equilibrium allocations and prices emerge in the real world. The advantage of this approach is that it is more directly amenable to experimental study. An experimental investigation can thus directly build on our analysis. It has the potential to clarify the key mechanisms behind the economics of leverage. It can also help discriminate between competing theories. For instance it can help clarifying the question whether supply and demand theories of endogenous leverage are sufficiently rich to explain the phenomena of interest or whether we do need theories based on asymmetric information such as Shleifer and Vishny [1992], Bernanke et al. [1999] and Holmstrom and Tirole [1997].

Taking an example studied in Geanakoplos [2010] as a benchmark we study a continuous double auction market for collateralized debt. We use Geanakoplos' parametrization of preferences, endowments and uncertainty in the double auction. As behavioral rules for the agents we assume that agents only take utility improving actions but otherwise behave randomly. They do not attempt to take optimal actions, neither do they use any information about historically traded prices or the status of the order book. We find that the continuous double auction converges to prices and allocations that match the predictions of the abstract general equilibrium theory. The meaning of convergence in this context is that the double auction eventually reaches a state where no further trade occurs. In our setup such a state is always reached and the prices and allocations prevailing in this state are interpreted as the outcomes of the double auctions. These outcomes are then compared to what the theory predicts. Due to random bidding the process of course will not end always at exactly the same values of prices and allocations. But the values are usually fairly close. By averaging over a whole set of outcomes and by reporting the variation of this statistics we derive support for our claim that the double auction converges to prices and allocations that are close to the predictions made by the theory.

As in the theory we observe that assets that serve as collateral earn a collateral premium over and above their fundamental value. Also in line with the theory we find that in the double auction leverage emerges endogenously. While there are many different debt instruments available for trade there is significant trade only in the debt instrument that allows the highest leverage for the minimal risk.

We should say that this is a model that focuses on asset pricing and endogenous leverage only. While these two aspects are an important ingredient of the leverage cycle theory (see Geanakoplos [2010]), they are not the full-fledged theory. By themselves they cannot explain financial instability or help in judging the implications of leverage for the efficient allocation of risk in the economy at large. We believe it is interesting to study in isolation the asset pricing effects of collateralized lending and the endogenous determination of leverage, because these effects play such an important role in a more general theory of financial instability and financial crisis.

The paper is structured as follows. Section 2 gives a brief exposition of the collateral equilibrium theory using and referring to the specific example of Geanakoplos [2010]. The core of the paper is section 3 where we describe and analyze the simulated double auction for trading of real assets financed by collateralized loans. Section 4 concludes.

2. Leverage and Asset Pricing in General Equilibrium: The Geanakoplos Example

Geanakoplos [2010] gives an example of the general competitive theory of collateralized lending developed in Geanakoplos and Zame [2010]. In this section we give a brief exposition of this example, which will set the scene for our agent based simulations of double auctions. Consider a simple event tree with two dates $t = 0, 1$ and two states of the world, the up and the down state at time 1. There is continuum of risk neutral agents, indexed by $i \in [0, 1]$, and a single consumption good. The index i is the probability the agent i assigns to the up state. Agents with a higher i are more optimistic. There preferences are thus given by the utility function $u^i(x^i) = x^i_0 + i\, x^i_U + (1 - i)\, x^i_D$There is also one real asset paying 1 unit of the consumption good in the up state and 0.2 units in the down state. One can interpret these numbers as the time 1 value of the asset expressed in units of the consumption good.

All agents have one unit endowment of the consumption good at $t = 0$ and nothing in either the up or the down state. They all have an endowment of one unit of the real asset. All agents have also access to a riskless free storage technology which allows them to warehouse the consumption good at $t = 0$ and have it available without discount or dividend in both states U and D.

Equilibrium without collateralized debt Geanakoplos [2010] shows that when there is trade in the real asset only, without the possibility of borrowing and lending, in a competitive equilibrium every agent holds one of two kinds of portfolio. Pessimistic agents, or P-agents, with $i \leq i^*$, hold $1 + p$ units of the consumption good, and nothing else. (p denotes the asset price.) Optimistic agents, or O-agents, with $i \geq i^*$ hold $(1+p)/p$ units of the real asset, and nothing else. The agent i^* is indifferent between holding only the consumption good, or holding only assets, or any mixture. The asset price p, and the allocation parameter i^* are given by the solution of the following equations

$$i^* = \frac{1}{1+p} \tag{1}$$

$$p = i^* + (1 - i^*)0.2 \tag{2}$$

Solving these two equations yields $i^* = 0.596$ and $p = 0.677$. O-agents hold 2.477 units of the asset, whereas P-agents hold 1.677 units of the consumption good. Fig. 1 shows the equilibrium allocation.

Equilibrium with collateralized loans The key idea in modeling collateralized loans is to characterize them by their face value and their corresponding collateral requirement. Loans that differ in either face value or collateral requirement are economically different and have a different price.

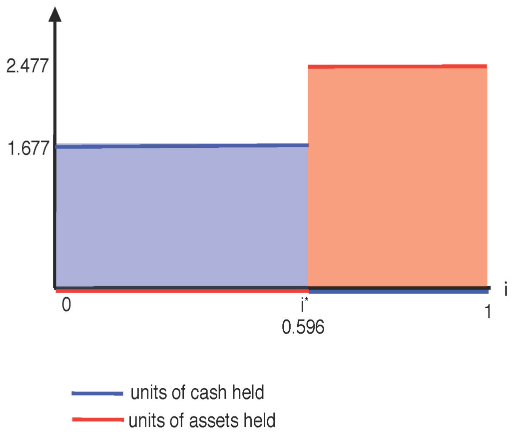

Figure 1: Equilibrium allocation without borrowing and lending.

More specifically, assume there are J different loans available. Their *face value* is a promise to deliver V^j units of the consumption good in every state at time $t = 1$. Their *collateral requirement* is that the borrower is required to hold one unit of the real asset as collateral for each unit of the collateralized loan.[2]

The payoffs of the collateral, and only these payoffs, back the loan. Whenever the collateral value at time 1 is lower than the face value V^j the agent will deliver the collateral instead of the face value. The agent who has issued the loan knows that a promise exceeding the collateral value will not be honored. In this case there will be a default and the collateral can be seized by the lender.

The collateral constraint drives the economics of collateralized borrowing and lending because it couples all decisions to buy and sell financial securities with decisions to buy and sell real assets. Thus real assets assume economically a double role: They serve as instruments of inter-temporal transfers of consumption but at the same time serve as collateral for financial promises. The liquidity of the collateralized debt market on the other hand is coupled with the stringency of collateral requirements. This interdependence creates a feedback loop between liquidity and leverage.

Geanakoplos and Zame [2010] define the concept of collateral equilibrium in the following way. A collateral equilibrium consists of an asset price p, a price q_j for each loan type, and a plan in which each agents specifies his time 0 holdings of the consumption good, the debt instrument, and of the real asset, such that (i) the utility each agent derives from his plan is maximal among the plans satisfying the budget constraint and the collateral constraint, (ii) the markets for each loan type, and for the asset clear at any time in any state.

In a collateral equilibrium agents make optimal decisions taking prices as given.

[2] Here we formulate the collateral requirement in terms of the amount of credit one can obtain per unit of collateral. Alternatively we could specify the loan contract by specifying the amount $c_j = 1/V^j$ of collateral needed per unit of face value.

Equilibrium requires that prices adjust such that all these decisions are compatible with the resource constraints of the economy at large: Long and short financial promises must be balanced, demand and supply for real assets and consumption must be balanced as well. The collateral requirement influences equilibrium by imposing constraints on feasible decisions of agents. Geanakoplos and Zame [2010] prove that for an economy with the structure described above a collateral equilibrium always exists.

In the Geanakoplos example, equilibrium can be explicitly calculated. For example in the case where there is one risk free loan only (i.e. a loan with face value $V^j = 0.2$), the asset price p, the price q of a riskfree loan, and the allocation parameter i^* are given by the solution of the following equations

$$i^* = \frac{p - 0.2}{0.8} \tag{3}$$

$$p = \frac{1 + q - i^*}{i^*} \tag{4}$$

$$q = V^j. \tag{5}$$

(0.2 is the value of the real asset in the down state, 0.8 is the difference of the values of the real asset in the up and in the down state.)

Solving these equations yields $i^* = 0.686$ and $p = 0.749$. O-agents hold 3.186 units of the asset, whereas P-agents hold 1.749 units of the consumption good. Fig. 1 shows the equilibrium allocation. Collateralized borrowing and lending increases the price of the real asset and concentrates it more among the most optimistic agents.

The equilibrium allocation is illustrated in Fig. 2. The P-agents are holding just the consumption good, or just the riskfree loan, or any combination of the two. The O-agents sell as many collateralized loans as allowed by the collateral constraint and buy as much of the real asset as possible.

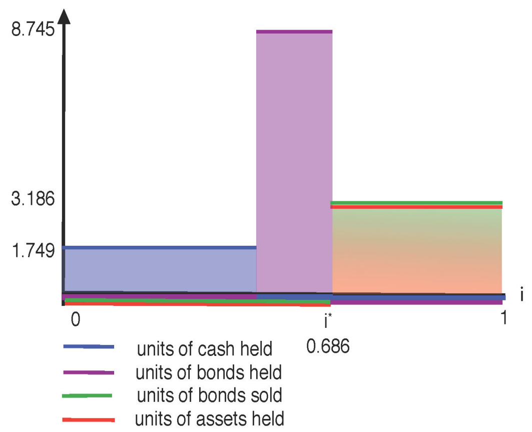

Figure 2: Equilibrium allocation with risk free borrowing and lending.

The resulting asset price (4) exceeds the asset price (2) in absence of collateralized loans by a premium that reflects its additional value as collateral. When more loans with higher face values are available for trade in addition to the risk free loan, Geanakoplos [2010] shows that in equilibrium only the risk free loan market will be active. All the other loans are priced but not traded in equilibrium. This is what Geanakoplos [2010] calls endogenous determination of leverage in equilibrium.

Unique selection of a loan is not a universal property of collateral equilibrium. Fostel and Geanakoplos [2011] show that it fails when either the asset, apart from paying a consumption good dividend, provides some utility in itself (like housing), or if three or more states are possible, rather than just the two states up and down. But debt contract selection does not depend on the infinite number of agents involved in this particular model. It also happens when the number of agents is finite.

Fig. 3 summarizes the asset pricing implications of the theory. It shows the asset price when in addition to the real asset a collateralized loan with face value V^j can be traded. The figure shows that the price of the real asset is highest when the riskless collateralized loan with maximal face value, $V^j = 0.2$, is traded. The dotted line in the figure shows the fundamental value of the real asset. It corresponds to its equilibrium price when no debt instruments are available. The picture illustrates that the function of the real asset as a collateral allows it to earn a premium over and above the fundamental value.

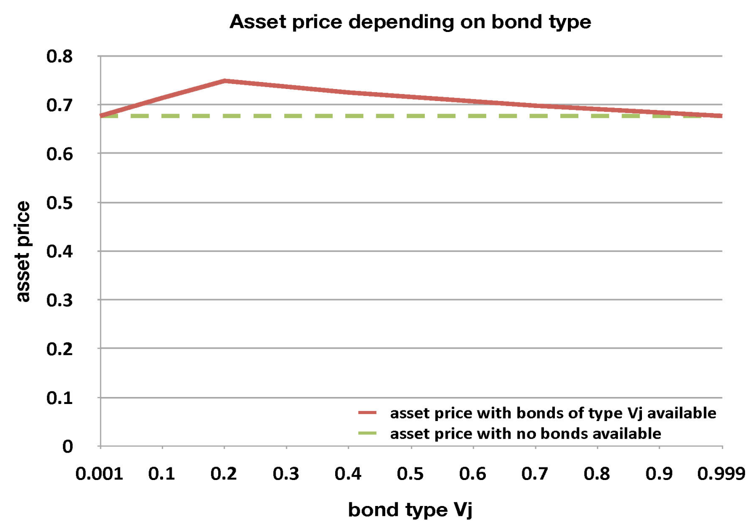

Figure 3: Asset prices and collateral value if one loan type V^j is available. Red: Asset price in dependence of V^j. Green: Asset price when no loans are available. The collateral value of the asset is the wedge between the red and the green line.

3. Leverage and Asset Pricing in Double Auctions

To conceptually relate the general equilibrium theory of leverage to economic mechanisms at work in reality and to empirically confront the question whether asset prices and leverage can be understood based on equilibrium principles we need to pin down specific institutions in which competitive market exchange takes place. In an actual trading institution it is not possible to look at prices as parameters. Prices must be choices of agents using the institution for trading. Based on the literature on experimental economics (see Smith [2008], Sunder [1995], Bossaerts [2002]) we choose the continuous double auction.

3.1. Double auctions without collateralised debt

Let us start with the case where there is no borrowing and lending but only consumption and trade in a real asset. In a double auction market trading proceeds in elementary time steps. Before the beginning of the double auction agents are endowed with a consumption good, which we will refer to in the following as cash because it is the unit of account. They are also endowed with a real asset whose liquidation value depends on the realization of a future state of the world after the auction has finished. Trades in the real asset are settled in cash. At each time step agents submit limit orders to buy or sell some part of the real asset at a certain price. The limit orders are stored in ascending order in the order book. All buy limit orders are stored in a list with the buy limit order with the highest bid price at the top of the list and all other limit orders below this highest bid price. All sell limit orders are also stored in a list with the sell limit order of the lowest ask price, at the bottom of the list and all other limit orders above this lowest ask price. When there is a buy order with a corresponding sell order that allows a utility improvement for both, a transaction is made and the orders are deleted from the book. The double auction runs for a given number of periods. Whether a trade is mutually beneficial depends on agent's preferences.

We want to use the Geanakoplos [2010] example based on general equilibrium principles as our benchmark. We therefore use the same parametrization of the economic environment for the double auction. Agents are risk neutral and hold heterogeneous believes about two states of the world. This induces reservation values which will depend on these subjective probabilities assigned to the states of the world. The bidding behavior of the agents in the double auction is modeled as in the paper by Gode and Sunder [1993]. Apart from agents knowing their reservation values, which amounts to knowing which trades are utility improving, agents behave in a random fashion.

To implement this model as a double auction we use a procedure described in Appendix A. If we simulate a double auction in accordance with these rules we are able to reproduce the well known findings of Gode and Sunder [1993]: The double auction "discovers" the competitive price and allocation predicted by general equilibrium theory.

Why does the double auction converge to the equilibrium price and allocation? The dynamics of the auction is such that in the beginning all traders are active. Trades takes place with highest probability of a match between asking offers of extreme pessimists and bidding offers of extreme optimists. Extreme pessimists and extreme optimists leave the auction first, either because they have sold all their assets, or because they have used all their cash to buy assets. In the end only traders close to the marginal agent are active since a match in that region takes place with lowest probability. This dynamics

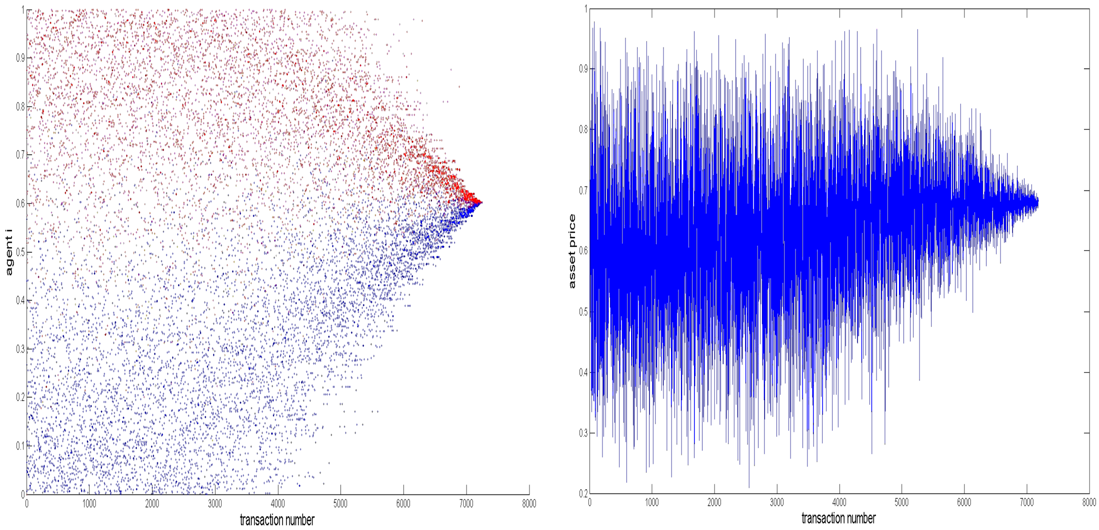

Figure 4: Evolution of active agents and transaction prices. Left: Identity of agents successfully bidding (red) and asking (blue). Right: typical evolution of transaction prices. x-axis: step number within auction.

of transacting agents is displayed in Figure 4. In the end, when both seller and buyer of assets are close to the marginal agent, prices of successful trades will be close to the reservation price of the marginal agent. This is equal to the equilibrium price. The right panel of Fig. 4 displays a typical price development. Allocation predictions of the theory are also roughly matched, compare the left and right panel in Fig. 5.

Moderate optimists and pessimists, placed near the marginal agent, hold slightly more assets or more cash than extreme optimists and extreme pessimists. The equilibrium prediction is that all optimists hold the same number of assets, and all pessimists hold the same amount of cash. A possible explanation of deviation is that the dynamics of the auction is such that all agents are active in the beginning, but only agents close to the marginal are active in the end. The extreme agents, which leave the auction earlier, have less time to reverse suboptimal early trades.

3.2. Double auctions with collateralised debt

If we extend this model to allow for collateralized loans we need to address two issues. The first one is that with financial contracts the problem is more complex because more markets are simultaneously active. This is conceptually easy but the simulation is more tedious because we have to keep track of matching and bidding across more markets simultaneously. What is more intricate is to capture how the activities in the collateralized debt market are coupled with activities in the market for the real asset that have to collateralize all loans. If asset buyers want to issue a collateralized loan using the asset as collateral, some coordination is required because they can borrow only if they have the collateral but to buy the collateral they need to borrow. If we allow for

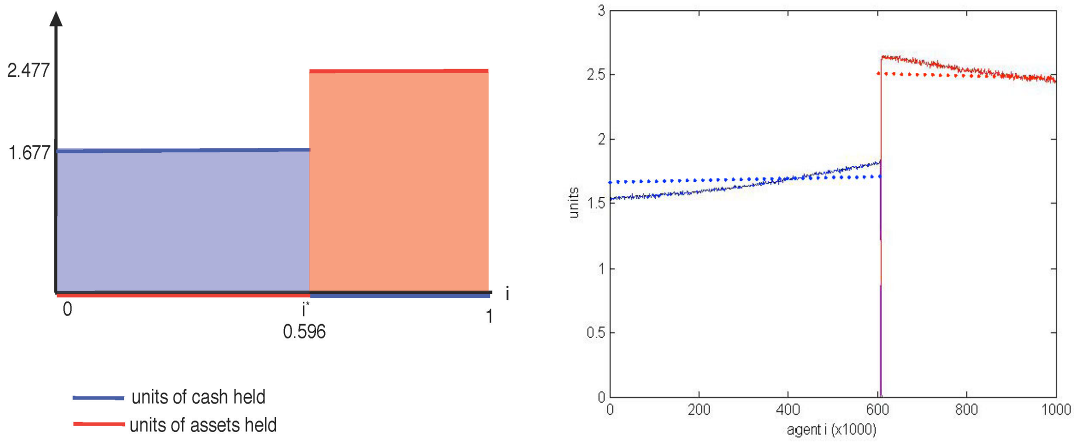

Figure 5: Equilibrium allocation vs. a typical final allocation from the double auction, when no collateralized loans are available. Left: equilibrium allocation. Right: Typical final allocation of a double auction, i.e. final wealth of 1000 agents, averaged over 30 replications. The dotted levels indicate equilibrium predictions for the amount of consumption good (blue) held by P-agents and assets (red) held by O-agents. x-axis: agent's i, y-axis: units of cash (blue) and of assets (red).

instance that an agent takes a collateralized loan and buys a real asset from somebody else using this real asset as collateral, we would need a special coordinating institution for this kind of exchange.

Such coordination is not required for bilateral exchanges. Our approach is that we bring the collateral problem back into the realms of bilateral exchange by considering a market where agents who sell the real asset simultaneously provides credit to the buyer who wants to acquire the real asset. In the general equilibrium model with one good the seller of the asset at the same time provides credit.

We now extend the double auction to allow for collateralized borrowing and lending. The details of the double auction are specified in Appendix B. It takes place in two markets. The first market is a market where the real asset is traded for cash. It functions exactly like the market described above in the previous subsection. The second market is a market for the exchange of collateralized loan financed real assets. Since the double auction needs bilateral matches we have to impose the restriction that agents do not give collateralized loans to agents who want to buy the asset from somebody else. Collateralised loans are given to agents to whom the asset is sold to enable them to finance the purchase of this asset.

In our simulation we impose on each agent a sequential use of these two markets. As long as resource constraints allow, agents who want to buy the real asset buy for cash. Agents who still want to buy the real asset after exhaustion of their cash, issue loans using the assets to be acquired as collateral (plus other assets they hold but did not yet pledge as collateral). Note that in this second phase asset purchases are financed purely by collateralized loans, without any cash.

The market for loan financed assets involves more complex bids because not only the price for the real asset has to be specified but also the price and the type of the loan. As in the market for real assets for cash the bidding behavior of agents is driven by potential utility improvements. Utility improvements for asset purchases financed fully or partly by sales of collateralized loans (i.e. by loans) take into account both the price of the asset and of the loan.

3.3. Results from an Agent Based Model

The results of the agent based model of the double auction can be summarized by two main points:

1. The double auction converges both to allocations and to prices close to the equilibrium predictions.
2. Leverage emerges endogenously through contract type selection in the market as predicted by the theory of collateral equilibrium.

Lets discuss these two results in turn.

Only one loan type is available A double auction with just the riskless loan $V^j = 0.2$ available converges both in prices and in allocation. The final allocation reached in the double auction has parameter values which are quite close to the equilibrium predictions, but the sharp allocation profiles of P-agents and O-agents are somewhat smoothened. These residuals are due to a an inefficiency of the double auction: towards the end the more optimistic P-agents hold some residual assets, and the more pessimistic of the O-agents hold some residual cash. An explanation of this residual inefficiency is given in Section 3.4. A graphical display which shows the allocation of the double auction in comparison with the theoretical allocation is given in Fig. 6.

The prices and allocations are summarized in Table 1

Endogenous Leverage We now run the double auction with two types of collateralized loans available, $V^1 = 0.2$ and $V^2 = 0.5$. Figure 7 shows a clear preference for the loan type $V^j = 0.2$. The left panel displays the final wealth of 1000 agents after a typical auction with loans $V^1 = 0.2$ and $V^2 = 0.5$. Almost all loans for financing assets are of type $V^1 = 0.2$. The right panel shows how the financing of asset purchases changes in the course of the auction. In the beginning asset purchases are finance by cash, towards the end they are financed primarily by $V^1 = 0.2$ loans. Asset purchases financed by $V^2 = 0.5$ loans hardly ever occur.

What is the reason for loan type selection in the double auction? In collateral equilibrium prices adjust so that lenders prefer loans with lower default risk, while borrowers are indifferent between taking a risky or a riskless loan. In the double auction the more risky loans are attractive to lenders only at prices which are so high that they are rarely matched by offers from borrowers.

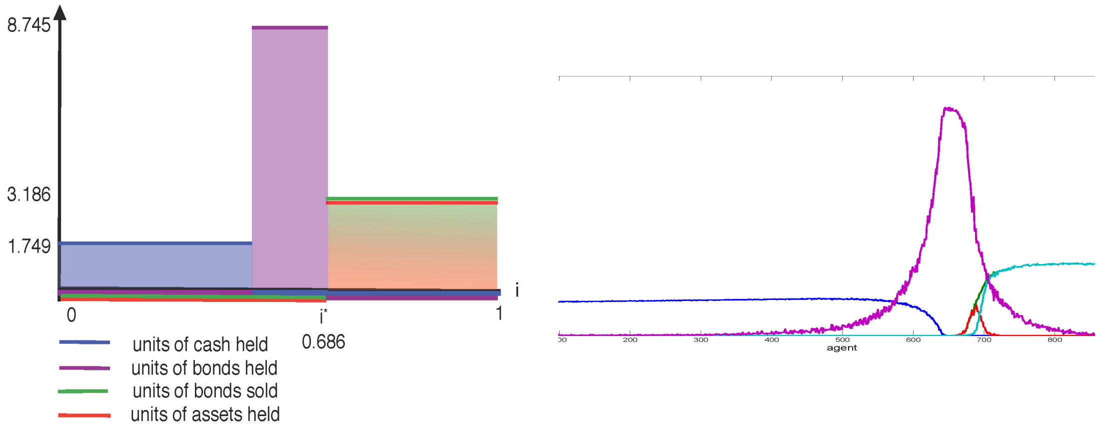

Figure 6: Equilibrium allocation and a typical final allocation from the double auction, when one risk free loan type (V^j=0.2) is available. Left: equilibrium allocation. Right: typical final allocation from the double auction, i.e. final wealth of 1000 agents for an auction with one loan type $V^j = 0.2$, averaged over 30 replications. x-axis: agent's i, y-axis: units of cash (blue), of assets (black, partly covered by green) of free assets (red), of loans bought (purple) and of loan sold (green).

prices		allocation		
asset price p	loan price q	marg. agent i^*	cash held by average P	assets held by average O
equilibrium				
0.749	0.200	0.686	1.749	3.186
end of double auction				
0.745	0.198	0.686	1.616	3.145
(0.006)	(0.002)	(0.003)	(0.002)	(0.021)

Table 1: Equilibrium state vs. final state of the double auction in a market with one collateralized loan type $V^j = 0.2$ available. The last two lines show averages and standard deviations (in parentheses) over the final transaction prices resp. final allocation parameters of 30 auctions.

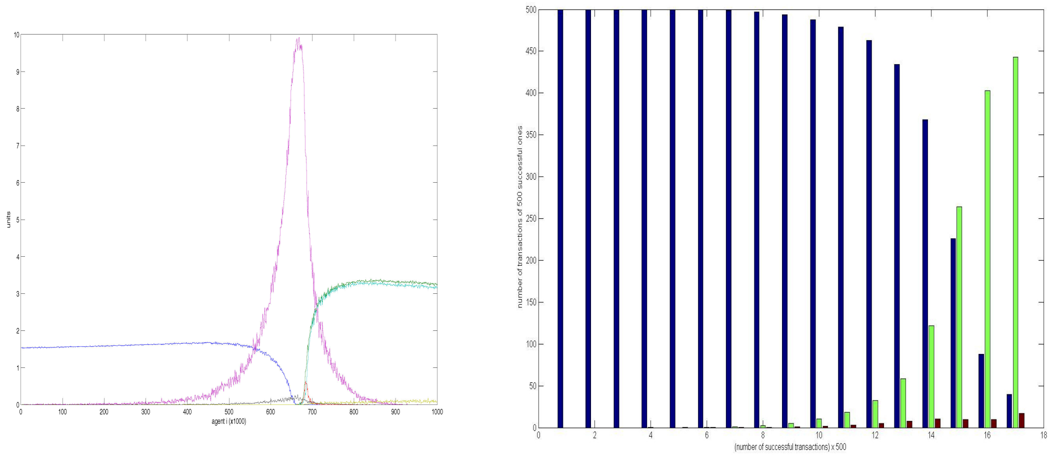

Figure 7: Selection between a risk free ($V^1 = 0.2$) and a risky ($V^2 = 0.5$) loan. Left panel: Final wealth of 1000 agents averaged over 30 replications. x-axis: agent's i, y-axis: units of cash (blue), of free assets (red), of loan $V^1 = 0.2$ bought (purple) and of loan $V^1 = 0.2$ sold (green); the respective values for loan $V^2 = 0.5$ can be seen close to the x-axis. Right panel: Traded loan types per 500 successful transactions: no loan (blue), loan $V^1 = 0.2$ (green), loan $V^2 = 0.5$ (brown) with 1000 agents.

3.4. Why and how does the double auction converge?

The intuitive reason for price convergence is apparent from Figure 4. The dynamics of the auction is such that in the beginning all traders are active. Trade takes place with highest probability of a match between asking offers of extreme pessimists and bidding offers of extreme optimists. Extreme pessimists and extreme optimists leave the auction first because they first made all the deals they wanted to make. The extreme pessimists are left only with cash, the extreme optimists are left only with assets all pledged as collateral for the loans they took. In the end only traders close to the marginal agent are active since a match meeting their reservation prices are of low probability. The range of prices acceptable for agents with i in a small interval is small. This effect yields price convergence. Convergence is to the reservation price of the marginal agent. For riskfree loans, this is equal to the equilibrium price by eq. (4).

Approximate allocation convergence. We have seen that allocation convergence approximates quite well the allocations of the theoretical equilibrium. The explanation of this convergence behavior comes from two crucial features built into the mechanism of the double auction:

1. The asset always flows in the direction from the lower i to the higher i.
2. The closer the i of asker and bidder around marginal agent i^*, the smaller is the likelihood of a successful match.

The first observation is obvious because a more pessimistic agent has a lower marginal valuation for the real asset, and thus a lower reservation price. His bid offer can never be matched by an ask offer of a more optimistic agent. A match is possible only between an ask offer of a more pessimistic agent and a bid offer of a less optimistic agent. Therefore, in every successful trade the asset is sold by the pessimist and bought by the optimist.

To evaluate quantitatively the second fact, note that the conditional probability of a match between an asker $i^a = i - \epsilon$ and a bidder $i^b = i + \epsilon$ given a fixed loan price q, where both i^a and i^b are within an ϵ interval of i is given as follows. Define $m^i := \mathbb{E}^i(A)/\mathbb{E}^i(V^j)$, then this probability is given by

$$Prob(i, \epsilon, q) = \left(\frac{q\,(m^{i^b} - m^{i^a})}{q\,m^{i^b} - 0.2} \right) \left(\frac{q\,(m^{i^b} - m^{i^a})}{1 - q\,m^{i^a}} \right) \tag{6}$$

The first expression gives the probability of a bidder to draw a successful matching price for the real asset and the other factor gives this probability for an asker, and both are multiplied because of independence of the two agents.[3] See Fig. 8 for the loan type $j = 0.5$ and $i = 0.8$.

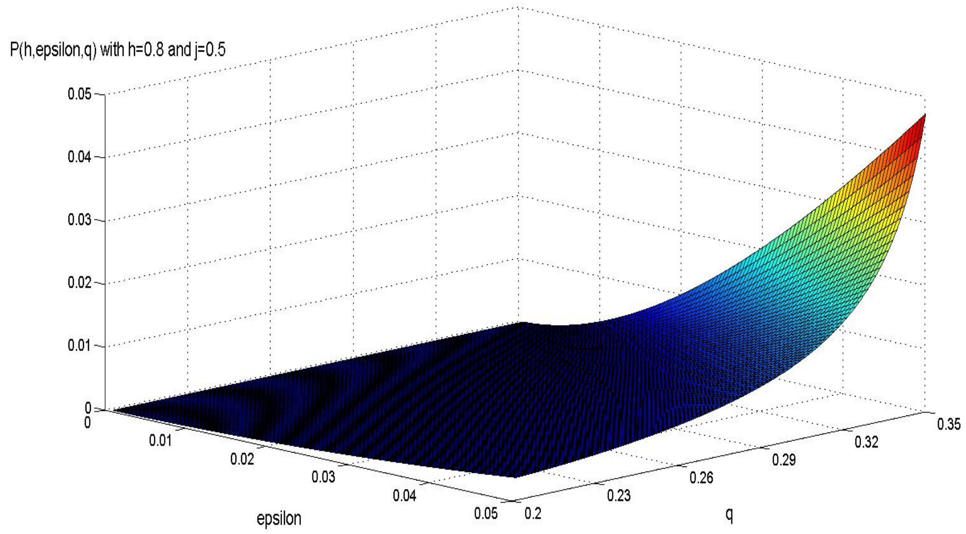

Figure 8: Conditional probability (z-axis) of a match between an asker $i^a = 0.8 - \epsilon$ and a bidder $i^b = 0.8 + \epsilon$ (x-axis: ϵ) given a fixed loan price q (y-axis)

The low probability of a match for agents close to $i = 0.8$ (the marginal agent for buying/selling an asset financed by the loan $j = 0.5$) explains why the risky loan is hardly

[3] For an asker achieving a utility gain, it is necessary that $p > q\,m^{i^b}$ holds, and for a bidder achieving a utility gain, it is necessary that $p < q\,m^{i^a}$ holds: this explains the numerator; the denominators represent the complete interval a bidder resp. an asker draws p from in order to achieve a utility gain.

ever traded. At the same time this striking decrease in matching probability is the very reason for the price convergence of the double auction.

4. Conclusions

This paper analyzes double auctions for collateralized debt. The aim of the analysis is to provide a deeper understanding of equilibrium theories of leverage and asset pricing. We hope that this deeper understanding is provided by a complementary perspective that looks more closely at processes and mechanisms that can potentially bring about equilibrium allocations and prices. This perspective at the same time suggest a framework by which the equilibrium theory of leverage and asset pricing can be investigated experimentally.

In our model agents with limited cognitive capacities and without information about historically traded prices or the status of the order book, exchange real assets and collateralized loans in a dynamic process by submitting limit buy and selling orders. We have demonstrated that this process model of market exchange converges to a state where no further trade takes place. This can be seen as a stable final state of the double auction. The final states of the double auctions are close to the equilibrium that would be predicted if one analyzed market exchange from the viewpoint of an abstract general equilibrium model. In particular some important features of the basic logic of competitive collateralized lending reemerge in the double auction: First, the real asset is traded at a price above its fundamental value to ensure that agents collectively stay within the constraints imposed by the need to secure loans by collateral. The price wedge or collateral value is highest for the riskfree loan of maximal face value (i.e. offering maximal leverage). Second, the market prices all collateralized loan types, but not all collateralized loans are traded in the double auction. In the case of the example discussed in our analysis the equilibrium contract selection will even be unique.

We believe that these results are important for two main reasons: They help us understand the circumstances when abstract equilibrium analysis might be appropriate to analyze potential outcomes of market exchange. While the double auction for real assets financed by collateralised loans is a purely fictitious institution, it sheds light on the key structural features of markets that might lead to equilibrium prices and allocations. Our analysis of convergence of the double auction showed that the institution works because it allows an unidirectional flow of the real asset from agents who value the asset low to those who value it high and that the probability of successful matches decreases as the auction goes on. This provides a useful benchmark against which we can study other abstract or real world institutions. The second reason is that the double auction provides an ideal institution to bring the analysis of general equilibrium models into the realms of an experimental investigation. We hope that our work provides a solid base to proceed to the experimental study of the competitive equilibrium theory of collaterlized lending.

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A. Double auction mechanism without collateralized loans

Organization The double auction is organized in a random sequence of order generation by agents. An order defines an amount of asset to be sold or bought, and a corresponding asset price. This amount is a part of an agent's initial asset endowment. Tradeable asset quantities are discretized to some (small) integral fraction δ_d of a unit: we take $\delta_d = 0.1$. At each step, the buying and the selling orders generated by the agent are matched with the already generated orders of other agents, and if an ask or bid hits the quote on the other market side an exchange of some part of the real asset for cash takes place.

Agents There is a finite number N_a of agents $i \in [0, 1]$, equidistantly distributed. The identity i of each agent describes the probability assigned by this agent that the real asset will pay 1 tomorrow. Thus $(1 - i)$ is his probability that the asset will only pay 0.2. The higher is are the agents who are more optimistic. The reservation prices of the agents are the expected payoff $\mathbb{E}^i(A)$ of the real asset with respect to these subjective probabilities of agent i.

Bidding In each trading period agents are requested in a random order without repetitions to submit limit orders, one to buy and one to sell the amount δ_d of the real asset. Sellers make a random price offer p^i from $U[\mathbb{E}^i(A), 1]$, called the ask price. This implies that the agents make only utility improving offers. If a selling agent does hold at least δ_d unit of the asset, the offer is placed. Otherwise, no offer is placed. This ensures that the no short selling constraint on the real asset is met. If an agent is buying he names a random price p^i from $U[0.2, \mathbb{E}^i(A)]$, called the bid price. Again this implies that the agents make only offers which would improve their utility. If the agent has not enough cash for buying δ_d units of the asset at the drawn price, he may place a buying offer with a quantity $\delta < \delta_d$. No offer is placed in case the agent ran out of cash. This ensures that the budget constraint of the agent is met.

Matching Whenever an agent submits his offers there is a search for a match in the list of best bid or ask offers in the order book in the current trading step t. Whether a match for the ask or for the bid offer is searched for at first, is decided randomly. A match occurs when the bid ask spread is non-negative. When a match has been found a transaction of the δ_d (or the minimum quantity of the ask and the bid offer) of the real asset against cash takes place at the price of the offer in the match that came in first. After a match all offers are deleted from the book. Matches at step t occur as long as the best open ask offer and the best open bid offer allow for a non-negative spread. If all agents have submitted an offer but no match has been made the auction proceeds to the next step. The next auction step $t + 1$ starts with an empty order book, and a new loop of bidding, matching and exchange is started.

Termination The double auction stops when an upper limit of trading periods with no matches that result in a trade has been observed.

B. Double auction mechanism with collateralized loans

Organization The double auction for cash and loan financed real assets is organized in a random sequence of order generation by agents. An order defines an amount δ_d (which is a part of an agent's initial asset endowment, see Appendix A) of assets to be sold or bought. At each step buying and selling orders are matched and if an ask or bid hits the quote on the other market side an exchange of the specified amount of loan financed real assets takes place.

Agents There is a finite number N_a of agents $i \in [0, 1]$, equidistantly distributed. The identity i of each agent describes the probability assigned by this agent that the real asset will pay 1 tomorrow. The higher is are the agents who are more optimistic.

Bidding In each trading period agents are requested in a random order without repetitions to submit a limit order to buy or a limit order to sell the amount δ_d of a cash financed real asset, in exactly the same manner for both the asker and the bidder side as described in subsection 3.1 for the auction without loans. When a bidding agent has no more cash, he may place an order for a loan financed asset. An asking agent may always place an order of selling a loan financed asset as long as he disposes of a positive amount of free, i.e. not collaterized asset. An offer of agent i to sell δ_d (or $0 < \delta_d$ if the agent has only δ free assets) of j-loan financed asset is a tuple (p^i, q^i, j, δ_d) (or (p^i, q^i, j, δ)). This offer can be placed if the agent has real assets that are not yet pledged as collateral and he has no more cash. The loan type j is chosen randomly from the set of all J available pairs of face value and collateral $(V^j, 1)$. The ask price p^i is randomly drawn from $U[\mathbb{E}^i(A), 1]$. Simultaneously a loan price q^i is randomly drawn from $U[0, V^j]$. Both p^i and q^i determine the amount z^i of loans required. Since the buyer of the real asset has no cash anymore the asset has to be fully financed by a loan and thus $z^i = \delta_d \cdot p^i/q^i$. An agent keeps drawing prices until he finds a price resulting in an utility improvement. An offer to buy δ_d (or $\delta < \delta_d$ if the agent has not enough free assets as collateral for financing δ_d assets) of j-loan financed asset is characterized by a tuple (p^i, q^i, j, δ_d) (or (p^i, q^i, j, δ)). The bid price p^i is randomly drawn from $U[0.2, \mathbb{E}^i(A)]$. Then q^i is randomly drawn from $U[0, V^j]$. Both p^i and q^i determine the amount z^i of loans required. Since cash is exhausted the asset is fully financed by a loan and thus $z^i = \delta_d \cdot p^i/q^i$. For that purpose, the agent keeps drawing random prices p^i and q^i until a utility improvement has been achieved, or until a maximum number of draws has been reached. To make sure that the collateral constraint is not violated the offer is eventually adjusted by reducing the asset amount and as a consequence the loan amount.

Matching The order book ranks bid and ask offers for loan financed assets in the bid and ask list. Ask orders that are matched by bids which lead to a non-negative bid ask spread for both the real asset and the loan are crossed. When a match has been found a transaction of the minimum amount (of the matching offers) of real asset against a loan takes place at the price of the offer in the match that came in first. The quantity of loans of either of the two offers may eventually be adjusted according to the adjustment logic described above. After a match all offers are deleted from the book. Matches at step t occur as long as the best open ask offer and the best open bid offer allow for a non-negative spread. If all agents have submitted an offer but no match has been made the auction proceeds to the next step. The next auction step $t + 1$ starts with an empty order book, and a new loop of bidding, matching and exchange is started.

Termination The double auction stops when an upper limit of trading periods with no matches has been observed.

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