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ESTIMATION OF THE TERM STRUCTURE OF INTEREST RATES

A PARAMETRIC APPROACH

Alois Geyer and Richard Mader

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Editorial

In this paper Alois GEYER, Professor at the University of Vienna, and Richard MADER, Head of the President´s Secretariat, Oesterreichische Nationalbank, investigate the information content of interest rates. Interest rate curves may be used for inflation and output forecasts, they may give useful indications about the differences in regional monetary stance and contain information about market expectations of future changes in interest rates. For comparisons between countries, however, it is important to use a common technique to estimate the term structure of interest rates. This paper presents the results of using parametric estimation models for the term structures of Austria, Germany, UK, USA, and Japan over the period 1993 to 1998. The frequently used Nelson/Siegel (1987) model produced reliable and reasonable estimation results over most of the six-year sample period and for all countries. However, in certain periods the fit was not completely satisfactory. For that reason, the number of parameters included was increased along the lines of the approach proposed by Svensson (1994). Experimentation with the Svensson model showed that the fit can be improved only at the expense of introducing problems of overparameterization. In addition it was found that lower levels of the goodness-of-fit need not be due to choosing the ‘wrong’ model but may be induced by a higher dispersion in market prices. This typically occurs when the term structure is flat or twisting. The Nelson/Siegel model was finally chosen because its performance generally was as good as the Svensson model, its estimation can be carried out far more quickly and it is less sensitive to outliers.

May 21, 1999

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A.Geyer and R.Mader

Abstract

Readily available information about the current term structure of interest rates, its level and recent trends in important countries has become a standard tool of monetary policy analysis. Interest rate curves can be used for inflation and output forecasts, they may give useful indications about the differences in regional monetary stance and contain information about market expectations of future changes in interest rates. This information can facilitate the implementation of monetary policy, for example by judging the timing of the central bank's market operations. For comparative purposes it is important to use a common technique to estimate the term structure for all countries. This report presents the results of using parametric estimating models of the term structure for Austria, Germany, UK, USA and Japan over the period 1993 to 1998.

Keywords: term structure of interest rates, estimation, econometric models

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1 Introduction

In the past, financial indicators played a subordinate role in formulating and implementing monetary policy in Austria. Among others two reasons were of primary importance. First of all, Austria followed a fixed exchange rate target linking the schilling to the Deutschmark. This strategy confined Oesterreichische Nationalbank (OeNB) to the interest rate policy of the anchor country rendering indicators to evaluate the monetary stance largely superfluous. Secondly, Austrian financial markets until recently lacked the degree of development which financial indicators require to contain sufficient forecasting power. It was not until the beginning of the nineties that deregulation and liberalization started to accelerate, thereby increasing the integration of national financial markets.

Membership in the European Central Bank System (ECBS), however, resulted in a profound reorientation of Oesterreichische Nationalbank's research focus. In evaluating the monetary policy stance and formulating its strategy the European Central Bank relies, inter alia, on a basket of financial indicators. In the first instance membership in the ECBS commits the OeNB – just as the other national central banks – to the regular contribution of, inter alia, reliable financial indicators required for the correct judgment of the national economic stance. But above all, the availability of financial and economic indicators for the European Monetary Union (EMU) as a whole enables the assessment of EMU's monetary policy and thus substantially determines the Bank's role in international monetary policy discussions. Therefore, the Bank's economic research focus is directed towards the EMU area.

Among financial indicators, the term structure of interest rates assumes a prominent place and provides a valuable source of information for policymakers. Interest rate curves can be used for inflation and output forecasts and often turn out to be good benchmarks for macroeconomic forecasts (see, e.g., Bernard/Gerlach 1996, Canova/De Nicolo 1997, Davis/Fagan 1995a, Estrella/Hardouvelis1991, Fama 1990, Mishkin 1991, Schich 1996, Smets/Tsatsaronis 1997). Moreover, they may give useful indications about the differences in regional monetary stance and whether these variations are expected to persist. Finally, the curves contain information about market expectations of future changes in interest rates (see, e.g., Davis/Fagan

1995b, Dziwura/Green 1996, Estrella/Mishkin1995). This information can facilitate the implementation of monetary policy, for example by judging the timing of the Bank's market operations.

The correct representation of the term structure of interest rates is the basis for further research in this area. Since spot and forward rates cannot be observed directly in the market – except in the case of a liquid strips market – estimation techniques have to be used to extract the term structure of interest rates. Apart from the USA and UK, where a long tradition of term structure estimation exists, most EMU central banks started to develop their own interest rate models in the nineties. According to a survey conducted by the Bank for International Settlements, most central banks rely on parametric models for the approximation of interest rates used for macroeconomic policy analysis. The results to be described in this paper are based on the Nelson/Siegel (1987) model. Over most of the six-year sample period and for all countries considered the model produced reliable and reasonable estimation results. Results of comparisons with the Svensson (1994) model are reported, too.

Section 2 of the paper reviews the basics of bond price calculations, describes the fundamental features of the Nelson/Siegel and Svensson approaches, and explains basic principles of model estimation. Section 3 describes the data and country-specific adjustments as well as the details of the estimation procedure. It further contains econometric results, describes some interesting cases and finally presents estimates of the term structure. Section 4 concludes the paper.

2 Basic principles of modeling the term structure

2.1 Bond pricing and definitions of interest rates

Before entering into the technical discussion of estimating the term structure, a brief review of the terminology used as well as the concept of bond pricing is presented.

A bond is the obligation of a debtor to provide a stream of future cash-flows – the coupon and redemption payments – at predetermined dates in the future. These conventional straight bonds represent the major instrument in the government bond markets for which the term structure is estimated. However, these markets also contain bonds with special features such as call or put options or multiple redemption dates. These instruments' characteristics are reflected in coupon effects requiring a special treatment (see Section 3.1).

The valuation of conventional government bonds is straightforward. Since all future payments are known, the bond price is simply the sum of discounted cash-flows – their present value. First, consider a zero-coupon bond. This is a bond with only one future payment C that occurs m periods in the future (e.g. at date $t+m$ if t is the present date). The price P of this bond and its cash-flow are related by the equation

$$P = C\exp(-mr).$$

The interest rate r used for discounting the payment C is called the spot-rate or zero-bond rate for a maturity of m periods. Continuous compounding using exponential rates is frequently used instead of arithmetic rates as in

$$P = C(1 + r')^{-m}$$

in order to simplify subsequent calculations.

Now consider a bond with a series of cash-flows $C_1, C_2, ..., C_J$ that are paid $m_1, m_2, ..., m_J$ time periods in the future. The final payment C_J is assumed to include the redemption payment. The price of such a bond can be obtained by discounting each cash-flow using the spot-rate r_j for the corresponding time-to-payment (or payment interval) m_j:

$$P = \sum_{j=1}^{J} C_j \exp(-m_j r_j) \,.$$

Note: The payment interval m_j is sometimes called maturity although the term maturity only refers to the remaining lifetime of the bond, which is m_J.

r_j as a function of maturities m_j defines the term structure of interest rates, or zero-coupon yield curve.

In principle, the spot-rates r_j can be obtained from a set of zero-bonds with maturities that equal the payment intervals of the coupon-bond. The practical difficulties with obtaining spot-rates from observed bond prices are described below.

Another frequently used relation between cash-flows and bond prices is based on the yield-to-maturity (or redemption yield). In this case a single interest rate y is used for discounting all future cash-flows:

$$P = \sum_{j=1}^{J} C_j \exp(-m_j y) \,.$$

The yield-to-maturity (or simply the yield) can be interpreted as an average interest rate assuming that all cash-flows are reinvested at the same rate y during the lifetime of the bond. This is equivalent to assuming a flat term structure with identical spot-rates for each maturity. If spot-rates increase with maturity, however, this will be underestimated by the yield-to-maturity. Conversely, yields overestimate a downward sloping spot-rate curve. Note that the term ‘yield curve’ is frequently used to designate zero-coupon yield curves (i.e. spot-rate curves) but must not be confused with a yield curve based on redemption yields.

The present term structure of spot-rates has implications for future interest rates, too. The following presentation of these so-called implied forward rates is made in terms of one-period rates but is not confined to that time interval.

First consider the simple case of a spot-rate for a maturity of two years. An investment worth V_t in period t yields a value of

$$V_{t+2} = V_t \exp(2r_{t,2})$$

in period $t+2$. The spot-rate $r_{t,2}$ can be broken down into two components: the one-year spot-rate that prevails now ($r_{t,1}$) and the (implied) forward-rate $f_{t,2}$ – that is the one-year spot-rate that has to prevail in one year (during the second year from now) in order to yield the same value of the investment in period $t+2$:

$$\exp(2r_{t,2}) = \exp(r_{t,1})\exp(f_{t,2}) .$$

The forward-rate is also indexed by t in order to stress its dependence on the current time period. Note that the forward-rate is not necessarily the one-year spot-rate that will prevail in one year. According to the (unbiased) expectations theory of the term structure, however, the forward-rate is equal to the expected value of the future spot-rate. Therefore forward-rates are particularly important from a monetary policy perspective. However, the expectations hypothesis does not account for risk-, term- or liquidity premia, which strongly affect implied forward rates. The existence of such premia cannot be rejected empirically. Therefore, forward-rates should be interpreted cautiously.

In general, the following relation holds:

$$\exp(mr_{t,m}) = \exp((m-1)r_{t,m-1})\exp(f_{t,m}) ,$$

or simply

$$mr_{t,m} = (m-1)r_{t,m-1} + f_{t,m} .$$

The forward-rate for maturity m can be interpreted as a marginal interest rate. As an example, suppose that the four- and five-year spot-rates are currently 3.5% and 4%, respectively. The forward rate is the one-year spot-rate that is implied to prevail in four years from now (during the fifth year):

$$f_{t,5} = 5 \cdot 4\% - 4 \cdot 3.5\% = 6\%.$$

In case of a downward sloping term structure (e.g. assuming the four- and five-year spot-rates are 4% and 3.5%, respectively) the forward-rate is 1.5%. This shows that the forward-rate curve is above (below) the spot-rate curve if the spot-rates are upward (downward) sloping.

In order to estimate the term structure of interest rates, one needs observed bond prices and the terms of each bond which determine the timing and size of future cash-flows. In bond markets prices are quoted as clean prices. If a transaction takes place, the seller also receives accrued interest for holding the bond over the period since the last coupon payment. The price including accrued interest is called the dirty price which represents the market value of a bond.

In principle, accrued interest A_t is calculated as a fraction of the coupon C:

$$A_t = a_t C$$
$$a_t = (1 - n_t / 365),$$

where n_t is the number of days since the last coupon payment. The computation of the fraction a_t depends on market conventions (details are explained in Section 3.1).

2.2 Parametric models of the term structure

The term structure is defined as a continuous function of maturity. This allows for assigning spot-rates to any maturity in order to price a payment at any date in the future. However, the term structure cannot be directly observed using bond price data. In practice, two problems must be solved in order to estimate the term structure. First, only a finite number of bonds is traded at any one point in time and their maturities provide only a discrete set of points of the term structure. Second, the majority of bonds are coupon bonds which do not allow for a direct calculation of a unique set of spot-rates using the equation

$$P = \sum_{j=1}^{J} C_j \exp(-m_j r_j) .$$

The term structure can be estimated, however, from observed coupon-bonds by assuming a parametric function relating spot-rates and time-to-maturity. Nelson and Siegel (1987) have suggested a flexible function for the forward-rate that can be used to obtain a corresponding function for the spot-rate. The forward-rate for maturity m is given by

$$f(m,\beta) \; = \beta_0 + \beta_1 \exp\left(\frac{-m}{\tau_1}\right) + \beta_2 \frac{m}{\tau_1} \exp\left(\frac{-m}{\tau_1}\right) .$$

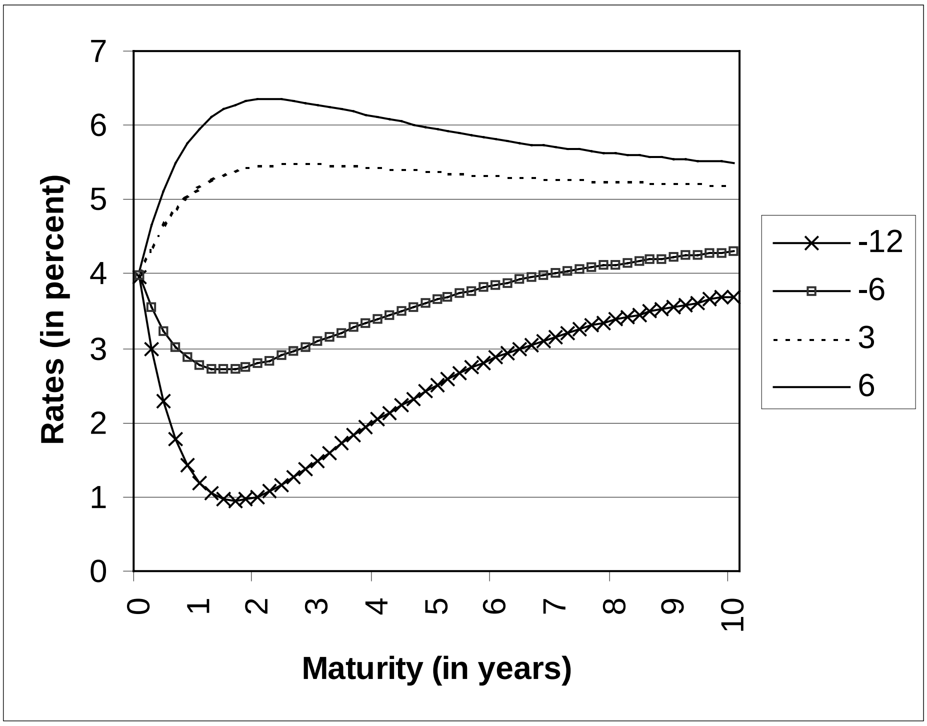
β = $\{\beta_0, \beta_1, \beta_2, \tau_1\}$ are the parameters which determine the shape of the forward-rate curve and need to be estimated from observed prices. The notation $f(m,\beta)$ is used to stress the functional dependence of the forward-rate on maturities and on parameters. The corresponding function for spot-rates is given by

$$r(m,\beta) = \beta_0 + \beta_1 \left(\frac{1-\exp(-m/\tau_1)}{(-m/\tau_1)}\right) + \beta_2 \left(\frac{1-\exp(-m/\tau_1)}{(-m/\tau_1)} - \exp(-m/\tau_1)\right) .$$

The parameters β_0 and β_1 can be interpreted as follows: β_0 is the limit of the spot-rate as the maturity tends to infinity. In other words, it is a long-term interest rate (in the limit forward- and spot-rates coincide). If the maturity tends to zero the spot-rate converges to the sum $\beta_0 + \beta_1$. This can be interpreted as an instantaneous interest rate. This further implies that $(-\beta_1)$ can be interpreted as the spread between long- and short-term interest rates. The parameters β_2 and τ_1 determine the shape of the curve, there is no direct economic interpretation for them.

The possible shapes of the Nelson/Siegel spot-rate curves are monotonically rising or decreasing, U-shaped (regular and inverted) and S-shaped. Figure 1 gives an impression of the flexibility of the model. The chosen parameters are $\beta_0 = 5$, $\beta_1 = -1$, $\tau_1 = 1$, and β_2 is set equal to –12, –6, 3, and 6, respectively.

Figure 1: Spot-rates implied by the Nelson/Siegel model.



Svensson (1994) has proposed an extension of the Nelson/Siegel model that allows for even more flexibility. Whereas the Nelson/Siegel model can have only one hump (a local maximum or minimum), the Svensson extension allows for two humps. For that purpose two additional parameters are required and the spot-rate function is given by:

$$r(m,\beta) = \beta_0 + \beta_1\left(\frac{1-\exp(-m/\tau_1)}{(-m/\tau_1)}\right) + \beta_2\left(\frac{1-\exp(-m/\tau_1)}{(-m/\tau_1)} - \exp(-m/\tau_1)\right) +$$
$$+\beta_3\left(\frac{1-\exp(-m/\tau_2)}{(-m/\tau_2)} - \exp(-m/\tau_2)\right).$$

2.3 Principles of model estimation

The key to understanding the estimation process is that for a given set of parameters, cash-flows and payment intervals (C_j and m_j), the Nelson/Siegel (or Svensson) model (in short, the spot-rate model) implies a theoretical price $\hat{P}$

$$\hat{P} = \sum_{j=1}^{J} C_j \exp(-m_j r(m_j,\beta)),$$

where $r(m_j,\beta)$ is the spot-rate for maturities m_j (or cash-flows due m_j periods in the future) implied by the parameters of the model. This price needs not conform to the observed price of the bond with the given cash-flows. Obviously, the model parameters can be modified in such a way that observed and theoretical prices coincide. However, there are several bonds traded on any one day. To find a set of parameters that matches the observed and theoretical price of each individual bond is not possible, except for very special circumstances. Therefore, the goal is to find parameter values that provide a compromise. Based on the principle of least-squares, the parameters can be chosen such that the sum of squared differences between observed and theoretical prices for all observed bonds is minimized:

$$\sum_{i=1}^{N} (P_i - \hat{P}_i)^2 \rightarrow \min,$$

where P_i is the i-th out of N bonds observed on a particular day.

Estimating parameters based on price errors is not generally preferred, however. Policy makers and economic discussions typically focus on interest rates rather than prices. Therefore, it seems natural that the parameters of the spot-rate model are estimated on the basis of interest rate errors. For that purpose the yield-to-maturity is used. The yield y of a single bond can be easily calculated from

$$P = \sum_{j=1}^{J} C_j \exp(-m_j y)$$

using an iterative search procedure (e.g. Newton's method). Since the term structure model implies a theoretical price $\hat{P}$, the corresponding theoretical yield $\hat{y}$ can be obtained from

$$\hat{P} = \sum_{j=1}^{J} C_j \exp(-m_j \hat{y}) .$$

Note that $\hat{P}$ depends on the parameters of the spot-rate curve. In other words, during the parameter estimation process a theoretical yield $\hat{y}$ is determined for each bond and each set of parameters.

Given observed yields y_i and theoretical yields $\hat{y}_i$ for a set of N bonds, the parameters of the term structure model can be estimated by minimizing the sum of squared yield errors:

$$\sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \to \min .$$

Estimation based on fitting yields has a further advantage. Minimizing price errors may lead to large yield errors for short maturities. For an explanation of this effect, note that prices with short maturities are relatively insensitive to the precise level of interest rates: $\exp(-mr)$ is close to one if m is close to zero, irrespective of r. Therefore, rather large yield errors would not be adequately reflected in price errors. According to the least-squares principle large errors are weighted more strongly than small errors. Thus, minimizing price errors would attribute too little weight to short maturities. Therefore, if the focus is on interest rates, the appropriate fitting criterion should be based on yield errors.

3 The term structure of interest rates in Austria, Germany, UK, USA and Japan

3.1 Data description and accrued interest calculation

The estimation of the parameters of term structure models proceeds in two steps – first, data collection and preparation, and second, estimation of parameters.

Clean prices for each trading day in the period January 1993 to July 1998 were obtained from Datastream. Only government bonds, which do not contain any credit risk, were selected. For each bond a set of descriptive information is provided (issuing date, redemption date, coupon rate, coupon dates, etc.). This information is used to exclude certain bonds, either because they are callable or convertible. The prices of such bonds reflect the embedded options and are therefore not comparable to prices of standard bonds. In addition, bonds with more than one redemption payment and bonds maturing in less than three months are excluded. Prices of such short-running bonds are considered unreliable because of their low liquidity. Moreover, tax-effects were not taken into account because of the difficulties associated with defining a valid tax-rate (see McCulloch 1975 and Schaefer 1982).

For each of the remaining bonds cash-flows and payment intervals are calculated. Dirty prices are obtained by adding accrued interest to clean prices given the specifications of the bond. In this context country-specific conventions have to be taken into account that can be briefly summarized as follows:

In Austria several conventions for the settlement period prevailed during the estimation period. Before January 9, 1998 the settlement date was the Monday two weeks after a transaction. Since January 16, 1998 the settlement period has been three days, i.e. a transaction is settled on the third working day after a transaction. For transactions between January 9 and January 16, 1998 the settlement date was January 21. Until the end of 1996 an ex-dividend period had to be taken into account. During this period a bond does not entitle to the payment of the next coupon and the accrued interest is negative. For coupon payments due between the first and the ninth day of a month the ex-dividend period starts on the third Monday of the previous month. For coupon payments between the 10th and the 24th day of a month the ex-dividend period starts on the first Monday in that month. For other coupon dates the ex-dividend period starts on the third Monday of the month. The calculation of the fraction a_t is done on the basis of 360 days per year. The time between the settlement day and the last coupon date is calculated in such a way that a month is assumed to have a maximum of 30 days.

In Germany coupons are paid once a year and the settlement period is two days. An ex-dividend period had to be taken into account before 1994. The fraction a_t is based on the same assumptions as in Austria.

In the UK coupons are paid twice a year and the settlement period is one day.
In addition, an ex-dividend period had to be taken into account. a_t is always based on 365 days per year.

In the USA coupons are paid twice a year and a settlement period of one day has to be taken into account. The calculation of the fraction a_t is based on the actual number of days between the settlement day and the last coupon date. This interval is divided by the actual number of days between the last and the next coupon payment.

In Japan several conventions for the settlement period prevailed during the estimation period. Coupons are paid twice a year and the fraction a_t is based on using either 365 or 366 days a year.

The number of bonds used in the estimation varies from country to country. Moreover, their number is not equally distributed across maturities. Table 1 contains the average number of bonds used in the estimation procedure by maturity. In general the coverage in each country is reasonable. In the USA the majority of bonds is for maturities up to five years. In contrast to Germany and Austria sufficient information about long-term bonds with maturities above 10 years is available for the USA, UK and Japan. In the UK and Japan the observations are equally distributed across the maturity range up to 10 years. Most of the bonds in Germany are traded in the segment up to five years.

Table 1: Average number of bonds used in the term structure estimation.

Maturity	Austria	Germany	UK	USA	Japan
0-1	4	11	2	22	5
1-2	6	19	4	34	10
2-3	7	16	4	23	10
3-4	8	15	4	19	9
4-5	9	13	3	19	9
5-6	7	7	3	6	9
6-7	7	7	2	6	9
7-8	6	6	2	6	8
8-9	5	5	2	6	8
9-10	4	4	2	5	9
10-20	1	3	10	6	11
20-30			2	21	15
Total	64	105	39	172	109

3.2 Estimation procedure

Although bonds with non-standard features were excluded, there is still the possibility of outliers in the data. Outliers are bonds with exceptionally high or low yields compared to other bonds with similar maturity. Such cases may bias the estimation results, in particular when a flexible functional form is used. Therefore outliers are identified using the following procedure (see Bianchi 1997). First, a smoothed curve is fitted to the observed yields using Cleveland's (1979) smoothing algorithm. Second, yields of bonds that are 'too far' from the smoothed curve are considered to be outliers and excluded. The decision regarding outliers is based on the interquartile range and a parameter whose value has been fixed on the basis of experiments.

The principles of the estimation procedure have been described above. A Newton-Raphson search procedure is used to minimize the sum of squared yield errors. In order to start the iterative procedure, starting values for the parameters have to be provided. The interpretation of β_0 and β_1 explained above makes it easy to specify starting values for these parameters. β_0 is the long-term interest rate implied by the model. Therefore, the smoothed yield with the longest maturity is used as the starting value for β_0. The difference between the smoothed yields with the longest and shortest maturities is used as a starting value of the spread parameter β_1. For β_2 there is no specific economic interpretation, it is initially set to –0.01. This corresponds to an upward sloping spot-rate curve with very little curvature (see Figure 1).

As regards the choice of starting values for τ_1 the following procedure is applied: Several reasonable starting values for τ_1 are specified. The estimation is carried out for each τ_1 in the set while the starting values of β_0, β_1, and β_2 are set as described above. The parameter estimates that provide the best fit are used. For the Svensson extension the same starting values are used. In addition, the initial values $\beta_3 = \beta_2$ and $\tau_2 = 1$ are used.

This choice of starting values is only applied when no estimates from the previous day are available or the date of the immediately preceding results in the database is more than one month ago. Whenever recent parameters are available, the estimation procedure is initiated with them.

Constraints are imposed on parameters and on spot- and forward-rate curves in order to exclude implausible or unrealistic estimation results. β_0 is constrained to be positive and less than 1, β_1**,** and β_2 must be in the range –1 to +1, and τ_1 must be positive and less than 20.

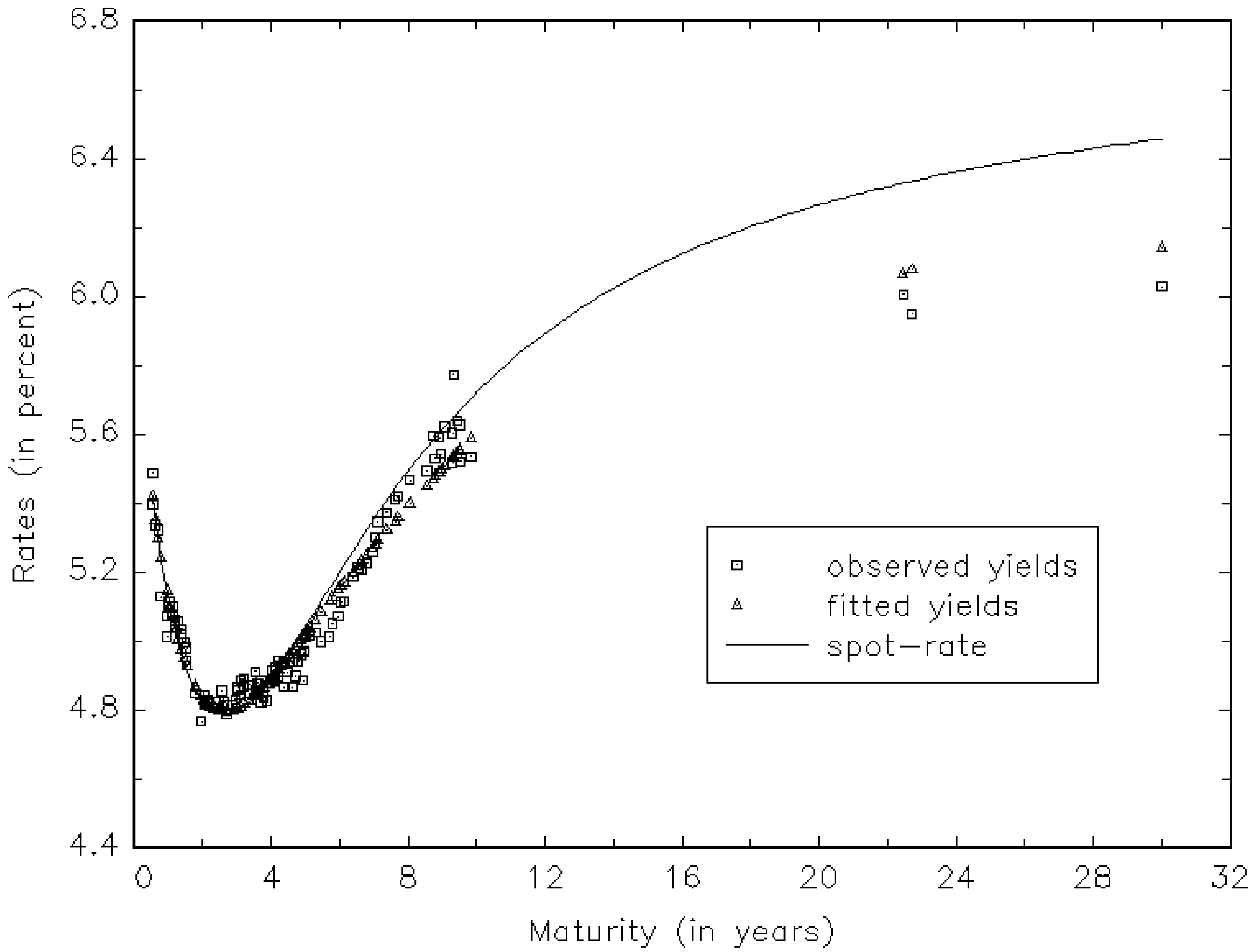
3.3 Econometric results

In general, the estimation procedure provides plausible curves, the scatter of observed yields across maturities is captured well, and convergence is achieved within reasonable time. Some interesting and important points will now be discussed by way of examples.

Figure 2 illustrates a typical result of the model estimation and, at the same time, allows to demonstrate the flexibility of the Nelson/Siegel model. Observed yields are indicated by squares and fitted yields – yields implied by the term structure model –

are indicated by triangles. There is a close correspondence between data and model, in particular for maturities of less than five years. The flexible functional form of the Nelson/Siegel model provides a reasonable approximation to the general shape of the observed yields. The spot-rate function implied by the estimated parameters provides a smooth curve, since it only depends on the maturity. Note that the curves of theoretical yields and spot-rates need not coincide. They coincide only for short maturities – when there are only a few outstanding payments.

Figure 2: Observed yields, fitted yields, and spot-rates. Model: Nelson/Siegel; Country: Germany; Date: January 3, 1994.



The goodness of fit can be measured by the coefficient of determination:

$$R^2 = 1 - \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \Big/ \sum_{i=1}^{N} (y_i - \bar{y}\ )^2,$$

where $\bar{y}$ is the mean of observed yields. R^2 measures how close the observed yields are scattered around the theoretical yields. The R^2 for the data in Figure 2 is 0.961. Figure 3 shows a case with a much smaller R^2 of 0.72. In this case, the fitted yields may also be considered a reasonable approximation of the general shape of the term structure. However, the scatter of observed points around the curve is much wider. This need not be an indication of a bad model resulting either from a wrong functional form or inferior parameter estimates. The data in Figure 3 is taken from the period when the Labour government had only recently come into power. Therefore, the low R^2 can be interpreted as the result of a high level of uncertainty in the market.

Figure 3: Observed yields, fitted yields, and spot-rates. Model: Nelson/Siegel; Country: UK; Date: July 31, 1997.

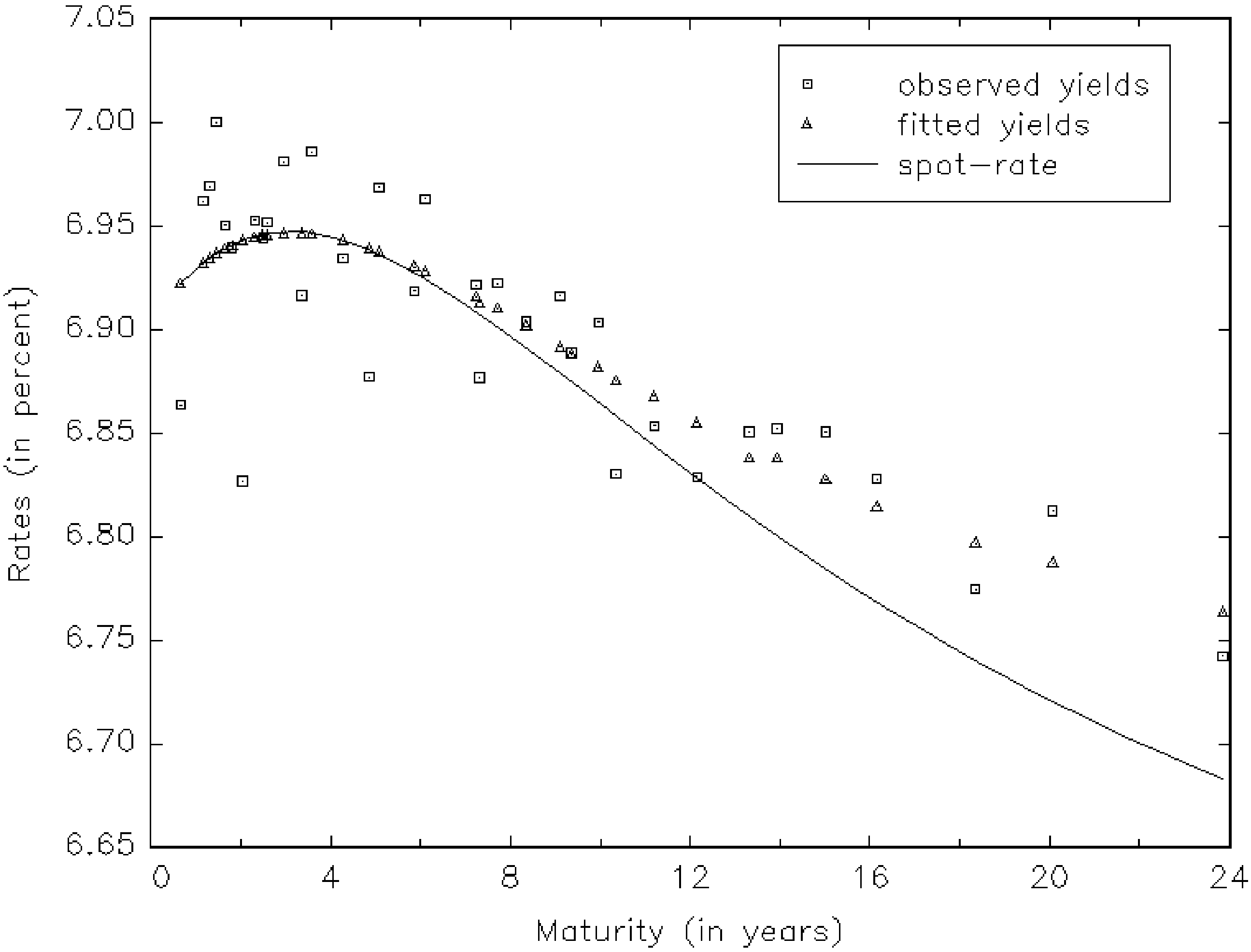


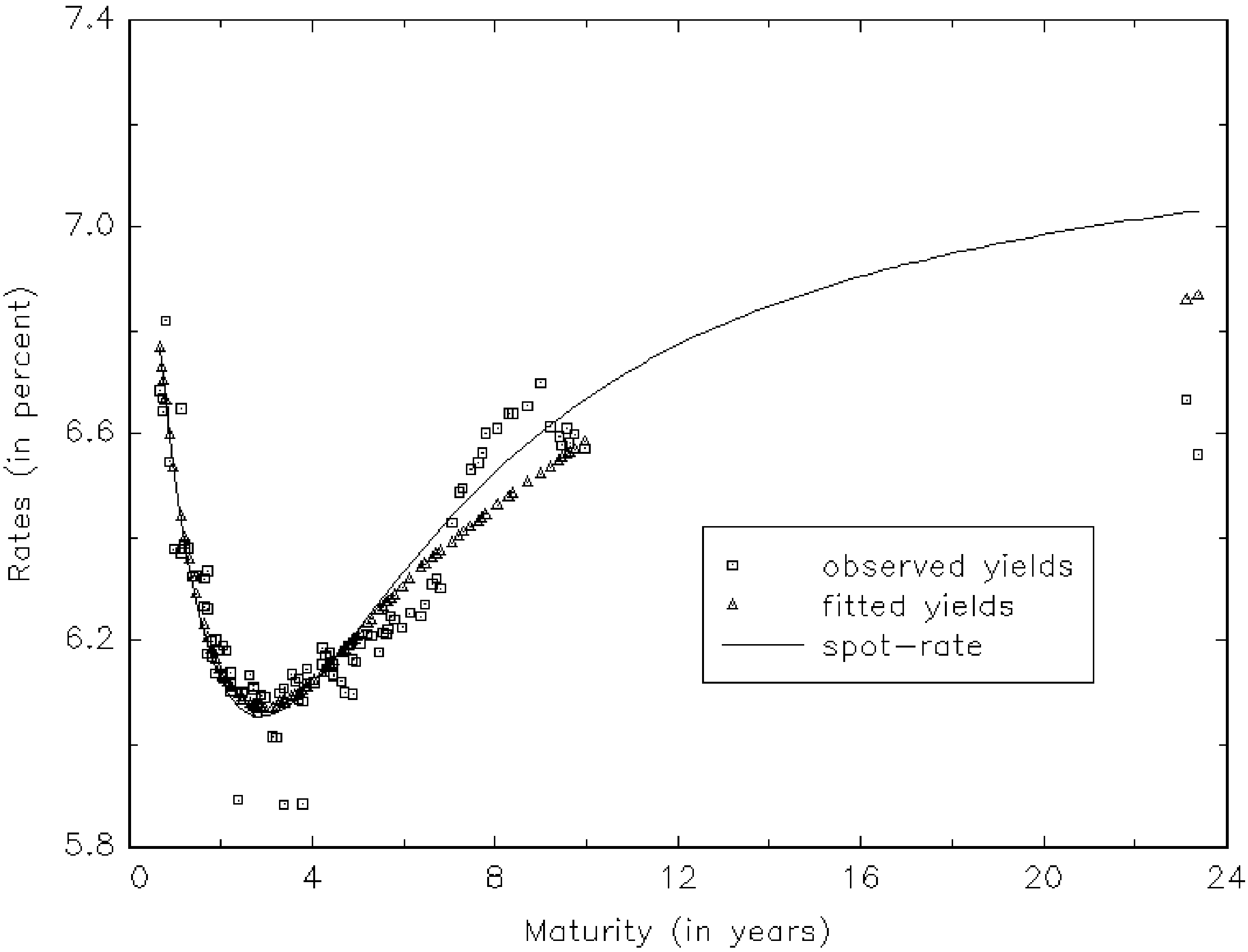
Table 2 contains yearly averages of R^2 for each country and allows for a summary judgment of the performance of the term structure model over time. In general, the fit is very good, except for Germany and Austria at the beginning of the sample period and except for UK in the middle of 1997. As shown above, a low R^2 need not be due to a weakness of the model or a problem in the estimation procedure. It rather reflects the dispersion of market prices and yields, whatever the reason may be. Generally speaking, R^2 is low whenever the term structure is flat (e.g. Germany and Austria at the beginning of 1993, USA in 1998) or the term structure is twisting (UK in the middle of 1997). These features will become more apparent from the presentation in section 3.4.

Table 2: Yearly averages of R^2

	1993	1994	1995	1996	1997	1998
USA	0.998	0.993	0.986	0.985	0.972	0.932
UK	0.979	0.962	0.984	0.995	0.867	0.988
Japan	0.991	0.990	0.985	0.994	0.996	0.997
Germany	0.847	0.966	0.991	0.997	0.998	0.996
Austria	0.804	0.936	0.961	0.986	0.987	0.967

The period from the middle of 1993 to the beginning of 1994 in Germany is another case worth while considering. In Figure 4 a S-shaped scatter of yields with a local minimum around a maturity of three years and a local maximum at around eight years can be observed. In addition, there are observations for two bonds with more than twenty years maturity. Yield and spot-rate curves may be considered too 'flat'.
This case raises the question whether the Nelson/Siegel model is not flexible enough since it cannot account for this particular shape with two humps.

Figure 4: Observed yields, fitted yields, and spot-rates. Model: Nelson/Siegel; Country: Germany; Date May 3, 1993.



This limited flexibility calls for considering the Svensson extension. Figure 5 shows that, in fact, the spot-rate of the Svensson model exhibits two humps. However, the second hump does not occur at a maturity of eight years. Instead, the model provides a better fit for the two bonds with very long maturities. As it turns out, even if the two bonds are excluded from the estimation, the Svensson model does not produce a second hump at around eight years (see Figure 6). Instead, the Svensson spot-rate is very similar to the Nelson/Siegel spot-rate.

Figure 5: Observed yields, fitted yields, and spot-rates. Model: Svensson; Country: Germany; Date: May 3, 1993.

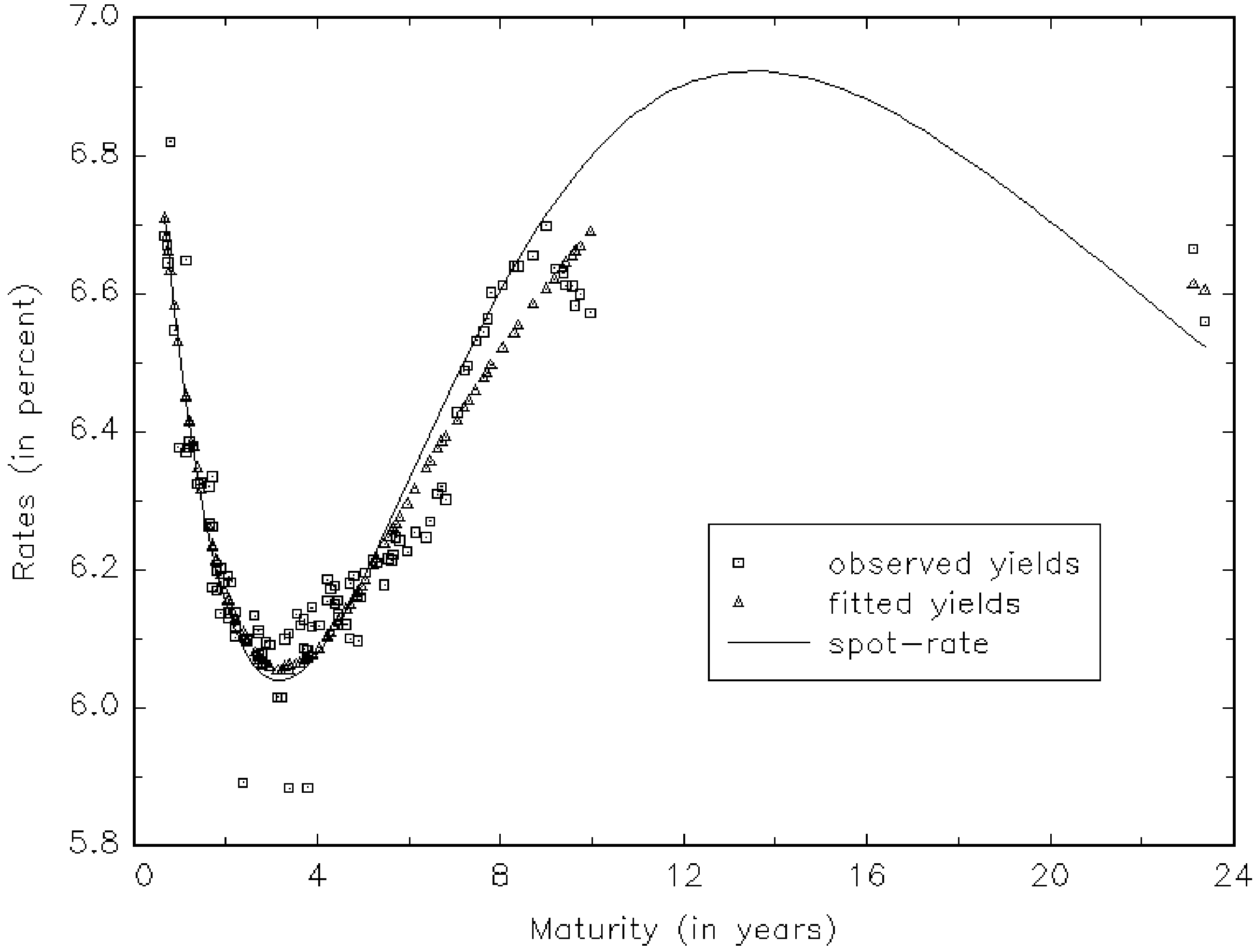
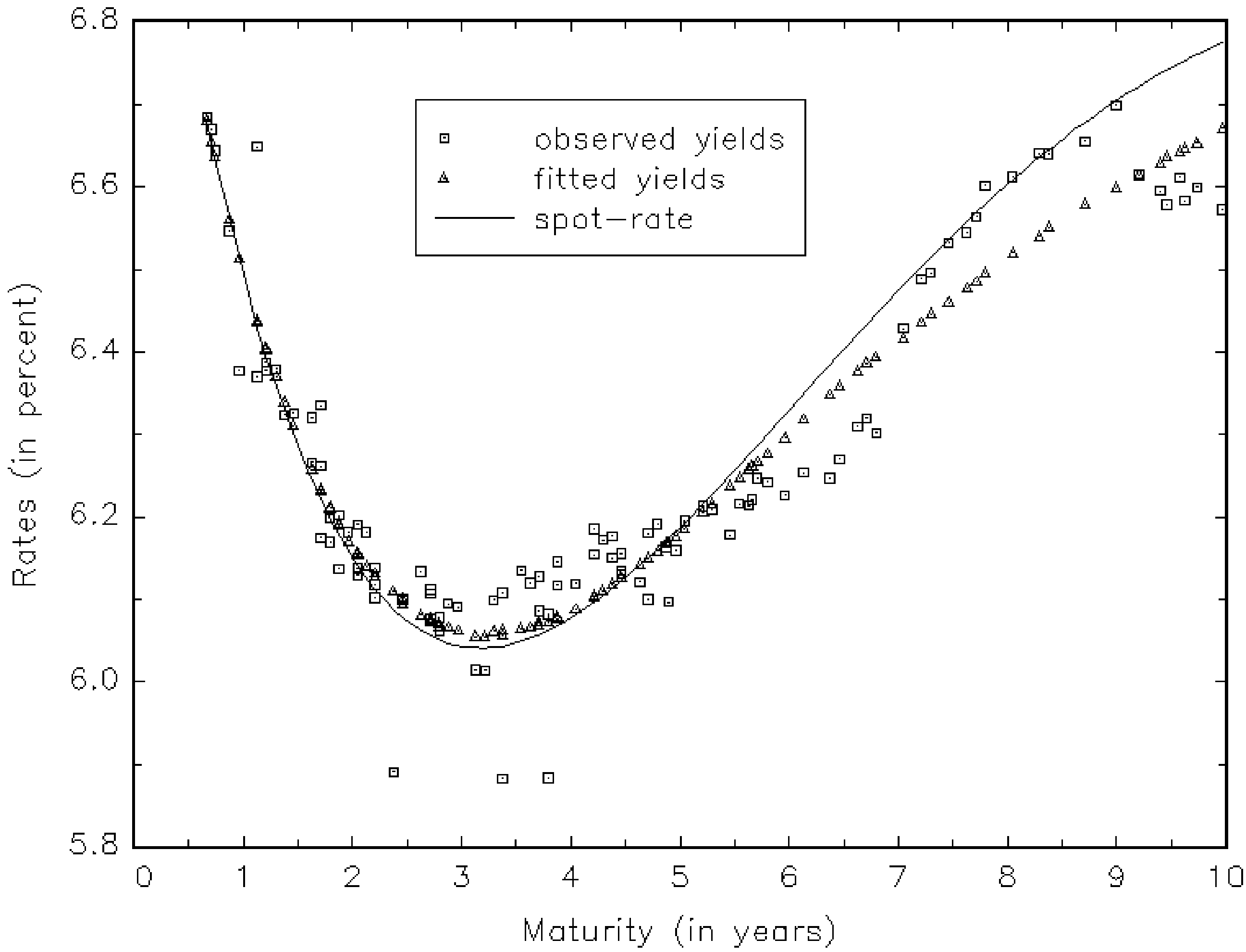


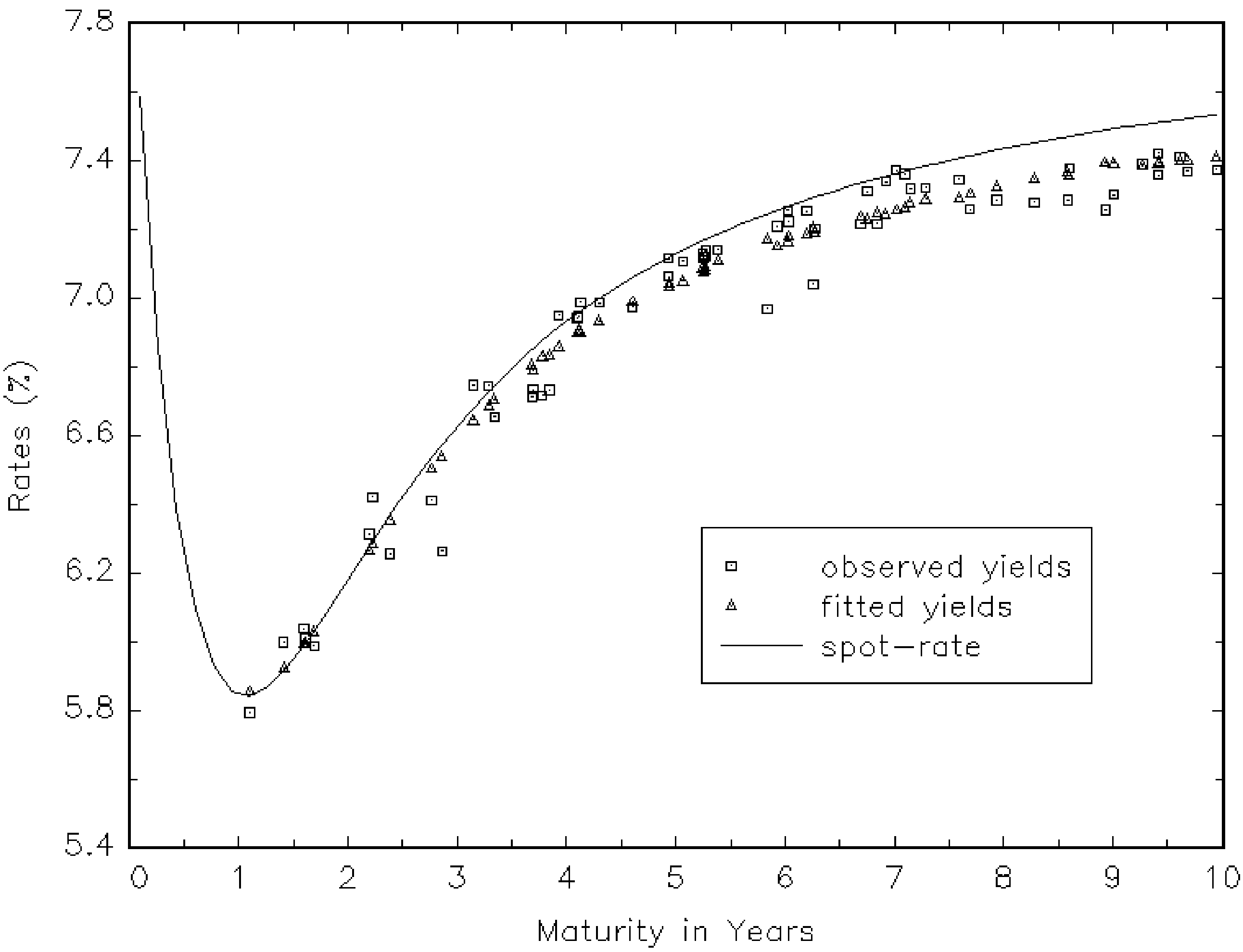
Figure 6: Observed yields, fitted yields, and spot-rates. Model: Svensson; Country: Germany; Date: May 3, 1993. Bonds with maturities over 10 years are excluded from the estimation.



For cases as shown in Figures 2 and 3 the added flexibility of the Svensson extension is not necessary. In general, it provides a fit that is almost identical with the Nelson/Siegel model. This result is confirmed in the study by Schich (1996) where only a slightly better fit of the Svensson model is found. This can be explained by the larger number of parameters in the model. However, when the term structure has a 'simple' shape, it has two disadvantages: First, the parameter estimation process takes much longer since two additional parameters are fitted. Moreover, convergence is harder to achieve since the two parameters are not really necessary. Second, the Svensson model is more prone to outliers and may reflect spurious features of the term structure.

The danger associated with the flexibility of models with many parameters must always be taken into account, even when the Nelson/Siegel model is used, and apparently provides a good fit to the data. Problems may arise when spot-rates are calculated for short maturities that are not well represented or not at all represented by the data. Figure 7 shows a case where the fit to observed data may be judged to be adequate. However, if the spot-rate is calculated for maturities lower than the range of maturities in the sample, it implies a shape for which there is no empirical support at all.

Figure 7: Observed yields, fitted yields, and spot-rates. Model: Nelson/Siegel; Country: Austria; Date: February 1, 1995.

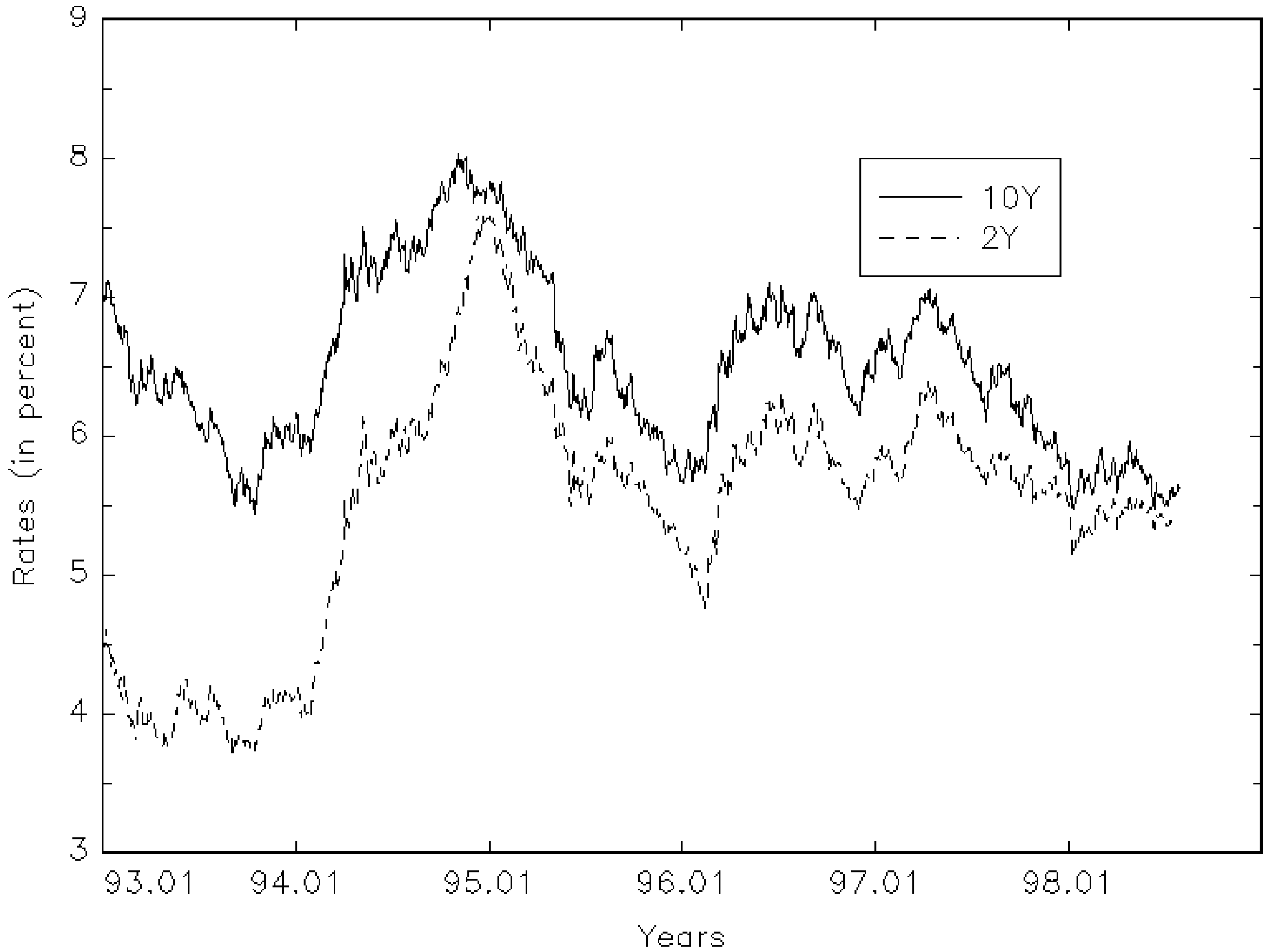


3.4 Term structure estimates

Over the sample period a variety of different shapes of estimated spot-rate curves can be observed. However, some general tendencies can be summarized: For most of the time term structures are monotonically increasing. Not surprisingly, the short end of the term structure is more volatile than the long end. Flat or inverse term structures are rare. In the UK an inverse term structure can be found as from the middle of 1997. U-shaped or S-shaped patterns are very rare, too, and persist only over short term periods. An exception is Germany's U-shaped term structure from 1993 until the beginning of 1994. Country-specific details are reported in the following.

Figure 8 compares ten-year and two-year spot-rates implied by the Nelson/Siegel model for the USA. The term structure is upward sloping throughout the sample period, although with varying flatness. The spread narrowed strongly during 1993 and 1994, and remained stable until 1997. Since the beginning of 1998 the term structure became very flat again, together with a fall in the interest rate level.

Figure 8: Ten-year and two-year spot-rates. Model: Nelson/Siegel; Country: USA.



Ten-year and two-year spot-rates for the UK are shown in Figure 9. The level of the long rate fell steadily since the middle of 1994. As in the USA, the spread has narrowed during 1993 and 1994, but increased again in 1996. As from the middle of 1998 an inverse term structure can be observed. Until the end of the sample period (July 1998) there are no signs that the term structure may become upward sloping again.

Figure 9: Ten-year and two-year spot-rates. Model: Nelson/Siegel; Country: UK.

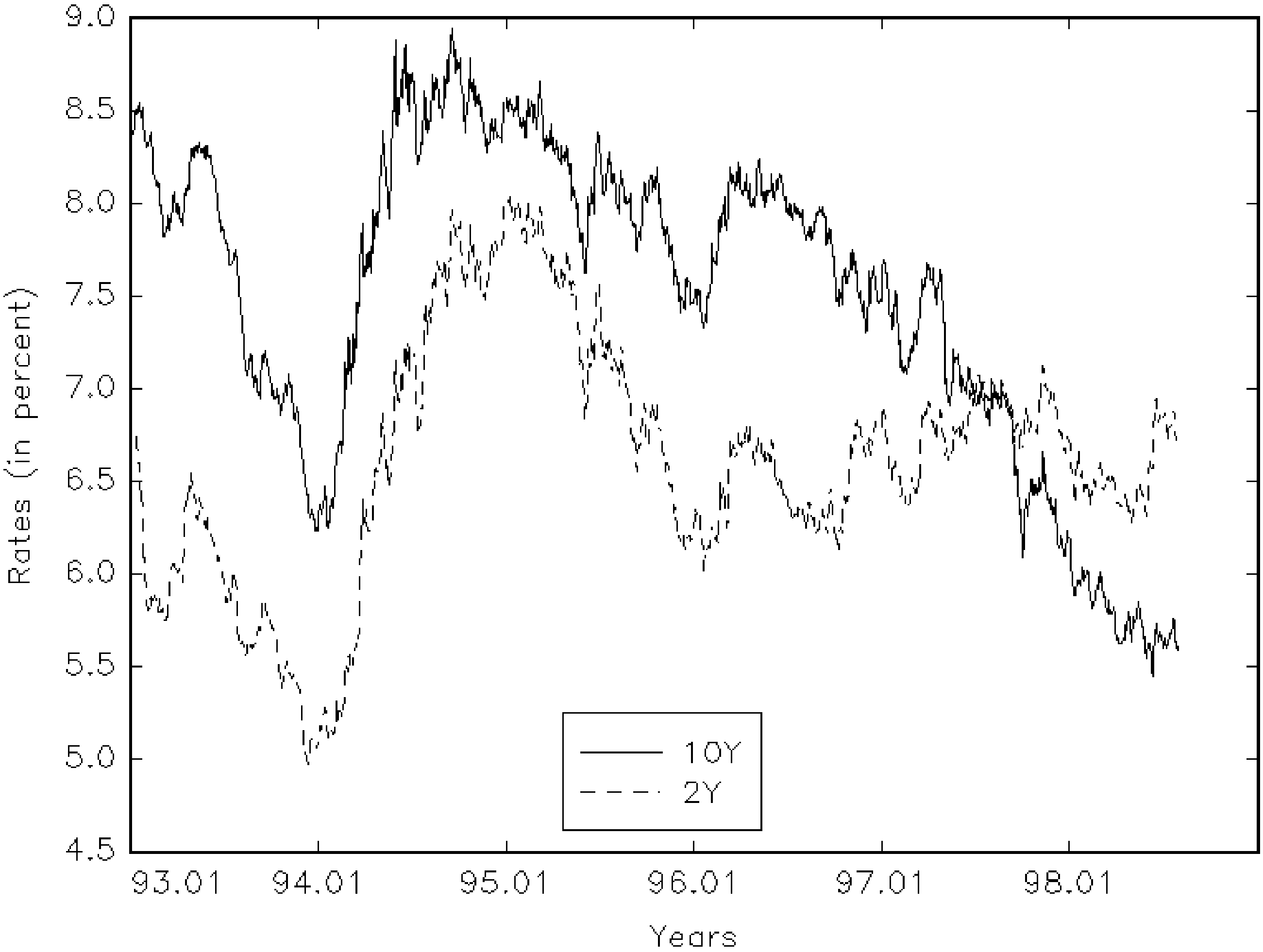
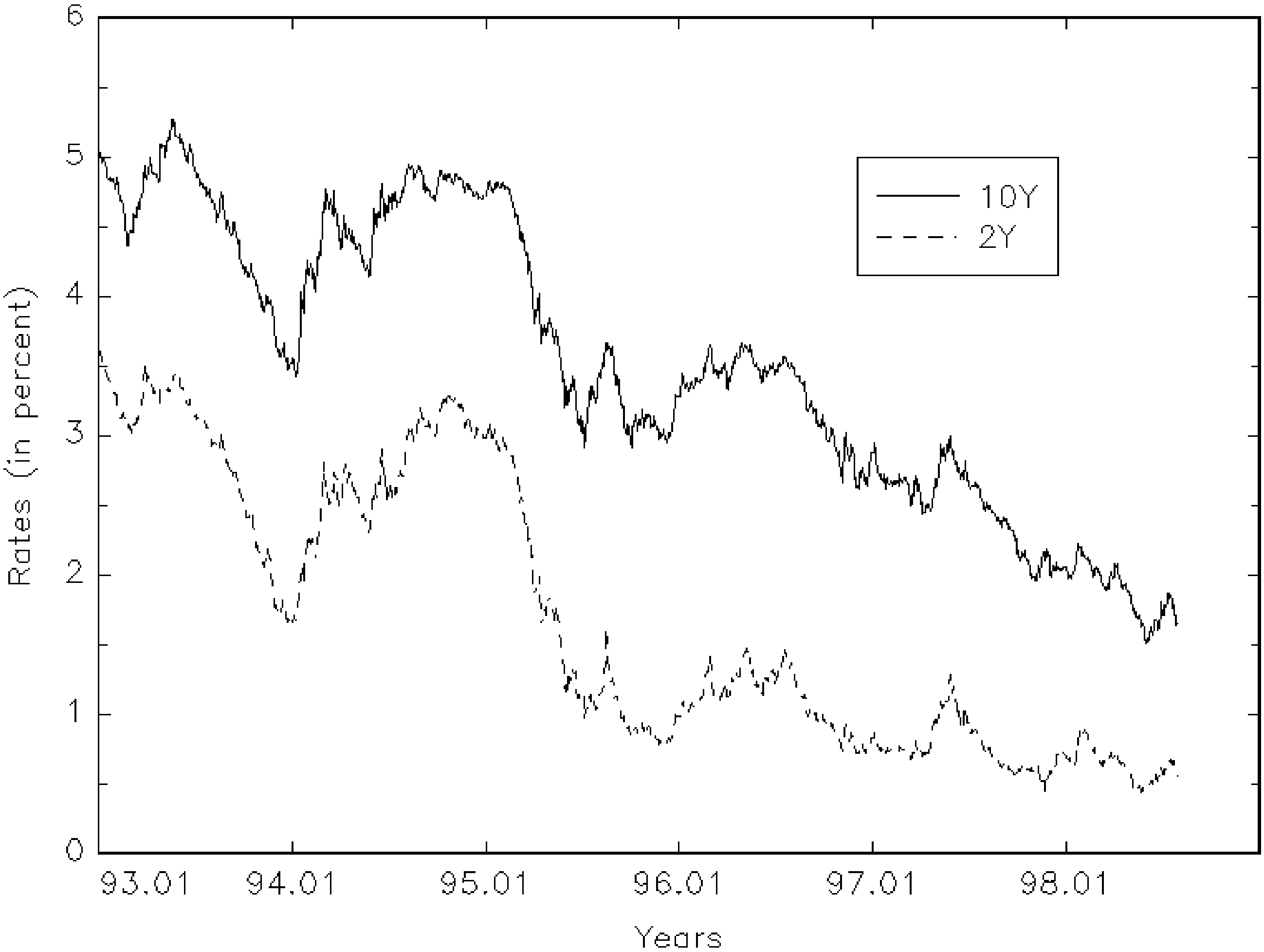


Figure 10 shows the almost continuous downward trend in Japanese interest rates. Over the sample period the spread hardly changed, a slightly narrowing tendency can be observed as from the beginning of 1998.

Figure 10: Ten-year and two-year spot-rates. Model: Nelson/Siegel; Country: Japan.



Falling interest rates as from the middle of 1994 can also be observed in Germany (see Figure 11). The term structure is upward sloping throughout. However, other than in the USA or the UK, the spread widened, and only as from the end of 1997 the term structure tended to become flatter. A very similar situation prevails in Austria (see Figure 12).

Figure 11: Ten-year and two-year spot-rates. Model: Nelson/Siegel; Country: Germany.

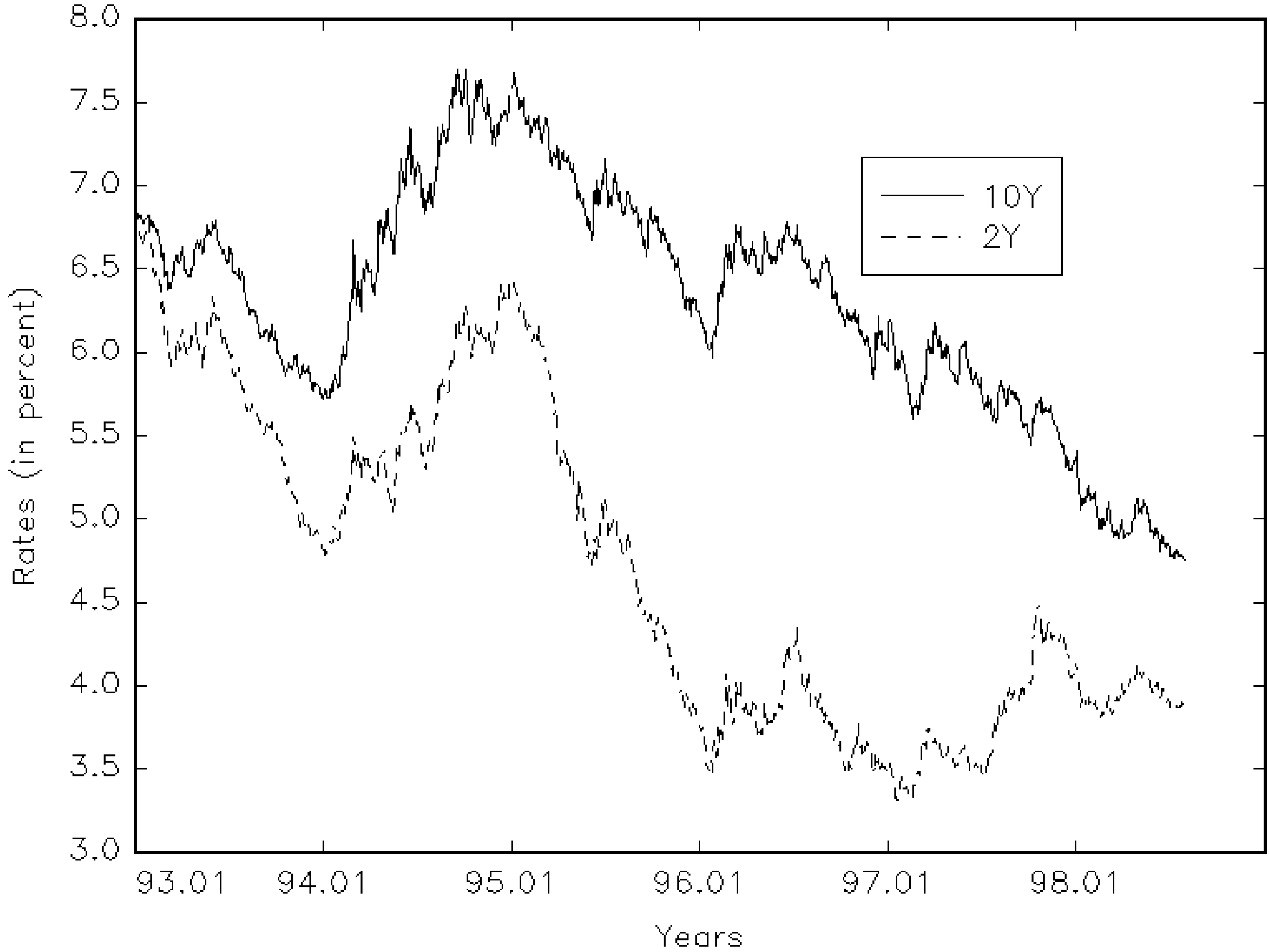
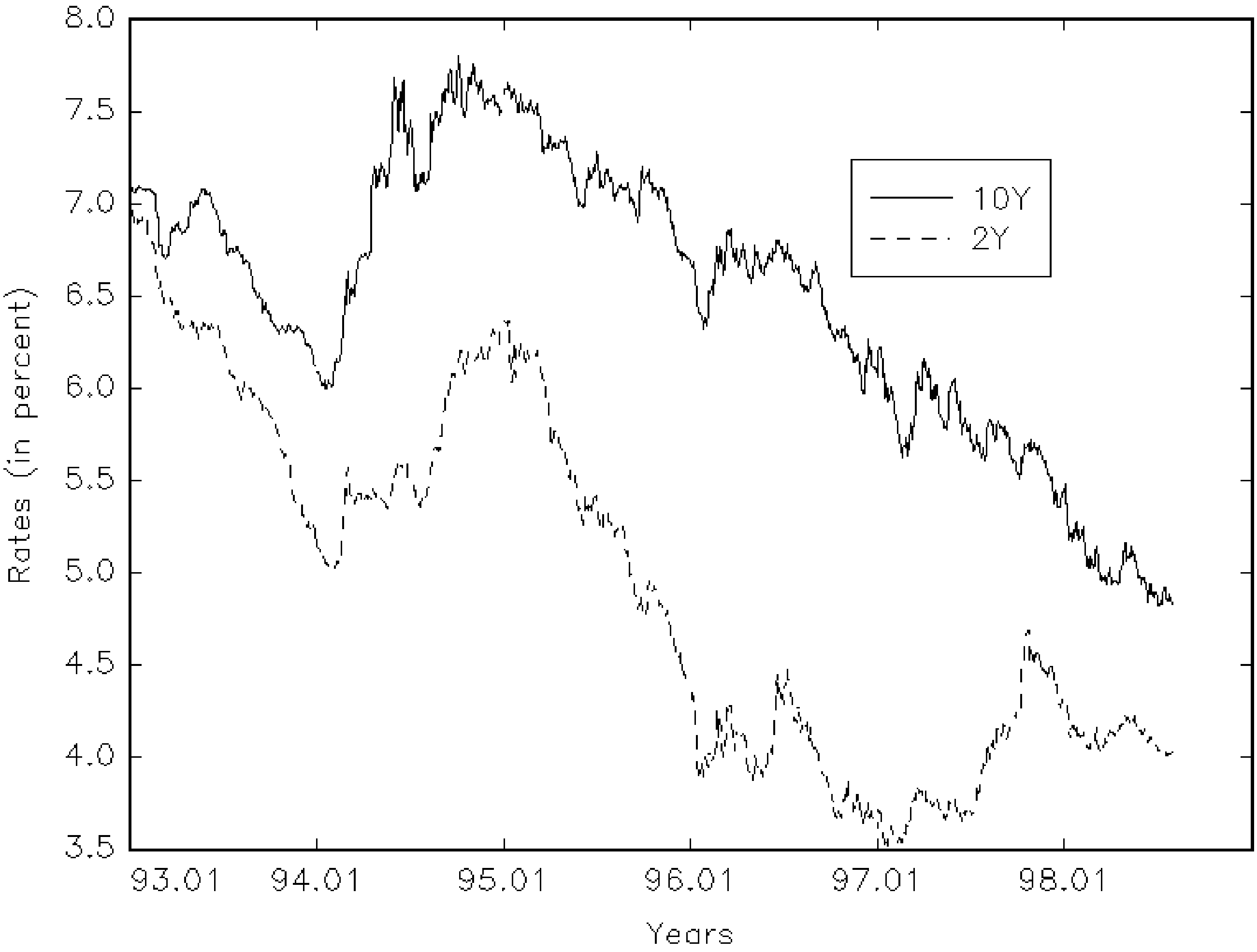


Figure 12: Ten-year and two-year spot-rates. Model: Nelson/Siegel; Country: Austria.



4 Conclusions

Estimates of the term structure of interest rates provide important information for monetary policy making. They can be used for inflation and output forecasts, give useful indications about the differences in monetary stance and contain information about market expectations of future changes in interest rates. Most EMU central banks started to develop their own interest rate models in the nineties. This paper describes the parametric modeling approach used to estimate the term structure for Austria, Germany, UK, USA and Japan over the period 1993 to 1998. Using a common approach to estimate the term structure for all countries has the advantage of facilitating inter-country comparisons.

The Nelson/Siegel (1987) model was used as the principal estimation technique. Over most of the six-year sample period and for all countries considered the model produced reliable and reasonable estimation results. However, in certain periods the fit was not completely satisfactory. For that reason, the number of parameters included was increased along the lines of the approach proposed by Svensson (1994). Experimentation with the Svensson model showed that the fit can be improved only at the expense of introducing problems of overparameterization. In addition it was found that lower levels of the goodness-of-fit need not be due to choosing the ‘wrong’ model but may be induced by a higher dispersion in market prices. This typically occurs when the term structure is flat or twisting. The Nelson/Siegel model was finally chosen because its performance generally was as good as the Svensson model, its estimation can be carried out far more quickly and it is less sensitive to outliers.

While implementing the term structure model the Austrian National Bank established a database for five important countries, consisting of observed prices and yields, as well as parameter estimates for each trading day from 1993 to 1998. This database will be continuously updated and extended in the future on a regular (daily) basis. Because of the link to Datastream, the Austrian National Bank is in a position for the first time to obtain up-to-date term structure estimates for several countries on a daily basis. In addition, the database provides the foundation for future research activities along the lines mentioned in the introduction (e.g., analysis of factors affecting the term structure, forecasts of interest rates, inflation and output).

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