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BANK CAPITAL, LIQUIDITY AND SYSTEMIC RISK

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Editorial

Juergen Eichberger and Martin Summer analyze the impact of capital adequacy regulation on bank insolvency and aggregate investment. They develop a model of the banking system that is characterized by the interaction of many heterogeneous banks with the real sector, interbank credit relations as a consequence of bank liquidity management and an insolvency mechanism. This allows the authors to study the impact of capital adequacy regulation on systemic risk. In particular, they can analyze the impact of regulation on contagious defaults arising from mutual credit relations. They show that the impact of capital adequacy on systemic stability is ambiguous and that systemic risk might actually increase as a consequence of imposing capital constraints on banks. Furthermore they analyze the indirect consequences of capital adequacy regulation that are transmitted to the real economy by their impact on equilibrium interbank rates and thus the opportunity costs of liquidity within the banking system.

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Bank Capital, Liquidity and Systemic Risk *

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Abstract

We analyze the impact of capital adequacy regulation on bank insolvency and aggregate investment. We develop a model of the banking system that is characterized by the interaction of many heterogeneous banks with the real sector, interbank credit relations as a consequence of bank liquidity management and an insolvency mechanism. This allows us to study the impact of capital adequacy regulation on systemic risk. In particular we can analyze the impact of regulation on contagious defaults arising from mutual credit relations. We show that the impact of capital adequacy on systemic stability is ambiguous and that systemic risk might actually increase as a consequence of imposing capital constraints on banks. Furthermore we analyze the indirect consequences of capital adequacy regulation that are transmitted to the real economy by their impact on equilibrium interbank rates and thus the opportunity costs of liquidity within the banking system.

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1 Introduction

The policy discussion about banking regulation during the past two decades has been mainly concerned with capital adequacy. This focus was reinforced by the refinement of existing capital adequacy rules by the Basel Committee which forms the core of the regulatory reform known as "Basel II". In the debate about how to reform the existing framework questions concerning the general rationale of capital adequacy have been moved to the background. Moreover, whether such regulation can actually serve as a safeguard against financial crises - as it is often claimed in policy debates - has perhaps received insufficient attention. In favor of capital adequacy the literature advances two different arguments. On the one hand, capital adequacy is seen as an instrument limiting excessive risk taking of bank owners with limited liability and, thus, promoting optimal risk sharing between bank owners and depositors. On the other hand, capital adequacy regulation is often viewed as a buffer against insolvency crises, limiting the costs of financial distress by reducing the probability of insolvency of banks. Irrespective of the viewpoint taken, one usually finds a general reference to financial stability, suggesting that capital adequacy regulation provides a safeguard against systemic crises. The mechanism linking capital adequacy and systemic risk remains however usually unexplained. In this paper we provide a new framework in which the dependence between the buffer stock view of bank capital and systemic risk can be discussed more precisely.

Given the extensive literature on capital adequacy regulation (see for instance Freixas and Rochet (1997)), it may appear surprising that the impact of this particular regulatory policy on financial stability has not been analyzed more rigorously. Yet most models in the banking literature deal with a single bank's decision problem and the incentives of the different claim holders of the bank, in particular of bank owners and managers. With a single bank it is clearly impossible to describe the two major sources of systemic risk: correlated portfolio positions in the banking system and domino-effects in consequence of interbank exposures. Apart from an early paper by Hellwig and Blum (1995) there are few attempts in the literature on capital regulation to move from a single-bank to a system perspective. While financial stability and the macroeconomic consequences of solvency regulation are studied in Gorton and Whinton (1995), Gersbach and Wenzelburger (2002), the issue of how financial linkages transmit, and possibly amplify, financial crises has received little attention (see however the papers by Goodhart, Sunidrand, and Tsomocos (2003) and Cifuentes, Ferucci, and Shin (2003)).

This paper provides a framework that can shed light on this issue. In particular, we analyze the impact of capital adequacy requirements on default probabilities and the systemic risk from contagion effects and correlated exposures. We consider a simple one-period model with heterogeneous banks that finance risky loans of enterprises. When banks make their loan decisions, their deposits and equity are already determined. If banks want to increase loans beyond what they can finance from their deposits and equity, they can obtain additional liquidity from banks with a surplus of funds through a competitive interbank market. The competitive interbank rate determines both the opportunity costs of a bank's loans to firms and the credit positions among banks participating in the interbank market.

Shocks to a bank's corporate loan portfolio may cause insolvencies. The resulting shortfalls of payments are settled in a clearing system that establishes ex-post consistency of all financial claims. This model captures two sources of systemic risk, an exposure of banks to common risk factors and the transmission of insolvencies via the interbank market.

Into this framework we introduce a capital adequacy rule in order to study how this regulation affects the interbank rate and default probabilities. We find that the consequences of capital adequacy regulation for systemic stability are not clear cut. Considering also the indirect effects of capital adequacy regulation, we can show that a capital adequacy regime, while boosting the capital buffer of individual banks, may increase the risk of contagious insolvencies in the banking system as a whole.

The paper is organized as follows. Section 2 describes the model, section 3 studies equilibrium and analyzes its properties. In section 4 we focus on capital adequacy and its impact on systemic stability. The final section 5 concludes the paper. All proofs are gathered in an appendix.

2 A Model of the Banking System

In this section we develop a model that allows us to analyze the role of bank capital for the stability of a banking system. In addition, we study the indirect effect of capital adequacy regulation on the capacity of banks to provide finance for firms. Three features characterize our model. Firstly, we study the interaction of heterogeneous banks in a *banking system*, rather than a representative institution. Secondly, we model the impact of bank activities on the real economy. Our model contains an explicit description of the investment behavior of firms financed by the banking system. Thirdly, we consider systemic risk from mutual credit obligations among banks. The interbank loan arrangements may cause domino effects of insolvencies. In this framework, the role of bank capital in mitigating exposure to such risks is investigated.

Consider a set of banks, $\mathcal{J} = \{1, ..., J\}$, which operate in a sequence of periods, $t = 1, 2, ...$ In each period banks obtain deposits, make loans to firms and either raise additional funds or lend to other banks in an interbank market. In our model we analyze decisions and equilibrium for the case $t = 0, 1$, leaving the analysis for a richer dynamics for future research. Deposits are raised at $t = 0$ and will become due at the end of the period, $t = 1$. Equity of the bank was provided by the bank's owners in the past. Within a period banks make loans to their customers and finance these loans by returns on their equity, new deposits and through an interbank market. If the value of deposits and equity exceeds the value of loans, banks offer their excess funds in the interbank market. Demand for these funds comes from other banks who wish to provide more loans to their customers than they can finance by their equity and deposits. At the end of the period, shocks are realized and claims are settled. After the realization of the shock to the loan portfolio, deposits have to be payed back and interbank settlements take place. Banks who do not default have a non-negative surplus which is their equity value of the next period. In the following we describe the details of this model.

2.1 The Real Sector: Loan Demand of Firms and Investment

In this section we study the loan market of a typical bank $j \in \mathcal{J}$ in $t = 0$. Each bank has a group of firms as customers. Firms in each group depend on their home bank for finance of their investment projects. Hence, each firm belongs to the customer base of one particular bank. Firms within the customer group of a bank are characterized by a continuum of different productivities $q \in [0, M]$[1], ranging from the lowest productivity $q = 0$ to maximal productivity $q = M$. While the productivity parameter is private knowledge of the firms, it is common knowledge that productivities are uniformly distributed on $[0, M]$. Investment projects of firms require a fixed input of 1 unit of capital at the beginning of the period and yield a risky payoff at the end of the period. The payoff of a firm with productivity q is described by the function $f : \{0, 1\} \times \mathbb{R}_{++} \times [0, M] \to \mathbb{R}_+$:

$$f(s_j, A, q) = s_j \cdot A \cdot q \tag{1}$$

where $A \in \mathbb{R}_{++}$ denotes an aggregate productivity parameter for the firm group and s_j a random variable indicating group-specific success or failure of projects. We assume that $s_j \in \{0, 1\}$ for all $j \in \mathcal{J}$ and that the probability of success is given by $\Pr(s = 1) = \rho$ with $\rho \in (0, 1)$. Firms within a group are distinguished only by their productivity parameter q. In particular, we do not model firm-specific shocks. All firms in a bank's customer group suffer the same shock s_j. We view s_j as a shock to the bank's loan portfolio[2].

Firms have limited liability and for each firm the opportunity costs of not undertaking a project are normalized to zero. This is a restrictive assumption, since the outside opportunities of firms are likely to depend on the loan conditions which other banks offer. Relaxing this assumption is conceptually easy but requires a more complex description of the loan market than is required for our analysis.

[1] This setup builds on work by Gersbach and Wenzelburger (2002).

[2] It is not difficult to generalize this setup to include firm-specific shocks. Since we are interested in bank failures and how these failures are transmitted through the interbank loan market, including such generality would complicate the model without affecting its message.

We assume that only those firms take a loan that expect a strictly positive profit at the loan interest rate R_j which its bank charges. Since banks can not observe the productivities of the firms in their customer base they must offer a uniform loan interest rate R_j for all firms in their group. A firm of productivity q facing a loan interest rate R will choose to take a loan if its expected profit,

$$\Pi(R_j, q) := \rho \ \ \max[f(1, A, q) - R_j, 0], \tag{2}$$

strictly exceeds the payoff from the outside option of 0. The individual firms' decisions generate a loan demand function $Q(R_j)$ for each bank's customer group.

Proposition 1 ***: Loan Demand***

1. *For each $R_j \in \mathbb{R}_+$ there exists a unique critical productivity $q^*(R_j) \in [0, M]$,*

$$q^*(R_j) = \min\left\{\frac{R_j}{A}, M\right\}. \tag{3}$$

2. *Firms with productivity $q > q^*(R_j)$ take a loan and all firms with productivity $q \leq q^*(R_j)$ take the outside option.*

3. *The aggregate loan demand function of the group is*

$$Q(R_j) := M - q^*(R_j). \tag{4}$$

Proof: See Appendix.

2.2 The Banking System: Loan Supply and Interbank Market

Since each firm group finances its projects by loans from only one bank we can choose the same index for banks as for firm groups. Each bank has an initial equity $e_{0j} \in \mathbb{R}_{++}$. The initial equity results from the settlement of past loans after shocks of the previous period have been realized, deposits have been paid and interbank claims have been cleared. Each bank also has a stock of deposits d_{0j} that require a (gross) interest rate D in period $t = 1$. The equity e_{0j} is positive because banks with negative equity are insolvent and exit the banking system.

Banks maximize expected profits from loans to their firm group. They monopolistically quote a loan interest rate R_j and supply loans to satisfy the loan demand according to their loan demand functions (4). The market power of banks results from the client-specific relationship with its firm group. Banks can use their funds $(e_{0j} + d_{0j})$ to finance loans. If they need additional funds they can enter a competitive interbank market where they can raise funds at a gross interest rate I from other banks that don't lend all of their initial wealth to firms in their own group. The interbank market allows banks to competitively exchange funds at $t = 0$. To denote interbank positions l_j we introduce notation that will be useful for the following discussion. Define $l_j^+ := \max\{l_j, 0\}$ and $l^- := -\min\{l_j, 0\}$ to denote long and short positions. When choosing their loan interest rate R_j and their interbank positions l_j^+, l_j^- banks are subject to the following budget constraint:

$$\mathcal{B}_j(e_{0j}, d_{0j}) := \{(R_j, l_j) \in \mathbb{R}_+ \times \mathbb{R} \mid Q(R_j) + l_j^+ - l_j^- \leq e_{0j} + d_{0j}\}. \tag{5}$$

Shocks to the technologies of firms, and hence to the J banks whose customers they are, will induce one of 2^J states $s = (s_1, ..., s_J) \in \mathcal{S} := \{0, 1\}^J$, where s_j indicates whether bank j has a positive return on its loans, $s_j = 1$, or not, $s_j = 0$. Since the shocks are independent the probability of a state (which is the probability of a profile of individual shocks s_j) can be described by a binomial distribution with parameters ρ and J. We will suppress these parameters which stay constant throughout this paper and denote the probability of a state s by $\pi(s)$. [3]

If a bank finances its loan portfolio with an interbank loan and if the customers of this bank suffer a negative shock, then the realized value of the bank is negative since $-I \cdot l_j^- < 0$. The bank is insolvent and some banks with positive positions in the interbank market cannot realize the full return on their interbank investment l_j^+. A clearing system, which we describe in the next section, will redistribute such losses among the banks participating in the interbank market. If in a state $s \in \mathcal{S}$ some banks fail, the return rate of the lending banks will be less than I. Denote by $\delta(s) \in [0, 1]$ the discount on the contracted interest rate I which a lending bank suffers, then lending banks receive an actual return rate of $\delta(s) \cdot I$ on their interbank loans. Below, we will assume that all lending banks are treated equally by the clearing system. Hence, the discount quota $\delta(s)$ will be the same for all lending banks.

Denoting by $s_j(s) \in \{0, 1\}$ the projection of the vector s on its j -th component, i.e., the shock which hits firm j in state s, one can write the expected profit of a bank as

$$\sum_{s \in S} \pi(s) \cdot \left[s_j(s) \cdot R_j \cdot Q(R_j) + \delta(s) \cdot I \cdot l_j^+ - I \cdot l_j^- - D \cdot d_{0j}\right] \tag{6}$$

[3] Clearly $Pr(s_j = 1) = \sum_{\{s \in S | s_j = 1\}} \pi(s) = \rho$.

This expression can be simplified to

$$\rho \cdot R_j \cdot Q(R_j) + \overline{\delta} \cdot I \cdot l_j^+ - I \cdot l_j^- - D \cdot d_{0j}, \tag{7}$$

where $\overline{\delta} := \sum_{s \in S} \pi(s) \cdot \delta(s)$ denotes the average discount across states on the contracted loan interest rate I.

Banks choose a loan interest rate and an interbank position (R_j, l_j) within its budget set $\mathcal{B}_j(e_{0j}, d_{0j})$ to maximize expected profit:

$$V(R_j, l_j | I, \overline{\delta}, D) := \rho \cdot R_j \cdot Q(R_j) + \overline{\delta} \cdot I \cdot l_j^+ - I \cdot l_j^- - D \cdot d_{0j}. \tag{8}$$

When setting the loan rate R_j the bank behaves like a monopolist. Contrary to the standard monopoly problem, however, marginal costs are determined in the interbank market. The interbank market rate I will be determined competitively and will indirectly determine the loan rates R_j of the banks. If there is at least one state where a bank fails, then the return on a loan to the interbank market $\overline{\delta} \cdot I$ will be strictly less than I, the return which has to be paid when making a loan to the interbank market. Hence, a bank will never borrow and lend from the interbank market in the same period. Therefore $l_j^+ \cdot l_j^- = 0$.

The bank's objective function (8) deserves a further comment. We assume that banks treat the state-dependent outcome of the clearing mechanism $\delta(s)$ as parametric in their planning. In particular, we exclude that a bank strategically considers that its own activity in the interbank market may affect this outcome. One can interpret this as the assumption that banks treat the insolvency risk of other banks as beyond their control and consider only an average discount on the contracted loan rate I. In an equilibrium these beliefs about $\overline{\delta}$ will be rational. The following proposition characterizes the optimal decisions of banks.

Proposition 2 : ***Characterization of Optimal Bank Decision***

Define $I_j^0 := 2\rho A \cdot \max\{0, \frac{M}{2} - (e_{0j} + d_{0j})\}$. *For each* $I \in \mathbb{R}_{++}$ *and each* $\overline{\delta} \in [0,1]$ *the bank problem (8) has a unique optimum* $(R_j^*(I,\overline{\delta}), l_j^*(I,\overline{\delta}))$. *This optimum is characterized as follows:*

For $I \leq I_j^0$ *(Interbank borrowing):*

$$R_j^*(I,\overline{\delta}) = \frac{MA}{2} + \frac{I}{2\rho}$$

$$l_j^*(I,\overline{\delta}) = (e_{0j} + d_{0j}) - \frac{M}{2} + \frac{I}{2\rho A}$$

For $I_j^0 < I < \frac{1}{\overline{\delta}} I_j^0$, *(No Interbank Activity:)*

$$R_j^*(I,\overline{\delta}) = \frac{MA}{2} + \frac{I_j^0}{2\rho}$$

$$l_j^*(I,\overline{\delta}) = (e_{0j} + d_{0j}) - \frac{M}{2} + \frac{I_j^0}{2\rho A}$$

For $\frac{1}{\overline{\delta}} I_j^0 \leq I$, *(Interbank lending):*

$$R_j^*(I,\overline{\delta}) = \min\left\{\frac{MA}{2} + \frac{\overline{\delta} \cdot I}{2\rho}, MA\right\}$$

$$l_j^*(I,\overline{\delta}) = \min\left\{(e_{0j} + d_{0j}) - \frac{M}{2} + \frac{\overline{\delta} \cdot I}{2\rho A}, (e_{0j} + d_{0j})\right\}$$

Proof: See Appendix.

Proposition 2 is illustrated in Figure 1. If I exceeds ρAM, the monopoly bank must raise its interest rate to a level where demand falls to zero. The economically interesting situation occurs if $I < \rho MA$. If $\overline{\delta} < 1$ holds, then there are three cases to distinguish. Since the opportunity costs of lending to the interbank market, $\overline{\delta} \cdot I$ are lower than I, the bank will charge a higher loan rate $R_j^*(I,1)$, if it has to borrow from the interbank market in order to finance its loans to the firms, $R_j^*(I,1) > R_j^*(I,\overline{\delta})$. If the optimal amount of loans to its customer firms at cost of I, $Q(R_j^*(I,1))$, exceeds the bank's liquid funds $e_{0j} + d_{0j}$, then it will lend from the interbank market to finance its loans. If the bank's funds $e_{0j}+d_{0j}$ are greater than the loans it wishes to make to its customers at the cost of $\overline{\delta} \cdot I$, $Q(R_j^*(I,\overline{\delta}))$, then the bank will lend its excess funds to other banks in the interbank market. If the bank's funds fall between these two benchmarks, $Q(R_j^*(I,1)) \leq e_{0j} + d_{0j} \leq Q(R_j^*(I,\overline{\delta}))$, then the bank will neither lend nor borrow, but charge its customers the rate R_j necessary to make loans equal to its liquid funds.

In figure 2 we show the interbank rate I for a given expected repayment rate $\overline{\delta}$. For

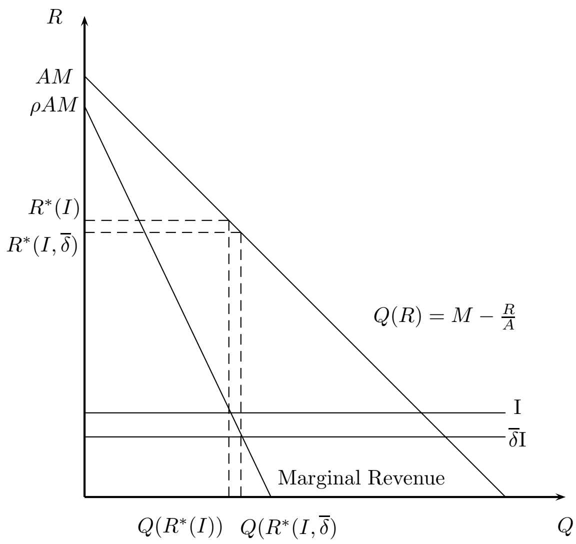

Figure 1. The optimal bank decision.

the diagram we assume that $\frac{M}{2} > e_{0j} + d_{0j} > 0$ holds.

2.3 Resolving Bank Insolvencies: The Clearing System

In each period, banks may borrow or lend in the interbank market. Aggregate claims and liabilities are denoted by $L^+ := \sum_{j \in \mathcal{J}} l_j^+$ and $L^- := \sum_{j \in \mathcal{J}} l_j^-$, respectively. The interest rate I adjusts to clear the market, $L^+ - L^- = 0$

At the end of the period $t = 1$, when shocks $s \in \mathcal{S}$ are realized, situations can arise where some banks suffer a negative shock, $s_j = 0$. These banks are not able to fulfill their liabilities both towards the interbank market and their depositors. Furthermore the failure of a counterparty in the interbank market can even lead to the insolvency of banks who get the full return on their loan portfolio.

We assume that debts are settled through a centralized *clearing system* that makes claims ex-post consistent by redistributing the value of insolvent institutions among its creditors. Insolvent banks exit the banking system. Banks that have invested in the interbank market have a claim on the clearing house. Banks that raise funds from the interbank market have a liability to the clearing house. The clearing house has no independent income or reserves. Hence, each insolvency of a bank implies that the clearing house can only partially fulfill its obligations. As a result of insolvencies the value of these

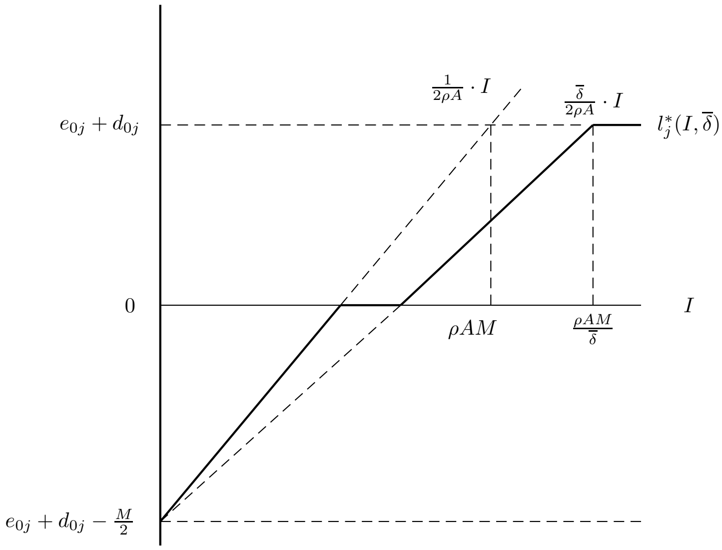

Figure 2. The optimal decision for interbank exposure.

lenders' claims must be reduced. We assume that banks with a claim on the clearing house get payed proportionally.

Given equilibrium interest rates $(I, R_1, ..., R_J)$, denote the amount of funds, which bank j has available in state s for repayment to the clearing house by $y_j(s)$. Denote their actual payment to the clearing house in state s by $p_j(s)$. Hence the total amount available to the clearing house to repay its loans is

$$P(s) = \sum_{j \in \mathcal{J}} p_j(s) \tag{9}$$

To fully specify the clearing mechanism we have to be precise about how different debt claims, which are in our case interbank liabilities and deposits, are treated in an insolvency. If deposits are senior to interbank liabilities this would mean

$$y_j(s) = \max\{s_j(s) R_j Q(R_j) - D \cdot d_{0j}, 0\} \tag{10}$$
$$p_j(s) = \min\{I \cdot l_j^-, y_j(s)\} \tag{11}$$

If we treat interbank liabilities senior to deposits[4] we have

$$\begin{aligned} y_j(s) &= s_j(s) R_j Q(R_j) \quad (12) \\ p_j(s) &= \min\{I \cdot l_j^-, y_j(s)\} \quad (13) \end{aligned}$$

Under both approaches an equilibrium will exists (see proof of proposition 3) and we can adopt whichever assumption the insolvency law imposes. For our analysis it is immaterial which clearing procedure we choose. We therefore work with the assumption which leads to the simplest possible analysis of the model. This is achieved by

Assumption 1 : Seniority of liabilities.

Interbank liabilities are senior to demand deposits.

Under assumption (1), given equilibrium interest rates $(I, R_1, ..., R_J)$, the maximal amount of funds, which banks have available for repayment to the clearing house, consists of the repayments from their loans. The funds $y_j(s)$ of bank j available in state s for repayment to the clearing house are given by (13). Denote the set of banks which borrowed from the interbank market by $\mathcal{J}^-$. These banks have a nominal debt of $I \cdot l_j^-$ to the clearing house. Their actual payment to the clearing house in state s is, however given by (13).

Hence, the clearinghouse has the amount given by (9) available for repayment of its loans $I \cdot L^+$. If all banks can settle their liabilities, $y_j(s) \geq I \cdot l_j^-$ then $P(s) = \sum_{j \in \mathcal{J}^-} p_j(s) = I \cdot L^- = I \cdot L^+$ holds and no bank will be insolvent. Moreover, from $p_j(s) \leq I \cdot l_j^-$ one concludes $P(s) \leq I \cdot L^+$.

The aggregate funds of the clearing house, $P(s)$, will be paid back proportionally to all lenders in the interbank market:

$$\delta(s) := \frac{P(s)}{I \cdot L^+}. \quad (14)$$

If in state $s \in \mathcal{S}$ there is no default, then the pro rata share $\delta(s) = 1$ obtains. In all other cases, the effective rate for borrowing in the interbank market I exceeds the lending rate $\delta(s) \cdot I$.

By absolute priority of debt, (interbank debt and deposits) and the limited liability of bank owners the value of bank j's equity $e_{1j}(s)$ at $t = 1$, equals

$$e_{1j}(s) = \max\left\{0,\ y_j(s) + \delta(s) \cdot I \cdot l_j^+ - p_j(s) - D \cdot d_{0j}\right\}. \quad (15)$$

[4] A complete list of cases would also allow to treat interbank liabilities and deposits in the same seniority class

The value of e_{1j} is bank j's new equity at $t = 1$. Banks with $e_{1j} = 0$ are insolvent and exit the banking system.

In the case of centralized clearing and indirect exchange via the clearing house, the clearing problem is very transparent. From (13) and (9) it follows immediately that we can always find a unique clearing vector $(p_1(s), ..., p_J(s), P(s))$ of payments. The vector of clearing payments is concave as the minimum function is a concave function. If we had a network of mutual obligations, where direct claims exist among individual banks, then the clearing problem would be more complex. Eisenberg and Noe (2001) provide such an analysis and characterize the unique clearing vector which satisfies *proportionality* and *absolute priority*. Our clearing house is a special case of the clearing system analyzed by Eisenberg and Noe (2001).

Assumption 1 has also implications for the average discount rate. We summarize this implication by

Lemma 1 : Average discount rate.

Given Assumption 1, if banks choose $(R_j^*, Q_j(R_j^*), l_j^*)$ *optimally, then*

$$\overline{\delta} = \sum_{s \in \mathcal{S}} \pi(s) \cdot \delta(s) = \rho$$

Proof: See Appendix.[5]

3 Temporary Equilibrium

A *temporary equilibrium* of this model is defined by a profile of optimal bank loan rates R_j^*, loan demands $Q(R_j^*)$ induced by the critical productivity threshold q_j^*, interbank positions l_j^*, and a market clearing interbank rate I^* such that all banks maximize their expected profit and the interbank market clears. In addition, we require expectations of firms about the repayment rate on interbank loans $\overline{\delta}$ to be consistent with the actual repayment rates realized by the clearing house. Formally, we state the following definition of a temporary equilibrium.

[5] Lemma 1 depends on the seniority rule of Assumption 1. For an optimal choice $(R_j^*, Q_j(R_j^*), l_j^*)$, we have $R_j^* \cdot Q_j(R_j^*) - I \cdot l_j^{*-} \geq 0$. This guarantees that $y_j(s)$ is either zero or $I \cdot l_j^{*-}$. If deposits had priority over interbank liabilities, then this result would no longer hold.

Definition 1 ***Temporary Equilibrium.***

A temporary equilibrium is a list $((R_1^*, ..., R_J^*), I^*)$ *of interest rates, a list* $\big((l_j^*, q_j^*)_{j\in\mathcal{J}},\big)$ *of interbank positions, loan demands and repayment quotas* $(\delta^*(s))_{s\in\mathcal{S}}$ *such that:*

1. *For each* $j \in \mathcal{J}$

$$q_j^* = \max\{q_j \in [0, M] \mid \Pi(R_j^*, q_j) = 0\}.$$

2. *For each* $j \in \mathcal{J}$

$$(R_j^*, l_j^*) = \underset{(R_j,l_j)\in\mathcal{B}_j(e_{0j},d_{0j})}{\operatorname{argmax}} V(R_j, l_j^-, l_j^+ | I^*, \overline{\delta}^*, D).$$

3. *The interbank market clears,*

$$\sum_{j\in\mathcal{J}} (l_j^{+*} - l_j^{*-}) = 0.$$

4. *Expectations about repayment are rational,*

$$\overline{\delta}^* = \sum_{s\in\mathcal{S}} \pi(s) \cdot \delta^*(s)$$

In a temporary equilibrium banks choose their loan policies and their interbank position optimally. Simultaneously, firms decide on their optimal investment behavior. Finally the interbank rate has to be set such that the interbank positions of the banks clear. Notice that, in a temporary equilibrium, expectations about the repayment ratio from the interbank market are correct.

Existence of a non-trivial temporary equilibrium requires some additional assumptions about the distribution of the banks' funds, $(e_{0j} + d_{0j})_{j\in\mathcal{J}}$, relative to the maximal private demand for loans $J \cdot \frac{M}{2}$. Clearly, if the banks' total funds $\sum_j (e_{0j} + d_{0j})$ would exceed the maximal demand for loans from the firms, $J \cdot \frac{M}{2}$, then there would be excess lending to the interbank market, which would meet no borrowers. Hence, the market equilibrium rate would be zero, $I = 0$. In addition, we need some assumption about the dispersion of the banks' funds in order to rule out no trade equilibria, i.e., situations where the optimal position of all banks would be zero for an interest rate. Such a situation would be an equilibrium, but an equilibrium without trade. The following assumption rules out these cases.

Assumption 2 ***Distribution of initial funds***

$$\begin{array}{llcc} \textit{(i)} & \sum_j (e_{0j} + d_{0j}) & < & J \cdot \frac{M}{2} \\ \textit{(ii)} & \overline{e}_0 + \overline{d}_0 & \geq & \frac{M}{2} \end{array}$$

where $\overline{e}_0 + \overline{d}_0 = \max\{e_{0j} + d_{0j} | \; j \in \mathcal{J}\}$.

The first condition in 2 guarantees that there will be an excess demand for interbank loans for interest rates which are low enough. From the second part of assumption 2 we know, that there is at least one bank that has enough funds to satisfy its customers loan demand and still funds available to lend in the interbank market. This is a sufficient condition for uniqueness of equilibrium.[6]

Proposition 3 ***Existence of Temporary Equilibrium***
Given Assumptions 2 and 1 there exists a unique $(I^*, \overline{\delta}^*) \in \mathbb{R}_{++} \times (0,1]$ *such that* $\sum_{j=1}^{J} l_j^*(I^*, \overline{\delta}^*) = 0$. *Furthermore, in a temporary equilibrium* $0 < I^* < \rho MA$ *and* $\overline{\delta}^* > 0$.

Proof: See Appendix.

The temporary equilibrium allocation and rates enter the clearing system. Once shocks have been realized, they determine the value of the banks, which is their equity in period $t = 1$. The equity positions e_1 open a new stage of bank activities and interbank transactions. The following example illustrates the temporary equilibrium concept and the clearing system.

Example 1 ***: A Simple Two Bank System***
Assume that we have two banks, thus $\mathcal{J} = \{1, 2\}$. *Let* $M = 2$, $A = 1$ *and* $\rho = \frac{7}{8}$. *The initial equity and deposit positions are given by* $(e_{01}, d_{01}) = (0.12, 0.1)$ *and* $(e_{02}, d_{02}) = (0.08, 0.575)$. *Let the (gross) deposit rate be given by* $D = 1.3$. *It is easy to check that the following values form a temporary equilibrium:*

[6] To show only existence, this assumption can be dropped. If assumption 2-(ii) is not fulfilled an equilibrium will exists but it is not neccessarily unique.

Bank 1:	$(R_1^*, Q_1^*, l_1^*) = (1.6, 0.4, -0.18)$,
Bank 2:	$(R_2^*, Q_2^*, l_2^*) = (1.525, 0.475, 0.18)$,
Interbank rate	$I^* = 1.05$,
Expected default rate:	$\overline{\delta}^* = 0.87$.

Notice that banks expect a repayment ration less than 1*. Bank 2, which has more funds available, has lower opportunity cost than bank 1. Therefore, it makes more loans to its firms than bank 1.*

The clearing problem is trivial in this case because there are only two banks. However this setup is sufficient to show the main aspects of risk exposure in this model. There are exactly four states of the world for the banking system as a whole. The following table shows these states and the actual payments to the interbank clearing system.

$s = (s_1, s_2)$	$\pi(s)$	$p_1(s)$	$p_2(s)$
$(1,1)$	0.76	0.189	0
$(1,0)$	0.11	0.189	0
$(0,1)$	0.11	0	0
$(0,0)$	0.02	0	0

In states $(1,1)$ *and* $(1,0)$*, the repayment ratio is* $\delta(1,1) = \delta(1,0) = 1$*, while it is zero in the other states. Hence,*

$$\overline{\delta}^* = \sum_{s \in S} \pi(s) = 0.87$$

which confirms that banks have rational expectations about the repayment.

Banks bear risk by the direct exposure to the risk factors $s_j(s)$ *and by indirect exposures to those risk factors via interbank linkages. Thus both sources of systemic risk, common exposures and domino effects are present. The risk factors in our example are shocks to technology. If both firm groups are successful* $(1,1)$*, an event that occurs with probability* 0.76*, banks have no problem. Bank* 1 *, the interbank borrower, gets a net profit after paying back its interbank liabilities and his liabilities towards depositors of* $e_1 = 0.321$ *whereas the second bank makes a profit of* $e_2 = 0.165$*. These are the funds that the owners of both banks could provide as equity in period* $t = 1$*. In state* $(0,0)$*, where both firm groups fail, an event that occurs with probability* 0.02*, both banks also fail. Bank* 1 *has no income and can neither pay its depositors nor its interbank liabilities and therefore bank* 2 *has no income as well.*

The remaining states, $(1,0)$ *and* $(0,1)$ *are both critical for bank* 2*. In the first case bank* 1 *earns enough revenue from its loan portfolio to pay back its interbank liabilities. The full repayment which bank* 2 *obtains from its interbank lending in this case is, however,* *not sufficient to pay off its depositors and it fails as a direct consequence of the* shock to its loan portfolio.

In the second case bank 1 *fails and defaults on its interbank loans. The firms that have borrowed from bank* 2 *are successful. However its losses on its interbank assets leave bank* 2 *with a negative net value. Thus, bank* 2 *indirectly fails by a* domino effect *that occurs through the network of interbank linkages.*

Note that under slightly more favorable conditions bank 2 *would be able to withstand the shock from the default of its counterparty. If the deposit rate were lower, say at* $D = 1.25$, *the bank would not become insolvent. It could survive because the revenues generated from the loan portfolio would be large enough to accommodate the default of its interbank creditor.*

4 Financial Stability and Capital Adequacy

For more than two decades the debate about banking regulation has been dominated by proposals for the design and refinement of capital adequacy rules. Capital adequacy regulation requires banks to hold risky assets in a fixed proportion to their equity. These proportions may of course differ across asset risk classes. Indeed, recent regulatory developments are mainly concerned with adequate refinements of capital requirements according to the varying risks associated with different assets of a bank.

Hellwig and Blum (1995) review the two main rationales for this instrument of banking regulation. According to one view, the protection of depositors is enhanced by setting aside sufficient equity as a buffer for shocks. A second view concentrates on the incentive effects of the capital structure. Capital adequacy is seen as an instrument which reduces moral hazard incentives of bank owners arising from the difficulties depositors face controlling the investment policies of banks (Freixas and Rochet (1997)). Both arguments have been discussed in Hellwig and Blum (1995) in great detail.

The model of Section 2 provides a tool for analyzing the buffer stock argument. Risky assets of a bank fall into one single class, the loan portfolio, while interbank loans, the second class of assets considered, bear no direct risk. Interbank loans are however subject to the indirect effect of defaults on other banks' risky assets.

In a temporary equilibrium, the banks' budget constraints are binding. Defaults of debtor banks on their loans in the interbank market will reduce the returns of creditor banks. Such losses may trigger further defaults as illustrated in Example 1. In this section, we will investigate whether capital adequacy requirements can protect the banking system from contagious defaults. Moreover, we will asses the cost of such regulation in terms of its effect on the investment behavior of firms.

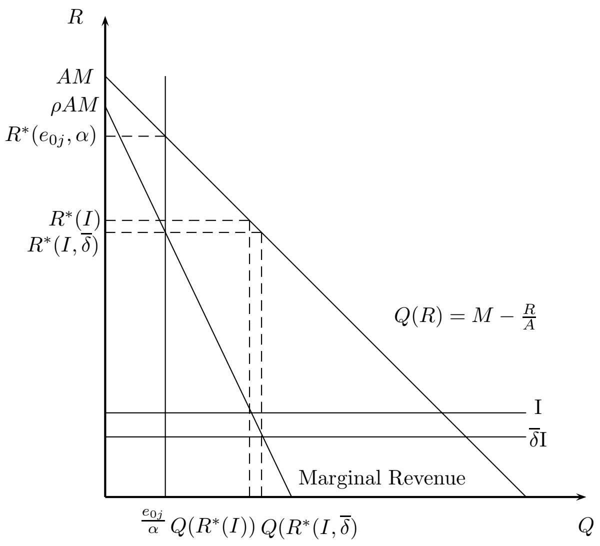

Figure 3. The optimal bank decision with capital adequacy.

4.1 Temporary Equilibrium with Capital Adequacy

Capital adequacy regulation determines the percentage α of the loan portfolio that has to be covered by equity[7]:

$$\alpha \cdot Q_j(R_j) \leq e_{0j}. \tag{16}$$

Maximizing $V(R_j, l_j^-, l_j^+|I, \overline{\delta}, D)$ subject to the budget constraint (5) and the capital adequacy constraint (16), one obtains the optimal bank decision with a capital adequacy constraint. Figure 3 illustrates the optimal bank decision in case of a binding capital constraint.

[7]We would like to acknowledge discussions with Eva Terberger which helped us shape the capital adequacy constraint.

Proposition 4 ***Optimal Bank Decision with a Capital Adequacy Constraint*** *Define with* $I_j^0 := 2\rho A \cdot \max\{0, \frac{M}{2} - (e_{0j} + d_{0j})\}$. *For each* $I \in \mathbb{R}_{++}$*, each* $\overline{\delta} \in [0,1]$ *and each* $\alpha \in [0,1)$ *the bank problem has a unique optimum* $(R_j^*(I, \overline{\delta}|\alpha), l_j^*(I, \overline{\delta}|\alpha))$ *given by:*

$$\begin{aligned}
&\textit{For } I \leq I_j^0 : \\
&R_j^*(I,\overline{\delta}|\alpha) = \max\left\{\frac{MA}{2} + \frac{I}{2\rho}, A\left(M - \frac{e_{0j}}{\alpha}\right)\right\} \\
&l_j^*(I,\overline{\delta}|\alpha) = (e_{0j} + d_{0j}) - \min\left\{\frac{M}{2} - \frac{I}{2\rho A}, \frac{\alpha - 1}{\alpha} e_{0j} + d_{0j}\right\} \\
&\textit{For } I_j^0 \leq I \leq \tfrac{1}{\overline{\delta}} I_j^0 : \\
&R_j^*(I,\overline{\delta}|\alpha) = \max\left\{\frac{MA}{2} + \frac{I_j^0}{2\rho}, A\left(M - \frac{e_{0j}}{\alpha}\right)\right\} \\
&l_j^*(I,\overline{\delta}|\alpha) = (e_{0j} + d_{0j}) - \min\left\{\frac{M}{2} - \frac{I_j^0}{2\rho A}, \frac{\alpha - 1}{\alpha} e_{0j} + d_{0j}\right\} \\
&\textit{For } \tfrac{1}{\overline{\delta}} I_j^0 \leq I : \\
&R_j^*(I,\overline{\delta}|\alpha) = \min\left\{\max\left\{\frac{MA}{2} + \frac{\overline{\delta} \cdot I}{2\rho}, A\left(M - \frac{e_{0j}}{\alpha}\right)\right\}, MA\right\} \\
&l_j^*(I,\overline{\delta}|\alpha) = \min\left\{(e_{0j} + d_{0j}) - \min\left\{\frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A}, \frac{\alpha - 1}{\alpha} e_{0j} + d_{0j}\right\}, (e_{0j} + d_{0j})\right\}
\end{aligned}$$

Proof: See Appendix.

From Proposition 4 one sees immediately that the loan demand function $l_j^*(I, \overline{\delta}|\alpha)$ is continuous in all its variables. Hence, an equilibrium exists as in Proposition 3 provided there is excess demand for interbank loans at an interest rate $I = 0$. At a high capital adequacy requirement, say of $\alpha = 1$, no bank would want to finance loans to its customers by loans from the interbank market. Hence, in order to guarantee existence of a temporary equilibrium, the capital adequacy constraint α must not be too tight:

$$\alpha \leq \overline{\alpha} := \frac{\sum_j e_{0j}}{\sum_j (e_{0j} + d_{0j})}.$$

With this constraint on the tightness of capital adequacy regulation, which we will assume to hold throughout this paper, existence of a temporary equilibrium is guaranteed. For a given capital adequacy regulation policy $\alpha \leq \overline{\alpha}$, we denote the temporary equilibrium with capital adequacy by

$$\left(\left(R_j^*(\alpha), l_j^*(\alpha), q^*(\alpha)\right)_{j \in \mathcal{J}}, I^*(\alpha), \overline{\delta}^*(\alpha)\right).$$

4.2 Direct and Indirect Consequences of Capital Adequacy Regulation

How does capital adequacy affect equilibrium? The explicit description of heterogeneous banks and the interbank linkages allow us to analyze direct and indirect consequences of the regulatory policy.

The immediate impact of a capital adequacy constraint can be deduced from Proposition 4. If the capital adequacy constraint of firm j binds, then the interest rate for this bank's loans will increase, its loans to firms will decrease, and its interbank position will also increase with α,. Let us summarize these effects in the following

Proposition 5 *Let $\alpha, \alpha' \leq \overline{\alpha}$ and consider the corresponding temporary equilibria with a capital adequacy constraint. If $\alpha > \alpha'$, then*

$$\begin{aligned} R_j^*(I, \overline{\delta}|\alpha) &> R_j^*(I, \overline{\delta}|\alpha') \\ Q(R_j^*(I, \overline{\delta}|\alpha)) &< Q(R_j^*(I, \overline{\delta}|\alpha')) \\ l_j^*(I, \overline{\delta}|\alpha) &> l_j^*(I, \overline{\delta}|\alpha'). \end{aligned}$$

Proof: Follows from Proposition 4.

These effects capture the logic of the regulation. Capital adequacy constraints reduce exposure to the risk of the regulated asset. Notice that, for a binding constraint, these effects depend neither on the interbank refinance rate I nor on the expected return quota $\overline{\delta}$.

A capital adequacy constraint limits the amount of bank capital invested in a particular asset. Since it does not specify an alternative investment, it is neither a reserve requirement nor a buffer guaranteeing that the bank can satisfy its creditors from the returns on its equity. The degree of protection achieved with a capital adequacy constraint depends crucially on the returns which banks can earn on other investments. Returns on other assets will be affected by the investment decisions of banks in response to a binding capital adequacy constraint. Systemic effects on the returns of other assets are often neglected, when the effectiveness of regulation is assessed. Such systemic effects may lead us to reconsider the aptitude of capital adequacy regulation as a tool for improvement of a banks' risk exposure.

In our model, the only alternative investment opportunity of banks is the interbank market. Equation 5 shows that a capital-adequacy constrained bank will increase its position in the interbank market, i.e., increase its lending in this market or decrease its borrowing from it. Though the interbank market is not subject to direct risk from shocks, it may be indirectly affected if banks fail in response to bank-specific shocks. This was demonstrated in Example 1 where a bank, which did not suffer from a shock to its loan portfolio, had to default on deposits because of its exposure to the interbank market.

Secondary effects of changed positions in the interbank market following capital adequacy regulation need to be studied. We can show that the interbank market rate will fall in consequence of a higher capital adequacy constraint.

Proposition 6 ***Capital Adequacy and the Interbank Market***

Consider a temporary equilibrium with binding capital adequacy constraints for some firms. If the capital adequacy ratio α is increased, then the interbank interest rate $I^*(\alpha)$ falls. Formally, $\alpha' > \alpha$ implies

$$I^*(\alpha') < I^*(\alpha).$$

Proof: See Appendix.

Lower interbank rates induce unconstrained banks to increase loans to their firms. Hence, capital adequacy regulation may increase, rather than decrease, systemic risk in the banking system.

If the capital adequacy constraint is binding for banks that are interbank borrowers, then these banks have to limit loans to their firms and, consequently, reduce their borrowing from the interbank market. Banks that are facing a binding constraint and who are interbank lenders will offer more loans in the interbank market. Thereby they increase their exposure to defaults from other banks. The excess supply of funds in the interbank market will drive down the interbank loan rate. Lower interbank interest rates provides an incentive for banks, which are not constrained by the capital adequacy regulation, to extend lending to their firms. For these banks the risk from other banks' failures will increase.

The consequences capital adequacy regulation for the risk exposure of the banking system as a whole are therefore not clear. The interbank market allows banks to shift risk from banks with little equity to banks with more equity. Whether this reallocation of funds among banks increases the financial stability of the banking system depends on the distribution of deposits. The following example illustrates this point.

Example 2 ***Capital Adequacy can increase Systemic Risk***
Assume again that we have two banks. Let $M = 2$, $A = 1$, $\rho = 0.979$ and $\alpha = 0.08$. The initial equity and deposit positions are given by $(e_{01}, d_{01}) = (0.024, 0.462)$ and $(e_{02}, d_{02}) = (0.1, 0.2)$. Let the deposit rate be given by $D = 1.3$. The following table contains the equilibrium values for the case without capital adequacy regulation.

Bank 1:	$(R_1^*(0), Q_1^*(0), l_1^*(0)) = (1.600, 0.4, 0.087)$,
Bank 2:	$(R_2^*(0), Q_2^*(0), l_2^*(0)) = (1.613, 0.387, -0.087)$,
Interbank rate	$I^*(0) = 1.20$,
Expected default rate:	$\overline{\delta}^*(0) = 0.979$.

It is easy to check that, at these equilibrium values, bank 1 *would not fulfill its capital adequacy constraint.*
If a bank's loan portfolio suffers a shock the bank will have no funds to repay its loans. Bank 2 *who is borrowing from the interbank market will fail in states* $(1, 0)$ *and* $(0, 0)$ *and pay back its full loan plus interest in the other states. The settlement payments* $p_i(s)$ *are listed in the following table.*

$s = (s_1, s_2)$	$\pi(s)$	$p_1(s)$	$p_2(s)$
$(1, 1)$	0.960	0	0.104
$(1, 0)$	0.019	0	0
$(0, 1)$	0.019	0	0.104
$(0, 0)$	0.002	0	0

In states $(1, 1)$ *and* $(0, 1)$, *the repayment ratio is* $\delta(1, 1) = \delta(0, 1) = 1$, *while it is zero in the other states. Hence,*

$$\overline{\delta}^* = \sum_{s \in S} \pi(s) = 0.979 \tag{17}$$

as the banks expected when making their decisions.

If we look at the four states of the system we see that in case of both firm groups being successful both banks will be able to honor their promises and to pay off their depositors. If both firm groups are in a bad state, clearly both banks fail. Note that bank 1 earns enough from its loan portfolio, $R^*(0) \cdot Q^*(0) = 0.64$ *in order to repay depositors,* $D \cdot d_{01} = 0.6$, *even if bank 2 were to default on its interbank promises. Although its profits would decline sharply as a consequence of such a bank failure, bank 1 would not become insolvent. On the other hand, if the loan portfolio of bank 1 fails, it will become insolvent even if bank 2 fully honors all its interbank promises. Hence, without a capital adequacy constraint, there are no contagious defaults. Banks fail only in direct consequence of their loan portfolio's risk.*

Consider now the case where the capital adequacy regime becomes effective. In this case bank 1 *hits its capital constraint. The new equilibrium can be calculated using proposition 4 and the definition of a temporary equilibrium. The equilibrium values for* $\alpha = 0.08$ *are listed in the following table.*

Bank 1:	$(R_1^*(\alpha), Q_1^*(\alpha), l_1^*(\alpha)) = (1.7, 0.3, 0.186)$,
Bank 2:	$(R_2^*(\alpha), Q_2^*(\alpha), l_2^*(\alpha)) = (1.514, 0.486, -0.186)$,
Interbank rate	$I^*(\alpha) = 1.00$,
Expected default rate:	$\overline{\delta}^*(\alpha) = 0.979$.

The expected default rate remains unchanged, because $p_2(s)$ *will again equal* 1, *in those states where bank 2's loan portfolio pays off, and* 0 *in the other states.*

While bank 1 *was able to absorb the failure of its interbank counterparty in the regime without capital adequacy* ($\alpha = 0$)*, it falls victim of a contagion effect in the new regime with* $\alpha = 0.08$*. At this capital adequacy rate* $R^*(\alpha) \cdot Q^*(\alpha) = 0.51$ *do not suffice to pay back its depositors,* $D \cdot d_{01} = 0.6$*. The probability of observing this domino effect is* $\pi(1, 0) = 0.019$ *with an capital adequacy constraints, while it is nil without such regulation. Hence, capital adequacy increases systemic risk.*

Example 2 illustrates the mechanism of risk transmission through the interbank market. Capital adequacy constraints set bank funds free for loans to other banks. These extra funds lower the interest rate I, in Example 2 from $I(0) = 1.2$ to $I(\alpha) = 1$. The cheaper funds of the interbank market induce unconstrained banks to lend more to their group of firms, $Q_2^*(\alpha) = 0.486 > 0.387 = Q_2^*(0)$. As a consequence, a failure of bank 2 will create a greater loss for the banking system. Bank 1 loses more funds in the interbank market and, because of the capital adequacy constraint, it receives less returns from lending to its group of firms. Thus, it can no longer honor its commitments to its depositors.

Systemic risk works through the interbank market which allows banks to exchange surplus funds. Constraining lending activities of banks by a capital adequacy constraint redirects funds to other banks whose constraints are not binding. This mechanism increases the risk exposure of these banks. Capital adequacy regulation encourages lending of banks with high equity at the expense of banks with little equity. If there is no positive correlation between the amount of equity a bank holds and the quality of its loan portfolio, then the risk from such rechanneling of funds may increase systemic risk.

4.2.1 Capital adequacy and loans to the banks' firms

The falling interbank rate decreases refinancing costs for the unconstrained banks and boosts credit in these firm groups. Thus there are structural effects that lead to the expansion of certain industry sectors in the economy, while other sectors face credit restriction. The net effect on aggregate investment is unclear. The depressed profits of constrained firms may have detrimental stability consequences as we have seen in Example 2.

While it is true that capital adequacy regulation can boost the capital buffers of an individual bank relative to its risky assets, the effects on the entire banking system are ambiguous. It is not clear which risk allocation will ultimately be achieved, once the indirect effects on refinancing costs and loan interest rates are taken into account.

A problem that has been widely discussed in the academic literature and among policy makers concerns potential procyclical effects of capital adequacy regulation (see Kashyap and Stein (2004)). While our model does not describe the dynamics of credit booms or credit crunches, it can shed some light on the debate. Capital adequacy regulation restricts credit and investment in the firm groups that are financed by banks with low equity values. The overall effect of the regulation is however ambiguous, because unconstrained firm groups can expand due to the indirect effect on the interbank rate and, hence, on refinancing costs.

Banks which face a binding capital adequacy constraint and whose firms are successful will end up with positive, but lower equity value than in a situation without regulation. Thus, in the following period, they are likely to be constrained again. Hence, capital adequacy constraints affects also the capacity of banks to build up equity value. This problem is most relevant for banking crises when capital constraints becomes binding for many banks simultaneously by adverse economic conditions. In such a situation many regulatory constraints are simultaneously hit and the interbank market may collapse [8]. A rigorous analysis of these issues requires a richer dynamic extension of our model.

4.2.2 Robustness of our results

Whether one considers the systemic risk displayed in our model as a relevant feature of actual banking systems depends on the acceptability of the assumptions of the model.

[8]For the proof of existence, this case was ruled out by Assumption 3.1 (ii).

There are a number of assumptions which serve only the purpose of a simpler exposition. For example, it would be possible to consider differing characteristics of the banks' groups of firms. One could also allow firms to seek funds from other banks at some extra cost or by selling some debt, as long as banks retain some comparative advantage as loan providers for their own groups of firms. In fact, empirical studies suggest that firms cannot easily substitute other sources of finance for bank credit because of asymmetric information (Freixas and Rochet (1997)) and credit constraints (Hoshi, Kahiap, and Scharfstein (1991)). Moreover, in many countries bank finance is the most important sources of external finance for firms.

We have refrained from generalizing firms' behavior, because it is inessential for the point of systemic risk through the interbank market which forms the core of our paper. The effect of capital adequacy regulation in this paper rests on two features of our model which we wish to discuss in more detail.

1. Other assets. Would systemic risk via the interbank transactions remain as important, if banks had other investment opportunities than loans to firms or interbank lending? In particular, if banks could hold a safe asset as reserves for losses. We believe that the answer to this question is yes.

If there was a completely safe asset which banks could hold as reserve, it would necessarily have to pay a lower return as lending in the interbank market. With risk-neutral banks such an investment possibility would be dominated by lending to other banks. If banks were risk-averse, banks may decide to hold some fraction of their funds in the safe asset with inferior return. This fraction would depend on the degree of risk aversion of the banks. Even with risk aversion, however, banks would increase lending in the interbank market, if capital adequacy regulation would divert funds to this market and lower the interbank interest rate. A safe reserve asset together with risk aversion of the banks would moderate the redirection of funds through the interbank market. Yet, there is no reason to believe that it would destroy the interbank market.

Regulators may try to overcome the redirection of funds via the interbank market by imposing capital adequacy constraints also on interbank positions. By a capital adequacy constraint on both loans to firms and lending to the interbank market, they may force banks to invest funds that cannot go into their loan portfolio or the interbank market, in the safe asset. In this case, capital adequacy regulation would work like a reserve requirement. If regulation falls short of enforcing 100-percent reserve holdings for risky loans to firms, it would still be open to the systemic risk described in the previous section. Banks that would be constrained in their lending to firms, but not in their lending to the interbank market, would redirect their funds to the interbank market because of the higher returns. The additional loan supply in the interbank market lowers the interest rate and induces more lending to banks that do not face a binding capital adequacy constraint for their lending to firms. Thus, systemic risk will increase again.

2. New equity. A second important assumption is the fixed amount of bank equity.

One could imagine that, confronted with capital adequacy regulation, a bank would issue new equity in order to avoid being forced to cut back loans to its firms. Legal restrictions, which for a number of reasons make issuing new equity a complicated business in many countries, often rule out this option. Yet, even if there is no such impediment, it is questionable if such a policy would succeed. Issuing new equity tends to lower the equity value. If one interprets low equity values as a result of depressed economic conditions, then issuing new equity may suffer from adverse signaling effects (Greenbaum and Thakor (1995)). Hence, working with a fixed equity for banks appears to be reasonable assumption.

5 Conclusions

The analysis of the economic consequences of capital adequacy regulation for the financial stability of a system of banks requires a framework, which allows for heterogeneous banks, mutual credit relations and potential defaults. We have presented a simple model with these features. Our analysis shows that the effects of capital adequacy regulation for financial stability are ambiguous. While capital constraints limit credit exposures of banks with a weak equity base, the risk exposure of other institutions in the system may increase through higher risk from interbank exposures. The probability of contagious default can rise. Capital constraints have indirect effects on the allocation of aggregate funds among firms in the economy. It is not clear whether a capital adequacy regime will lead to a lower risk exposure of the banking system as a whole.

If regulation aims at the risk allocation in the entire banking system, then it has to depart from concentrating on individual bank balance sheets. A regulator who wants to use capital adequacy regulation to influence the risk exposure of the banking system finds himself in a similar position as a portfolio manager. Individual positions in a portfolio have to be judged on the basis of their contribution to the overall portfolio risk and cannot be analyzed in isolation. A system approach to banking regulation is in the beginning.[9] We hope that our model provides a useful starting point for analyzing some of the problems that arise with the regulatory control of systemic risk.

[9] Some methodological and empirical work has been done in this sense: See Lehar (2002), Elsinger, Lehar, and Summer (2002)

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Appendix: Proofs.

Proof of Proposition 1:

The payoff function $f(1, A, q) = Aq$ is strictly increasing in q. Hence, for any $R_j \leq AM$, we can find a $q^* \in [0, M]$ such that such that

$$\Pi(R_j, q^*) = 0. \tag{18}$$

Let $q^*(R_j)$ be the function implicitly defined by $\Pi(R_j, q^*(R_j)) = 0$,

$$q^*(R_j) = \min\left\{\frac{R_j}{A}, M\right\}. \tag{19}$$

Firms with quality $q \leq q^*(R_j)$ will reject the loan offer and firms with quality $q > q^*(R_j)$ will accept it. Since q is distributed uniformly on $[0, M]$, $q^*(R)$ defines the proportion of firms rejecting a loan. Hence, the aggregate loan demand function is given by

$$Q(R_j) := M - q^*(R_j). \tag{20}$$

Proof of Proposition 2:

The bank's optimization problem can be written as follows:

Choose (R_j, l_j^+, l_j^-) to maximize

$$\rho \cdot R_j \cdot \left[M - \frac{R_j}{A}\right] + \overline{\delta} \cdot I \cdot l_j^+ - I \cdot l_j^- - D \cdot d_{0j}$$

subject to

$$\begin{aligned}
(M - \frac{R_j}{A}) + l_j^+ + l_j^- &\leq e_{0j} + d_{0j} \\
(M - \frac{R_j}{A}) &\geq 0 \\
l_j^+ &\geq 0 \\
l_j^- &\leq 0
\end{aligned}$$

The constraint set is obviously closed and convex. The loan rate R_j is bounded above by AM and below by 0. Note that the interbank market acts like an income transfer technology between $t = 0$ and $t = 1$ for the individual bank. A bank can not be simultaneously long and short in the interbank market. Thus to consider boundedness of the constraint set it is sufficient to look at the boundedness of $l_j := l_j^+ + l_j^-$. By the budget constraint l_j is bounded above by $e_{0j} + d_{0j}$. and bounded below by $-(e_{0j} = (d_{0j})$ Therefore the budget set is compact. The objective function of the bank is continuous. These conditions imply the existence of an optimal solution to the bank problem. Since the objective function is strictly increasing in l_j^+ and l_j^- and concave in (R_j, l_j^+, l_j^-) the optimal solution will yield a unique maximum to the bank's objective function.

To characterize the optimal solution to the bank's problem we study the Lagrangian function and analyze first order conditions.

$$\begin{aligned} & \mathcal{L}(R_j, l_j^+, l_j^-, \lambda_j, \mu_j, \beta_j, \gamma_j) \\ = \; & \rho \cdot R_j \cdot \left[M - \frac{R_j}{A}\right] + \overline{\delta} \cdot I \cdot l_j^+ - I \cdot l_j^- - D \cdot d_{0j} \\ & +\lambda_j \left(e_{0j} + d_{0j} - (M - \frac{R_j}{A})) - l_j^+ + l_j^-\right) + \mu_j \left(M - \frac{R_j}{A}\right) \\ & +\beta_j \cdot l_j^+ - \gamma_j \cdot l_j^- . \end{aligned}$$

The first-order conditions are:

$$M\rho - \frac{2\rho R_j}{A} + \frac{\lambda_j}{A} - \frac{\mu_j}{A} = 0, \tag{21}$$
$$\overline{\delta} \cdot I - \lambda_j + \beta_j = 0, \tag{22}$$
$$-I + \lambda_j + \gamma_j = 0, \tag{23}$$
$$\lambda_j(e_{0j} + d_{0j} - (M - \frac{R_j}{A}) - l_j^+ + l_j^-) = 0, \tag{24}$$
$$\mu_j(M - \frac{R_j}{A}) = 0, \tag{25}$$
$$\beta_j \cdot l_j^+ = 0, \tag{26}$$
$$-\gamma_j \cdot l_j^- = 0. \tag{27}$$

From (22) we see that $\lambda_j = \overline{\delta} \cdot I + \beta_j > 0$. Therefore the budget constraint must be binding and from (24) we conclude that $l_j^+ + l_j^- = e_{0j} + d_{0j} - (M - \frac{R_j}{A})$. Now (22) and (23) imply that $\beta_j + \gamma_j = I \cdot (1 - \overline{\delta}) > 0$. Therefore the constraints on long and short positions can not bind simultaneously and we must have $l_j^+ \cdot l_j^- = 0$. Hence, we have to consider only three cases for possible configurations of β and γ.

Case (i): $\beta_j > 0$ and $\gamma_j = 0$.

Then $l_j^+ = 0$ and $\lambda_j = I > 0$. From the budget constraint, $l_j^- = e_{0j} + d_{0j} - (M - \frac{R_j}{A}) \leq 0$ and, therefore,

$$0 \leq e_{0j} + d_{0j} \leq (M - \frac{R_j}{A}). \tag{28}$$

From (25), we have $\mu_j \geq 0$. Hence either $\mu_j = [I - \rho AM] > 0$ and $R_j = AM$, or $\mu_j = 0$ and

$$R_j = \frac{AM}{2} + \frac{I}{2\rho}. \tag{29}$$

Thus, $R_j^*(I, \overline{\delta}) = \min\left\{\frac{MA}{2} + \frac{I}{2\rho}, MA\right\}$ and $Q(R_j^*(I, \overline{\delta})) = \max\left\{\frac{M}{2} - \frac{I}{2\rho A}, 0\right\}$. Substituting into $l_j^- = e_{0j} + d_{0j} - (M - \frac{R_j}{A})$ yields $l_j^- = e_{0j} + d_{0j} - \max\left\{\frac{M}{2} - \frac{I}{2\rho A}, 0\right\} \leq 0$. Hence, case (i) is valid for

$$0 \leq e_{0j} + d_{0j} \leq \frac{M}{2} - \frac{I}{2\rho A}. \tag{30}$$

In summary, if $\beta_j > 0$ and $\gamma_j = 0$ then the optimal solution to (21)-(22) is given by:

$$R_j^*(I, \overline{\delta}) = \min\left\{\frac{MA}{2} + \frac{I}{2\rho}, MA\right\}. \tag{31}$$

$$Q(R_j^*(I, \overline{\delta})) = \max\left\{\frac{M}{2} - \frac{I}{2\rho A}, 0\right\}. \tag{32}$$

$$l_j^- = e_{0j} + d_{0j} - \max\left\{\frac{M}{2} - \frac{I}{2\rho A}, 0\right\}. \tag{33}$$

Case (ii): $\beta_j > 0$ and $\gamma_j > 0$.

In this case, by (26) and (27) $l_j^+ = l_j^- = 0$. By (22) and (23) $I > \lambda_j > \overline{\delta} \cdot I$. From the budget constraint we have

$$R_j = A \cdot [M - e_{0j} - d_{0j}] \tag{34}$$

$$Q(R_j) = M - \frac{R_j}{A} = e_{0j} + d_{0j} > 0. \tag{35}$$

Hence, by equation (25), $\mu_j = 0$. Thus by (22) and (23), $\lambda_j = \overline{\delta} \cdot I + \beta_j = I - \gamma_j$ and equation (21), we see that this case applies if

$$\beta_j = \rho AM - 2\rho A\,(e_{0j} + d_{0j}) - \overline{\delta} \cdot I > 0 \tag{36}$$

$$\gamma_j = 2\rho A\,(e_{0j} + d_{0j}) - \rho AM + I > 0, \tag{37}$$

or

$$\frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A} > e_{0j} + d_{0j} > \frac{M}{2} - \frac{I}{2\rho A}. \tag{38}$$

In summary, if $\beta_j > 0$ and $\gamma_j > 0$ the optimal solution to (21)-(22) is given by:

$$R_j^*(I, \overline{\delta}) = A \cdot [M - e_{0j} - d_{0j}] \quad (39)$$

$$Q(R_j^*(I, \overline{\delta})) = e_{0j} + d_{0j} \quad (40)$$

$$l_j^* = 0. \quad (41)$$

Case (iii): $\beta_j = 0$ and $\gamma_j > 0$.

By (27) $l_j^- = 0$ and by (22) $\lambda_j = \overline{\delta} \cdot I > 0$. Thus by the budget constraint we know that, $l_j^+ = e_{0j} + d_{0j} - (M - \frac{R_j}{A}) > 0$.

By (25) we have $\mu_j \geq 0$.

If $\mu_j > 0$ $R_j = AM$ by (25). If $\mu_j = 0$ then by (22)

$$R_j = \frac{AM}{2} + \frac{\overline{\delta} \cdot I}{2\rho}. \quad (42)$$

Hence, $R_j^*(I, \overline{\delta}) = \min\left\{\frac{MA}{2} + \frac{\overline{\delta} \cdot I}{2\rho}, MA\right\}$ and $Q(R_j^*(I, \overline{\delta})) = \max\left\{\frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A}, 0\right\}$. Substituting into $l_j^+ = e_{0j} + d_{0j} - (M - \frac{R_j}{A})$ yields $l_j^+ = e_{0j} + d_{0j} - \max\left\{\frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A}, 0\right\} \geq 0$. Hence, this case obtains for

$$e_{0j} + d_{0j} \geq \frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A} \geq 0. \quad (43)$$

In summary, if $\beta_j = 0$ and $\gamma_j > 0$ then the optimal solution to (21)-(22) is given by:

$$R_j^*(I, \overline{\delta}) = \min\left\{\frac{MA}{2} + \frac{\overline{\delta} \cdot I}{2\rho}, MA\right\}. \quad (44)$$

$$Q(R_j^*(I, \overline{\delta})) = \max\left\{\frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A}, 0\right\}. \quad (45)$$

$$l_j^+ = e_{0j} + d_{0j} - \max\left\{\frac{M}{2} - \frac{\overline{\delta} \cdot I}{2\rho A}, 0\right\} \quad (46)$$

Proof of Lemma 1:

In any optimum of the bank $j \in \mathcal{J}$ we have $R_j^* \cdot Q^*(R_j^*) + I \cdot l_j^{*-} \geq 0$. To see this consider first the case $l_j^{*-} = 0$. In this case the inequality obviously holds. Now let $l_j^{*-} \neq 0$. Then by proposition (2) we have

$$\begin{aligned} R_j^* &= \frac{MA}{2} + \frac{I}{2\rho} \\ Q_j^* &= \frac{M}{2} - \frac{I}{2\rho A} \\ l_j^{*-} &= -\left((e_{0j} + d_{0j}) - \frac{M}{2} + \frac{I}{2\rho A}\right). \end{aligned}$$

Now we claim:

$$\begin{aligned} R_j^* \cdot Q_j^* - I \cdot l_j^{*-} &\geq 0 \\ \left(\frac{MA}{2} + \frac{I}{2\rho}\right)\left(\frac{M}{2} - \frac{I}{2\rho A}\right) &\geq -I\left((e_{0j} + d_{0j}) - \left(\frac{M}{2} - \frac{I}{2\rho A}\right)\right) \\ \left(\frac{M}{2} - \frac{I}{2\rho A}\right)\left[\left(\frac{MA}{2} + \frac{I}{2\rho}\right) - I\right] &\geq -I(e_{0j} + d_{0j}) \end{aligned}$$

which must hold in any optimum. The first expression on the right hand side is quantity, which is non negative. The expression in brackets is the difference between the loan rate and the marginal costs of borrowing funds from the interbank market, which is non negative in any optimal decision. Therefore the inequality always holds and we have indeed:

$$R_j^* \cdot Q^*(R_j^*) \geq I \cdot l_j^{*-}$$

Thus:

$$\begin{aligned} p_j(s) &= \min\{I \cdot l_j^-, y_j(s)\} \\ &= \min\{I \cdot l_j^{*-},\ s_j(s) \cdot R_j^* \cdot Q^*(R_j^*)\} \\ &= \begin{cases} I \cdot l_j^{*-} & \text{for} \quad s_j(s) = 1 \\ 0 & \text{for} \quad s_j(s) = 0 \end{cases}. \end{aligned}$$

Straightforward computations show:

$$\begin{aligned}
\overline{\delta} \quad : \quad &= \sum_{s\in\mathcal{S}} \pi(s)\cdot\delta(s) = \sum_{s\in\mathcal{S}} \pi(s)\cdot \frac{\sum_{j\in\mathcal{J}} p_j(s)}{I\cdot L^+} \\
&= \left[I\cdot \sum_{j\in\mathcal{J}} l_j^{*+} \right]^{-1} \cdot \left[\sum_{s\in\mathcal{S}} \pi(s)\cdot \sum_{j\in\mathcal{J}} p_j(s) \right] \\
&= \left[I\cdot \sum_{j\in\mathcal{J}} l_j^{*+} \right]^{-1} \cdot \left[\sum_{j\in\mathcal{J}} \sum_{s\in\mathcal{S}} \pi(s)\cdot p_j(s) \right] \\
&= \left[I\cdot \sum_{j\in\mathcal{J}} l_j^{*+} \right]^{-1} \cdot \left[\sum_{j\in\mathcal{J}} (-I\cdot l_j^{*-}) \cdot \sum_{s\in\{s|\ s_j(s)=1\}} \pi(s) \right] = \rho.
\end{aligned}$$

The last equality follows because $\sum_{j\in\mathcal{J}} l_j^{*+} - \sum_{j\in\mathcal{J}} l_j^{*-} = 0$ holds in equilibrium.

Proof of Proposition 3:

Consider the aggregate excess demand function

$$L(I,\overline{\delta}) := \sum_{j=1}^{J} \left(l_j^{+*}(I,\overline{\delta}) - l_j^{-*}(I,\overline{\delta}) \right). \tag{47}$$

The proof will proceed in two steps.

Step 1: *For every expected interbank discount rate there is a unique interbank market clearing rate.*

Case 1: $\overline{\delta} \in (0,1]$

Fix any expected interbank discount rate $\overline{\delta} \in (0,1]$ The aggregate excess demand function (47) is a continuous and non-decreasing function of I as the sum of continuous and non-decreasing functions $l_j^*(I,\overline{\delta})$ by proposition 2).

By assumption 2 (ii) there is a bank j with $e_{0j} + d_{0j} = \overline{e}_0 + \overline{d}_0 > \frac{M}{2}$. This bank's interbank position $l_j^*(I,\overline{\delta})$ is strictly increasing in I Hence, $L(I,\overline{\delta})$ must be strictly increasing as the sum of non-decreasing functions with a strictly increasing function.

We have $\lim_{I->\infty} - > \infty$ and By Assumption 2 $\lim_{I->0} - >, 0$ The intermediate value theorem then guarantees that there exists an $\widetilde{I}(\overline{\delta}) \in (0, \rho MA)$ with $L(\widetilde{I}(\overline{\delta}),\overline{\delta}) = 0$. Since $L(I,\overline{\delta})$ is strictly increasing in I by assumption 2 (ii), $\widetilde{I}(\overline{\delta})$ is unique.

case 2: $\overline{\delta} = 0$,

and define $I_j^0 := 2\rho A \cdot \max\left\{ \frac{M}{2} - (e_{0j} + d_{0j}), 0 \right\}$. For $I \leq I_j^0$, bank j's demand for interbank funds remains as before, $l_j^- := e_{0j} + d_{0j} - \frac{M}{2} + \frac{I}{2\rho A} \leq 0$. For interest rates $I > I_j^0$, however, bank j no longer wants to finance loans to its firms by borrowing from the interbank market, because it has excess liquidity, $e_{0j} + d_{0j} - \frac{M}{2} + \frac{I}{2\rho A} > 0$. Because $\overline{\delta} = 0$ holds, these banks are indifferent between holding the excess liquidity and lending it to the interbank market. Formally, for $\overline{\delta} = 0$, the optimal interbank position is a correspondence, rather than a function. Consider the following continuous selection from this correspondence,

$$l_j^*(I, 0) = \begin{cases} e_{0j} + d_{0j} - \frac{M}{2} + \frac{I}{2\rho A} & \text{for} \quad I \leq I_j^0 \\ e_{0j} + d_{0j} - \frac{M}{2} + \frac{I_j^0}{2\rho A} & \text{otherwise} \end{cases}.$$

The function $l_j^*(I, 0)$ is continuous. Moreover, $\lim_{I\to\infty} L(I, 0) > 0$ by Assumption 2 (ii). $L(0, 0) := \sum_{j=1}^J l_j^*(0, 0) = \sum_{j=1}^J \left[e_{0j} + d_{0j} - \frac{M}{2}\right] < 0$ by Assumption 2 (i) and $L(\rho MA, 0) := \sum_{j=1}^J l_j^*(\rho MA, 0) \geq \overline{e}_0 + \overline{d}_0 - \frac{M}{2} > 0$ by Assumption 2 (ii). Hence, there exists $\widetilde{I}(0) \in (0, \rho MA)$ such that the interbank market clears, $L(\widetilde{I}(0), 0) := \sum_{j=1}^J l_j^*(\widetilde{I}(0), 0) = 0$.

Step 2: *Existence of rational expectations equilibrium*

If Assumption 1 holds we know from Lemma 1 that

$$D(\overline{\delta}) := \sum_{s\in S} \pi(s) \cdot \frac{P(s|\overline{\delta})}{\widetilde{I}(\overline{\delta}) \cdot L^+(\overline{\delta})} = \rho,$$

Therefore there is a unique discount factor compatible with a temporary equilibrium and therefore equilibrium is unique.

To show that existence of equilibrium depends not on the seniority structure of liabilities consider a solution $\widetilde{I}(\overline{\delta})$ to

$$L(\widetilde{I}(\overline{\delta}), \overline{\delta}) = 0$$

and consider a sequence $\overline{\delta}^\nu$ in $[0, 1]$. Since $\overline{\delta}^\nu$ is bounded it contains a monotone subsequence, which is convergent. Let $\overline{\delta}^{\nu_k}$ denote this subsequence and δ^0 its limit. Since $\widetilde{I}(\overline{\delta}^{\nu_k})$ exist for all k by step (i) $\lim_{k\to\infty} \overline{\delta}^{\nu_k} -> \delta^0$ implies $\lim_{k\to\infty} \widetilde{I}(\overline{\delta}^{\nu_k}) \to \widetilde{I}(\delta^0)$ $\widetilde{I}(\overline{\delta})$ is continuous.

For all $s \in \mathcal{S}$ and all $j \in \mathcal{J}$, the function

$$y_j(s|\overline{\delta}) = \max\left\{0, \left[s_j(s) \cdot R_j^*(\widetilde{I}(\overline{\delta}), \overline{\delta}) \cdot Q(R_j^*(\widetilde{I}(\overline{\delta}), \overline{\delta}))\right] - D \cdot d_{0j}\right\}$$

is a continuous function of $\overline{\delta}$, as a composition of continuous functions. Similarly,

$$p_j(s|\overline{\delta}) = \min\{-\widetilde{I}(\overline{\delta}) \cdot \min\{l_j^*(\widetilde{I}(\overline{\delta}), \overline{\delta}), 0\}, y_j(s|\overline{\delta})\}$$

is a continuous function of $\overline{\delta}$. Finally,

$$L^+(\overline{\delta}) := \sum_{j\in J} \max\{l_j^*(\widetilde{I}(\overline{\delta}), \overline{\delta}), 0\}$$

and

$$P(s|\overline{\delta}) = \min\left\{\sum_{j\in J} p_j(s|\overline{\delta}),\ \widetilde{I}(\overline{\delta})\cdot\sum_{j\in J}\max\{l_j^*(\widetilde{I}(\overline{\delta}),\overline{\delta}),0\}\right\}$$

are continuous functions, as the sum and maximum or minimum of continuous functions.

By Assumption 2 (ii), for all $\overline{\delta}\in[0,1]$,

$$L^+(\overline{\delta}) = \sum_{j\in J}\max\{l_j^*(\widetilde{I}(\overline{\delta}),\overline{\delta}),0\} \geq \overline{e}_0 + \overline{d}_0 - \frac{M}{2} > 0$$

and, as shown in part (i) of this proof, $\widetilde{I}(\overline{\delta}) > 0$. Hence, we have $\widetilde{I}(\overline{\delta})\cdot L^+(\overline{\delta}) > 0$ for all $\overline{\delta}\in[0,1]$.

Finally, observe that, for all $\overline{\delta}\in[0,1]$,

$$0 \leq P(s|\overline{\delta}) \leq \widetilde{I}(\overline{\delta})\cdot\sum_{j\in J}\max\{l_j^*(\widetilde{I}(\overline{\delta}),\overline{\delta}),0\} = \widetilde{I}(\overline{\delta})\cdot L^+(\overline{\delta}).$$

We can conclude that the function

$$D(\overline{\delta}) := \sum_{s\in S}\pi(s)\cdot\frac{P(s|\overline{\delta})}{\widetilde{I}(\overline{\delta})\cdot L^+(\overline{\delta})},$$

defined by the interbank market clearing mechanism, is a continuous function $D:[0,1]\to[0,1]$. Invoking the intermediate value theorem again, $D(\overline{\delta})$ has a fixed point, $\overline{\delta}^* = D(\overline{\delta}^*)$. Moreover, $\overline{\delta}^* > 0$, since

$$\sum_{s\in S}\pi(s)\cdot P(s|0) > 0$$

and, therefore, $D(0) > 0$. Without any further information about the monotonicity of $D(\overline{\delta})$ we don't know however whether this fixed point will be unique.

Proof of Proposition 4:

Existence and uniqueness of a solution to the bank decision problem with a capital adequacy constraint follows from the same arguments as in the proof of Proposition 2. What remains to be shown is the characterization of the optimal solution.

In this case, the Lagrangian is

$$\begin{aligned}
& \mathcal{L}(R_j, l_j^+, l_j^-, \lambda_j, \mu_j, \varphi_j, \beta_j, \gamma_j) \\
= \; & \rho \cdot R_j \cdot \left[M - \frac{R_j}{A} \right] + \overline{\delta} \cdot I \cdot l_j^+ - I \cdot l_j^- - D \cdot d_{0j} \\
& + \lambda_j \left(e_{0j} + d_{0j} - (M - \frac{R_j}{A})) - l_j^+ + l_j^- \right) + \mu_j \left(M - \frac{R_j}{A} \right) \\
& + \varphi_j \left(e_{0j} - \alpha (M - \frac{R_j}{A}) \right) \\
& + \beta_j \cdot l_j^+ - \gamma_j \cdot l_j^- .
\end{aligned}$$

The first-order conditions are:

$$M\rho - \frac{2\rho R_j}{A} + \frac{\lambda_j}{A} - \frac{\mu_j}{A} + \frac{\alpha \varphi_j}{A} = 0, \quad (48)$$

$$\overline{\delta} \cdot I - \lambda_j + \beta_j = 0, \quad (49)$$

$$-I + \lambda_j + \gamma_j = 0, \quad (50)$$

$$\lambda_j (e_{0j} + d_{0j} - (M - \frac{R_j}{A}) - l_j^+ + l_j^-) = 0, \quad (51)$$

$$\mu_j (M - \frac{R_j}{A}) = 0, \quad (52)$$

$$\varphi_j \left(e_{0j} - \alpha (M - \frac{R_j}{A}) \right) = 0 \quad (53)$$

$$\beta_j \cdot l_j^+ = 0, \quad (54)$$

$$-\gamma_j \cdot l_j^- = 0. \quad (55)$$

If the capital adequacy constraint is not binding, $e_{0j} > \alpha(M - \frac{R_j}{A})$, then $\varphi_j = 0$ and the optimal solution is characterized by Proposition 2.

Assume that the capital adequacy constraint is binding then $\varphi_j > 0$ and $e_{0j} - \alpha(M - \frac{R_j}{A}) = 0$. Hence, $R_j = A\left(M - \frac{e_{0j}}{\alpha}\right)$, $Q(R_j) = \frac{e_{0j}}{\alpha}$ and $l_j = \frac{(\alpha-1)}{\alpha} e_{0j} + d_{0j}$.

The case $\varphi_j > 0$ obtains, if $\frac{MA}{2} + \frac{I}{2\rho} \leq A\left(M - \frac{e_{0j}}{\alpha}\right)$ and $l_j = \frac{(\alpha-1)}{\alpha} e_{0j} + d_{0j} < 0$ or $\frac{MA}{2} + \frac{\overline{\delta} \cdot I}{2\rho} \leq A\left(M - \frac{e_{0j}}{\alpha}\right)$ and $l_j = \frac{(\alpha-1)}{\alpha} e_{0j} + d_{0j} > 0$ holds.

Proof of Proposition 6:

By Lemma 1 $\overline{\delta}^*(\alpha) = \rho$ for all α.

Assume that the capital adequacy constraint is binding for at least one bank $j \in \mathcal{J}$. From Proposition 4 it is clear that $l_j^*(I, \overline{\delta}|\alpha)$ is a non-decreasing function of α. Moreover, if the capital adequacy constraint is binding for bank j, then $l_j^*(I, \overline{\delta}|\alpha)$ is strictly increasing in α. Hence, the aggregate excess supply function $L(I, \overline{\delta}|\alpha) := \sum_{j \in \mathcal{J}} l_j^*(I, \overline{\delta}|\alpha)$ must be strictly increasing in α.

In a temporary equilibrium the interbank interest rate $I^*(\alpha)$ and the expected return quota $\overline{\delta}^*(\alpha)$ must satisfy the equation

$$L(I^*(\alpha), \rho, \alpha) = 0.$$

Hence, we can conclude that, for any $\alpha' > \alpha$,

$$\begin{aligned} 0 &= L(I^*(\alpha), \rho, \alpha) \\ &< L(I^*(\alpha), \rho, \alpha'). \end{aligned}$$

From case (i) in the proof of Proposition 3 we know that the aggregate excess interbank market function $L(I, \overline{\delta}|\alpha)$ is continuous and strictly increasing in I. Thus equilibrium for α',

$$L(I^*(\alpha'), \rho, \alpha') = 0,$$

implies $I^*(\alpha') < I^*(\alpha)$.

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