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Bank Solvency Stress Tests with Fire Sales*

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Abstract

We present a simple and operational yet rigorous framework that combines current methods of bank solvency stress tests with a description of fire sales. We demonstrate the applicability of our framework to the EBA stress testing exercise. Fire sales are described by an equilibrium model which balances leverage improvements and drops in security prices. The differences in bank losses caused by fire sales are significant and go beyond the trivial fact that with deleveraging we will get bigger losses. It is shown that ignoring potential deleveraging effects can show institutions as resilient which are in fact fragile and thus create a false sense of resilience.

Keywords: Stress Testing, Fire Sales, Deleveraging, Systemic Risk

JEL-Classification Numbers: C18, C44, C60, G01, G32, M48

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Non-Technical Summary

While deleveraging has been widely acknowledged as one of the key amplifiers of financial distress the considerations of such mechanisms in bank solvency stress tests have not been widely adopted. One reason might be the concern that augmenting an already complex framework might overburden a practical stress testing exercise. We present a simple and operational yet rigorous framework that can be combined with actual stress testing procedures and is firmly rooted in empirical knowledge about price impact. We demonstrate the applicability of our framework to the EBA stress test and put the evaluation of losses using a traditional methods into direct comparison with an evaluation of losses that takes potential deleveraging effects into account. The data show that the differences are significant and go beyond the trivial fact that with deleveraging we will get bigger losses. It is shown that ignoring potential deleveraging effects can show institutions as resilient which are in fact fragile and thus create a false sense of resilience.

1 Introduction

When undertaking a solvency stress test for banks, the consequences of a stress scenario are judged by evaluating a financial loss function. The goal is finding out, whether in a hypothetical extreme economic environment banks have sufficient capacity to absorb the resulting financial losses and to continue operations. Bank stress tests currently evaluate losses by applying estimated risk parameter values to quantify potential future losses given a certain financial exposure. Amplification of these losses by the attempt of banks to improve their individual financial position by selling assets in distress are not considered. While it has often been acknowledged that in financial distress losses are often substantially amplified by such deleveraging effects, their consideration in actual stress tests have not yet been widely adopted.[1] The reason is, perhaps, that stress testers fear this might overburden an already complex framework.

In this paper we show a practical way to incorporate deleveraging effects into the evaluation of losses which is both simple and that can be combined with stress testing models currently in place very easily. We substantiate this claim by applying our framework to the published stress test data of the European Banking Authority to compare a stress test with and without taking deleveraging processes into account.

We present both conceptual as well as empirical results. The core of our conceptual result is that insights from market microstructure theory can be combined with a very simple yet credible behavioral model of bank reactions in distress to understand the impact of fire selling in terms of equilibrium ideas. We capture this concept by a constructive fixed point argument (Theorem 1) which gives us at the same time an algorithm to compute deleveraging impacts (Theorem 2).

In the data part of the paper we use our deleveraging framework to undertake a thought experiment. Using the published data of the 2016 stress test of the European Banking Authority (EBA) we ask: How would a loss assessment for banks under the traditional approach compare to a loss evaluation that takes potential deleveraging into account. We demonstrate that the outcome differs materially. Not only does it differ in the trivial way of leading to larger losses. We can also see that important institutions can get into trouble indirectly because assets sales take place in the entire banking system. Thus banks which look resilient in the traditional approach look

[1]An important institution that has taken fire sales into account in their stress testing practice is the Bank of England, who has taken a pioneering role: "The Bank has also considered the risks of amplification through sales of commonly held assets. The Bank has adapted the methodology developed by Cont and Schaanning [2016], which seeks to quantify a) the impact of the sales of traded securities on the prices of those securities, and b) the realised and mark-to-market losses that result from asset sales. Contagion occurs when one or more banks sell assets held by other banks, leading to a fall in asset values and mark-to-market losses for those banks". See Bank of England [2017][1, p. 41].

fragile when losses are evaluated more comprehensively and - as we believe - also more realistically. The data show that the order of magnitude in which losses can differ under the two approaches is non-negligible. The results suggest that the traditional approach might create an illusion of resilience which is in fact not as strong as it appears.

Fire sales and deleveraging have been very prominent topics in the literature of the last ten years. So we contribute to an already very large literature. Before we place our paper into the context of our most important references, let us explain where we see the new contribution of this paper.

Most importantly, we transform some of the abstract ideas from the huge theoretical literature into concepts that can be practically applied, easily combined with approaches already used and which are firmly rooted in the empirical research on market micro structure theory when it comes to the modeling of price impact. While most applied papers in the field are based on simulation, we give a rigorous analysis of potential deleveraging impact and point out potential pitfalls that might occur in a simulation based framework. For instance, a very popular assumption on deleveraging behavior - proportional selling of marketable assets - will not necessarily lead to a unique outcome. We show that still in such situations we can give lower and upper bounds for losses. Many theory papers on deleveraging often do not make the step to actual applications. In our paper we give an integrated view of both theory and actual stress testing data from the field. One way to see our main contribution is thus perhaps this: We synthesize and distill a large literature on fire sales and bank distress in a way which gives stress testers a framework which they can directly apply and integrate into what they already have. The code as well as all the compiled and raw data we use for this project is available to the interested reader in a GitHub repository.[2]

Our paper builds on Cont and Schaanning [2016]. It differs in two main respects. In Cont and Schaanning [2016] the results of deleveraging on price impact are based on pure simulation, while our paper embeds this approach into a theoretical framework, which perhaps both reveals more clearly what is going on and at the same time allows for a more efficient and cleaner computation of price impact. In the data part Cont and Schaanning [2016] are focused on triggers of deleveraging waves and the role of liquidity weighted overlapping portfolios. We focus, in contrast, directly on the EBA stress test and the difference we can see when a loss evaluation with and

[2] `https://github.com/Martin-Summer-1090/syslosseval`. From the GitHub repository one can download the package sources code as a zip archive by pressing the green right upper button which says "Code". The raw data and the R-scripts which compile the data-sets used in the paper are in a tar.gz archive in the folder "data-raw" (syslossevel_raw_data.tar.gz). You can find instructions how to untar and unzip this archive on Windows and Mac for example here: `https://www.uubyte.com/extract-tar-gz-bz2-on-windows-mac.html`; or on Linux here: `https://smarttechnicalworld.com/how-to-extract-unzip-tar-gz-file/`

without deleveraging is compared. Such an analysis of the data aiming at a direct comparison between the two approaches can not be found in their paper.

Our paper is also closely related to the work of Cont and Wagalath [2016], Braouezec and Wagalath [2018], Feinstein and El-Masri [2017], Detering et al. [2020], Aymanns et al. [2018] as well as Veraart [2020]. Pioneering papers of applied deleveraging analysis which are closely related are also Greenwood et al. [2015] as well as Duarte and Eisenbach [2013]. The most important papers in the market microstructure literature, which we use to base our impact model on are Kyle and Obizhaeva [2016], Bouchaud [2017] and Caccioli et al. [2012].

Section 2 sets up our model. Section 3 explains the key theoretical results of the paper. Section 4 is the data part of our paper with the direct comparison of the traditional approach to loss evaluation with our own. Section 5 contains conclusions. A detailed appendix contains formal proofs of the two theorems, an example of an instance of the model where multiple fire selling equilibria can occur as well as a detailed guide to the data sources with an explanation how we compile the data for our analysis.

2 The banking system and its exposure

2.1 Present state of the banking system

Given is a set $\mathcal{B}$ of banks labeled $b = 1, \ldots, B$. For each bank b we can observe the value of security and loan exposures on its asset side as well as the value of its equity on the liability side. Furthermore, we assume to have additional information about bank assets which allows us to categorize security holdings by mapping them to a given set $\mathcal{I}$ of different security classes labeled $i = 1, \ldots, I$ and loan exposures into a given set $\mathcal{J}$ of loan asset classes labeled $j = 1, \ldots, J$. By assumption, all of these figures can be observed at a given point in time denoted $t = 0$.

One way to compactly describe the exposures of the banking system at this observation period is to write the value of security exposures across banks and security classes at time t as a $B \times I$ matrix S^t_{bi}, $b = 1, \ldots, B$, $i = 1, \ldots, I$. The value of loan exposures across banks and across loan classes is given by the $B \times J$ matrix L^t_{bj}, $b = 1, \ldots, B$, $j = 1, \ldots, J$. Whether the values are market values or accounting values depends on the data source from which we retrieve the exposure values.

The equity holdings in the banking system at time t are described by a $B \times 1$ vector $\mathbf{e}^t$. We assume that at the observation time $t = 0$ we observe only solvent banks, so $\mathbf{e}^0$ is strictly positive. Sometimes it is convenient to write equity in matrix form as a $B \times B$ diagonal matrix with the values $\mathbf{e}^0$ on the main diagonal. In this case we write E^0 for the equity holdings at $t = 0$.

Assets	Liabilities
$S_b^0 \mathbf{1}_I$	D_b^0
$L_b^0 \mathbf{1}_J$	e_b^0

$a_b^0 = S_b^0 \mathbf{1}_I + L_b^0 \mathbf{1}_J$
$\lambda_b^0 = (S_b^0 \mathbf{1}_I + L_b^1 \mathbf{1}_J)/e_b^0$

Table 1: Balance sheet of bank $b \in \mathcal{B}$ at $t = 0$. The data provide information on S_b^0, L_b^0, and $\mathbf{e}_b^0$ for all banks b. Debt D_b is the aggregate residual figure $S_b^0 \mathbf{1}_I + L_b^0 \mathbf{1}_J - e_b^0$.

We denote by $\mathbf{1}_I$ an $I \times 1$ vector with all components 1 and similarly vector $\mathbf{1}_J$. Using these vectors we write $S_b^t \mathbf{1}_I$ for the total value of securities holdings of bank b and $L_b^t \mathbf{1}_J$ for the total value of loan holdings of bank b at time t.[3]

This description leads to a stylized balance sheet of a bank b at time $t = 0$, which is shown in Table 1. The value of assets and leverage for the entire banking system are given by the $B \times 1$ vectors

$$\mathbf{a}^0 = \left(S^0 \mathbf{1}_I + L^0 \mathbf{1}_J\right) \quad (1)$$

$$\boldsymbol{\lambda}^0 = \left(E^0\right)^{-1} \left(S^0 \mathbf{1}_I + L^0 \mathbf{1}_J\right) \quad (2)$$

2.2 Future state of the banking system

We fix a given future time horizon, denoted as $t = 1$. The value of the bank holdings in securities, loans and equity at this horizon is risky and unknown as of $t = 0$. Formally this is captured by assuming that the values of the components of $\mathbf{a}^1$ and $\boldsymbol{\lambda}^1$ at $t = 1$ are all random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$.

But for some states ω the values of assets and leverage additionally depend on potential market and bank reactions to these value changes. We try to capture behavioral reactions observed in the recent and also in previous financial crises. Factoring in behavior during distress is important for the evaluation of potential bank losses in a modern banking system. Especially the big banks, which are the main focus of regulators and the public, today depend very strongly on financial markets both for funding as well as for their asset management. There is a long and rich literature on mechanisms of loss amplification through capital markets. Some important references are Brunnermeier and Pedersen [2009], Shin [2010] and Geanakoplos [2009]. We attempt to model these effects in a way that can be brought to data and can

[3] The current regulatory regime is based on risk weighted assets. Risk weighting can be considered in our model by choosing vectors $w_I \in [0,1]^I$ and $w_J \in [0,1]^J$ of risk weights instead of $\mathbf{1}_I$ and $\mathbf{1}_J$. In the current discussion we ignore risk weighting.

at the same time be combined with more traditional loss evaluation models used in current stress tests.

When we talk about behavioral reactions it is important not to get confused about the time horizon at which uncertainty due to external shocks and further relevant loss events due to behavior take place. This is the simplest modeling choice we can think of: On the one hand we have the time period from $t = 0$ to $t = 1$ which is the time horizon for our random variables on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. It describes the external uncertainty banks face at $t = 1$. Once exogenous uncertainty is resolved, this can in certain states of the world trigger further behavioral reactions.

In the context of our model we want to focus on potential asset sales once leverage is higher than a threshold level. In the model we will not go into the details of exactly why leverage levels trigger asset sales. Our formulation is compatible with different deleveraging stories based on the level of actual leverage. One that focuses on the aim of banks is the motive to stay above some thresholds which banks themselves see as critical for their own operations as for instance in Cont and Schaanning [2016]. Our model would also be consistent with a story where upon the breach of some threshold other investors withdraw funding from the bank, because they become concerned with the viability of that institution, which must then sell assets to raise the cash to fill the funding gap. This is referred to as the interaction of market and funding liquidity by Brunnermeier and Pedersen [2009]. One example of the detailed modeling of such a mechanism is given, for example, in Cont and Wagalath [2013].

It is important to keep in mind that the value changes due to asset sales are realized after uncertainty at $t = 1$ is resolved. One way to think about this is that as long as shocks are not too strong banks can cope with their regular business and a static and purely stochastic description of balance sheets will capture most relevant events in terms of financial stability. Once this threshold is exceeded we can not abstract anymore from behavior and have to factor in its consequences for losses more precisely. Behavior in financial distress becomes amenable to somewhat realistic modeling, because in distress and at the typically short time horizon – let us call it τ – behavioral options are restricted to very few things a bank can do.

So our time line has in fact two periods. One period ranges from $t = 0$ to $t = 1$. In this period exogenous uncertainty is resolved. The value of securities holdings changes from $S_b^0 \mathbf{1}_I$ to $S_b^1(\omega)\mathbf{1}_I$. The value of loan holdings changes from $L_b^0 \mathbf{1}_J$ to $L_b^1(\omega)\mathbf{1}_J$. The value of equity changes from e_b^0 to $e_b^1(\omega) = e_b^0 + (S_b^1(\omega)\mathbf{1}_I + L_b(\omega)\mathbf{1}_J) - (S_b^0 \mathbf{1}_I + L_b^0 \mathbf{1}_J)$.

In the second period, from $t = 1$ to $t = 1 + \tau$ behavioral reactions and their effects on bank values unfold. Here τ is the expected time over which these behavioral reactions take place.[4]

[4]Note that this time structure is similar to the discrete time model in Cont and Wagalath

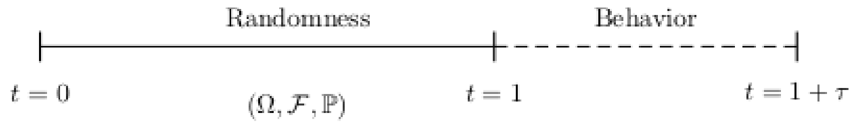

Figure 1: The value of positions is determined by exogenous uncertainty modeled by random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and by market and bank reactions. While exogenous uncertainty is resolved at $t = 1$ the consequences of behavioral reactions unfold over an expected time period τ starting at $t = 1$.

Assets	Liabilities
$S_b^1(\omega)\mathbf{1}_I$	D_b^1
$L_b^1(\omega)\mathbf{1}_J$	$e_b^1(\omega) = e_b^0 + (S_b^1(\omega)\mathbf{1}_I + L_b^1(\omega)\mathbf{1}_J) - (S_b^0\mathbf{1}_I + L_b^0\mathbf{1}_J)$
$a_b^1(\omega) = S_b^1(\omega)\mathbf{1}_I + L_b^1(\omega)\mathbf{1}_J$	
$\lambda_b^1(\omega) = (S_b^1(\omega)\mathbf{1}_I + L_b^1(\omega)\mathbf{1}_J)/e_b^1(\omega)$	

Table 2: Balance sheet of bank b at $t = 1$. The value of its total assets is $a_b^1(\omega)$ and its leverage is $\lambda_b^1(\omega)$.

We have now a new state of the banking system given by a new stylized balance sheet of a bank b at time $t = 1$, which is shown in Table 2. The value of assets and leverage for the entire banking system at $t = 1$ are given by the $B \times 1$ vectors

$$\mathbf{a}^1(\omega) = \left(S^1(\omega)\mathbf{1}_I + L^1\mathbf{1}_J(\omega)\right) \tag{3}$$

$$\boldsymbol{\lambda}^1(\omega) = \left(E^1(\omega)\right)^{-1}\left(S^1(\omega)\mathbf{1}_I + L^1(\omega)\mathbf{1}_J\right) \tag{4}$$

Assume that for some state ω and some bank b leverage changes from λ_b^0 to $\lambda_b^1(\omega)$. There is, however, a threshold λ^* above which the bank needs to sell some of its liquid positions, which in our model (as well as in a real situation) are securities, to rebalance its portfolio such that leverage is restored back to that threshold. We denote the share of marketable assets that are sold in the market by ban b by θ_b. As we have explained above, this is very much reduced form model in which we do not go into the details as to why exactly the selling takes place.

In this paper we focus on the analysis of how to model losses arising from fire sales. We do not model the exogenous stochastic shocks in any detail. We have described the assumed probability component up to now to make clear how our loss model is embedded in the wider stress testing framework.

[2013] where exogenous uncertainty unfolds from t_k to $*$ and behavior unfolds from $*$ to t_{k+1}.

Assets	Liabilities
$S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1-\theta_b)$	$D_b^0 - \theta_b S_b^1(1-\boldsymbol{\delta})$
$L_b^1 \mathbf{1}_J$	$e_b^1 - S_b^1 \boldsymbol{\delta}$

$$a_b^{1+\tau} = S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1-\theta_b) + L_b^1 \cdot \mathbf{1}_J$$
$$\lambda_b^{1+\tau} = \frac{S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1-\theta_b) + L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \boldsymbol{\delta}}$$

Table 3: Balance sheet of bank $b \in \mathcal{B}$ at $t = 1 + \tau$ with price impact. The value of its total assets is $a^{1+\tau}$ and its leverage is $\lambda^{1+\tau}$. The asset sale has an ambiguous effect on the new leverage. On the one hand, debt can be reduced with the proceeds from the sale. On the other hand, by the price impact the bank is facing a valuation loss on its assets at $t = 1 + \tau$ relative to $t = 1$. The loss affects the share of the assets sold as well as the value of the shares kept on the balance sheet. This loss has to be absorbed by equity and increases leverage. The total effect depends on the size of the price impact.

Since we do not use the stochastic component in the following we do not explicitly write the variables at $t = 1$ as functions of the state ω from now on to economize on notation.

At the time of fire sales the value of a security i exposure equals only $1 - \delta_i$ times its value before fire sales, where $0 < \delta_i < 1$ is the price drop due to fire sales. We assume that we can interpret the change in the value of the position as coming from an impact to prices. So we interpret δ_i as the price impact of fire sales on security i. The price impact on all securities is denoted by the vector $\boldsymbol{\delta} = (\delta_1, \ldots, \delta_I)$ which is the vector of fire sales discounts as percentage of pre-sales prices.

Fire sales discounts imply that security exposures do not have a value of $S_b^1 \mathbf{1}_I$ but $S_b^1(\mathbf{1}_I - \boldsymbol{\delta})$. This has consequences on the value of equity:

$$\begin{aligned} e_b^{1+\tau} &= S_b^1(\mathbf{1}_I - \boldsymbol{\delta})(1-\theta_b) + L_b^1 \mathbf{1}_J - (D_b^0 - \theta_b S_b^1(\mathbf{1}_I - \boldsymbol{\delta})) \\ &= S_b^1(\mathbf{1}_I - \boldsymbol{\delta}) + L_b^1 \mathbf{1}_J - D_b^0 \end{aligned} \tag{5}$$

Therefore the loss in equity as a result of fire selling is

$$e_b^{1+\tau} - e_b^1 = S_b^1(\mathbf{1}_I - \boldsymbol{\delta}) - S_b^1 \mathbf{1}_I = -S_b^1 \boldsymbol{\delta} \tag{6}$$

The balance-sheet mechanics of the delveraging process is shown in Table 3

If the bank is able to bring its leverage back to λ^* or lower it will stabilize. This might, however, turn out to be infeasible. While the outflow of debt reduces D_b and thus leverage, the loss on security values increases leverage. Thus it can happen that it is not be possible for the bank to reach the target, even if it sells its entire security portfolio.

3 Fire sales equilibrium

3.1 Individual bank behaviour

Let $\omega \in \Omega$ be a given realization of risk factors at $t = 1$ and assume there is a bank b for which $\lambda_b^1(\omega) > \lambda^*$. It is assumed that this bank will then begin to sell securities to achieve the target leverage λ^*.

The bank understands that selling assets at time $t = 1$ will have a price impact on the sold asset classes at $t = 1 + \tau$ expressed as a vector $\boldsymbol{\delta} \in [0, 1]^I$. From the viewpoint of the individual bank this impact vector $\boldsymbol{\delta}$ is regarded as a parameter in its decision. It is not the choice of the bank but is determined in the market.

A leverage threshold λ^* specifies the maximum leverage a bank is allowed to have. If the leverage of the bank is below λ^* the bank is fine and there is no further need for action. If the leverage is above λ^* the bank has to sell part of its security portfolio to bring its leverage back to λ^*.

Assume that some bank—not necessarily bank b —has to sell securities. Then we denote by $\lambda_{b,\min}$ the leverage after fire sales if bank b sold its whole security portfolio, and by $\lambda_{b,\max}$ the leverage if bank b sold no securities. Both $\lambda_{b,\min}$ and $\lambda_{b,\max}$ depend on the fire sale price discount $\boldsymbol{\delta}$.

Assumption 1. *If the leverage λ_b^1 of a bank b is larger than the required λ^*, it sells the same proportion θ_b of all types of securities it holds. $0 \leq \theta_b \leq 1$, assuming that bank b must not go short in securities ($\theta_b \leq 1$) and that it does not buy securities ($0 \leq \theta_b$) in a fire sale when it already violates the leverage constraint. We assume the loan portfolio cannot be sold on the time scale τ of fire sales. The proceeds from selling the proportion θ_b are used to reduce debt.*

Leverage after fire sales depends on the price impact $\boldsymbol{\delta}$ of fire sales (see Table 3). So do $\lambda_{b,\min}$ and even $\lambda_{b,\max}$, although the latter results from the bank not selling itself any securities. But the value of securities held by the bank is affected by fire sale price effects triggered by other banks.

We make the following assumption on bank behavior and fire selling, which details Assumption 1:

Assumption 2. *At $t = 1$ the bank decides about its participation in the fire sale. In this decision it assumes some price impact $\boldsymbol{\delta}$ of the fire sales. We assume the bank decides to sell the following proportion of its securities portfolio:*

$$\theta_b(\boldsymbol{\delta}) = \begin{cases} 1 & \textit{if } \lambda^* < \lambda_{b,\min}(\boldsymbol{\delta}) \textit{ or if } e_b^1 \leq 0 \\ 1 - \frac{\lambda^*(e_b^1 - S_b^1 \boldsymbol{\delta}) - L_b^1 \mathbf{1}_J}{S_b^1(\mathbf{1}_I - \boldsymbol{\delta})} & \textit{if } \lambda^* \in [\lambda_{b,\min}(\boldsymbol{\delta}), \lambda_{b,\max}(\boldsymbol{\delta})] \\ 0 & \textit{if } \lambda^* > \lambda_{b,\max}(\boldsymbol{\delta}) \end{cases} \quad (7)$$

The fire sale proportion in equation 7 can easily be computed from solving the expression for leverage at $1+\tau$ in Table 3 by setting $\lambda^{1+\tau} = \lambda^*$ and solving for θ_b. From 7 we can derive the expressions for $\lambda_{b,\min}$ and $\lambda_{b,\max}$ as

$$\lambda_{b,\min}(\boldsymbol{\delta}) = \frac{L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \boldsymbol{\delta}}, \qquad \lambda_{b,\max}(\boldsymbol{\delta}) = \frac{S_b^1(\mathbf{1}_I - \boldsymbol{\delta}) + L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \boldsymbol{\delta}}. \tag{8}$$

$\lambda_{b,\max}(\mathbf{0})$ is the leverage of bank b at $t = 1$.

If we focus on the idea that the breach of a trigger level of leverage initiates the fire-sale of securities, then the assumption that the banks strive to get back to a leverage target value seems reasonable. For this modeling decision it does not matter why exactly the breach of the trigger initiates security sales. The motives from sales could come from the banks' own attempt not to exceed a certain level of leverage. It could also come from the fact that other banks who are concerned with too high leverage withdraw funds and thereby force a sale of securities.

Since we are interested in a model which can be used in applications, we are interested in making the model as simple as possible. We are aware that the practical decision, which asset classes to sell and in which precise amounts is complex. Banks might consider the market liquidity, the risk weights (see Braouezec and Wagalath [2018]) and perhaps also other aspects.

A particularly simple assumption we make in this model is that banks sell a constant share of value uniformly over all security classes (Assumption 1). While this is unrealistic, in terms of modeling it leads to a great simplification of the description of bank behavior. The reason is that — as in Cont and Schaanning [2016]– this allows to determine the share from the goal to achieve a certain leverage target only.

Why do we choose such a simple and mechanical decision rule, when we could try modeling the decision rule of banks in a more general way? Our short answer is that given the very coarse precision of the data and a lack of systematic empirical evidence on how banks actually behave in an actual fire sale situation, it seems reasonable to use a simple rule, and make the model thus easier to use in actual stress tests.

The literature has considered more sophisticated behavioral models. For instance Braouezec and Wagalath [2018] consider an optimization problem, where banks choose an optimal liquidation strategy based on risk weights. In this case the decision requires the solution of a linear programming problem for each state of the world. Braouezec and Wagalath [2019] model the fire sale decision as a strategic equilibrium problem where banks choose a to play Cournot Nash quantity strategies with certain constraints. This also needs the solution to an optimization problem in each state of the world. Detering et al. [2018] assume a general sales function which depends in a systematic way on the ratio of losses to equity.

Our approach allows to determine the sales decision based on the constraint λ^* alone and leads thus to a very simple decision rule. Without any detailed evidence giving a basis to guide the modelling on bank securities selling behaviour under distress there is no clear "best" way how to model the security selling decision. Our approach leads to the simplest meaningful decision rule, we could think of. It can be easily implemented in applications. Given that a fire sale model is perhaps not able to give a quantification of effects beyond a rough estimate of orders of magnitude the level of behavioral detail in modeling the decision rule we choose in this paper, might be all that is needed.

3.2 Price impact

If each bank b sells a proportion $\theta_b(\boldsymbol{\delta})$ of its securities portfolio, the total volume of security i which is sold on the market is

$$q_i(\boldsymbol{\delta}) = \sum_b S^1_{b,i} \theta_b(\boldsymbol{\delta}), \tag{9}$$

where $S^1_{b,i}$ is the value of security position i held by bank b at time $t = 1$. Following the market micro-structure literature we assume that the price impact on a security asset class i is some function of the total volume of this security sold in the market. We denote this function by φ and make the following:

Assumption 3. *The price impact of selling a certain volume of security i is described by a function φ_i from $\mathbb{R}^I_+$ to $[0, 1]^I$ which we assume to have the following properties: $\varphi_i(0) = 0$, φ_i is strictly increasing and continuous and for all $i \in \mathcal{I}$, $\varphi_i < 1$, more specifically*

$$\varphi_i \left(\sum_b S^1_{b,i} \right) =: \delta_{i,max} < 1. \tag{10}$$

The precise shape of the price impact function φ is a question actively discussed in the market microstructure literature. For a recent overview see, for example, Bouchaud [2017].

We derive our specific impact function by relying on recent work of Kyle and Obizhaeva [2016]. They postulate and empirically test two market micro structure invariance principles. These invariance principles essentially claim that there exists a deeper structure in financial markets so that some salient characteristics of trades do neither depend on time nor the particular asset class. One of them relates to transaction costs. They show that market micro-structure invariance implies a transaction cost model where the percentage costs of trading of a particular asset is proportional to the product of volatility, two invariant constants and a general invariant price impact function.

The shape of that function can be determined only empirically. The invariance principles are shown to approximately hold for equities. However, the authors express belief that they likely hold for other asset classes including bonds. Following the empirical literature on price impact we assume that this function is a square root function (see Bouchaud [2017]).

$\delta_{i,max}$ is the price discount resulting when the maximal quantity of security i, namely the total holdings of all banks, is sold. Denote the vector $(\delta_{1,max}, \ldots, \delta_{I,max})$ by $\boldsymbol{\delta}_{\max}$. We assume that the maximal price discount is smaller than one. Even if all banks sell their full security holdings, the price of the securities will not be zero. This assumption ensures that $\theta_b(\boldsymbol{\delta})$ defined in (7) is well defined. The price impacts of all security sales are $\varphi(q_1, \ldots, q_I) := (\varphi_1(q_1), \ldots, \varphi_I(q_I))$.

Using the square root specification we get:

$$\varphi_i(q_i) = \sigma_i \kappa \sqrt{\frac{q_i}{ADV_i}} \tag{11}$$

where q_i is the aggregate volume in value terms (say Euro or Dollar) of security i sold in the market, κ is a constant of order unity independent of the asset class and ADV_i is the average daily volume (turnover) of security i.

Our equation (11) is consistent with Kyle and Obizhaeva [2016] equation (18) under the assumption that their constant $\kappa_0 = 0$. This last condition implies that spread costs are ignored. Unobservable quantities in their model are absorbed in our constant κ. Following the terminology of Kyle and Obizhaeva [2016] the quantity q_i is the "aggregate bet". In the invariance equations the original expressions contain bet volatility and bet volume. Both of these quantities are defined for the business time τ, in their terminology. Expressing these two quantities in terms of observable variables, daily returns volatility and average daily volume, bet volatility scales as $\sqrt{\tau}$ while bet volume scales as τ. As a result the τ dependence of price impact cancels out in the square root case.[5]

3.3 Fire sales equilibrium

To make a specific loss assessment for a given state of the world ω we determine the price discount by applying an equilibrium idea. Given a discount vector $\boldsymbol{\delta}$, we can think of our bank behavior equation (7) as a security supply decision by banks. We have explained why we prefer in the

[5] Cont and Schaanning [2016] use different impact functions based on exponential functions rather than the square root function. In their specification, therefore, τ has to be specified to pin down the price impact. We stick to the square root function because the literature presents some evidence that this function is actually often observed in the context of price impact events (see Bouchaud [2017]). Note that our results do not depend on the exact form of the impact function but only on Assumption 3. The impact functions used in Cont and Schaanning [2016] do - for instance - fulfill Assumption 3 and could therefore be used as well.

particular context to derive this supply decision from a simple leverage target rule rather than from optimizing behavior, more usually applied in economics and finance. The impact function φ can be thought of as an inverse demand function which describes the price reactions that can be expected in the security markets for a given volume sold. In a fire-sale equilibrium supply and demand balance. The ultimate price impact is the discount vector which achieves this balance. Note that a bank only knows its own fire sales behavior but not the behavior of other banks, which depends on their balance sheets. Our concept therefore applies a non-strategic, competitive-equilibrium idea.

Definition 1. *Given a state $\omega \in \Omega$ at $t = 1$ a fire-sale equilibrium is given by a pair $(\mathbf{q}^*(\boldsymbol{\delta}^*), \boldsymbol{\delta}^*)$ such that:*

1. *For every bank $b \in \mathcal{B}$:*

$$\theta_b(\boldsymbol{\delta}^*) = \begin{cases} 1 & \text{if } \lambda^* < \lambda_{b,\min}(\boldsymbol{\delta}^*) \text{ or if } e_b^1 \leq 0 \\ 1 - \frac{\lambda^*(e_b^1 - S_b^1 \boldsymbol{\delta}^*) - L_b^1 \mathbf{1}_J}{S_b^1(\mathbf{1}_I - \boldsymbol{\delta}^*)} & \text{if } \lambda^* \in [\lambda_{b,\min}(\boldsymbol{\delta}^*), \lambda_{b,\max}(\boldsymbol{\delta}^*)] \\ 0 & \text{if } \lambda^* > \lambda_{b,\max}(\boldsymbol{\delta}^*) \end{cases} \tag{12}$$

   with $\mathbf{q}^*(\boldsymbol{\delta}^*) = (S^1)^T \boldsymbol{\theta}(\boldsymbol{\delta}^*)$, where $(S^1)^T$ denotes the transposed security holdings matrix S_{bi}^1, $b = 1, \ldots, B$, $i = 1, \ldots, I$.

2. *Security supply equals security demand:*
   $(\varphi_1(q_1^*(\boldsymbol{\delta}^*)), \ldots, \varphi_I(q_I^*(\boldsymbol{\delta}^*))) = \boldsymbol{\delta}^*$

To make our equilibrium concept useful for an application we have to show that under our assumptions a fire sales equilibrium actually exists. We furthermore need to give a constructive procedure how such an equilibrium can be computed given our data. Let us start with existence first.

Theorem 1. *Given Assumptions 1 - 3 and $\lambda^* > 1$ a fire sale equilibrium exists.*

Proof. A proof is given in the appendix. □

The idea of the proof uses the fact that the map $\varphi \circ q$ turns out to be an order preserving self map on the complete lattice $D := \{\boldsymbol{\delta} = (\delta_1, \ldots, \delta_I) : 0 \leq \delta_i \leq \delta_{i,max}\}$. By the Knaster-Tarski fixed point theorem, we can then deduce that a fixed point exists and that the set of fixed points of $\varphi \circ q$ contains a minimal and a maximal element.

Note that our concept of a fire sale equilibrium also allows for the case, where no fire sales take place. In the appendix we give a proof that $\boldsymbol{\delta} = \mathbf{0}$ is a fixed point of $\phi \circ q$ if and only if all banks have $\lambda^* > \lambda_{b,\max}(\mathbf{0})$ in (7).

At first sight this result may appear of only limited use in a stress test application. While in this paper we focus on the evaluation of losses, in a fully fledged stress test, we also have a risk factor distribution in the background.

So if we associate to each state of the world $\omega \in \Omega$ a corresponding behavioral reaction, we can not say what the ultimate loss will be in this state, since there are in general several possibilities.

Still, we argue that the order theoretic setup of the model provides additional structure that is useful, because it allows for cases where we do not have a unique fixed point, to give upper and lower bounds for the fire-sale losses. This allows us to assess the losses by giving a lower (optimistic) and an upper (pessimistic) bound. Whether a particular loss scenario ends up in a unique fixed point or in several fixed points can not be said in general. In the appendix we give an example which demonstrates that we can not hope for uniqueness of the fixed point in general, given our setup.

3.4 Computing a fire sales equilibrium

Based on a version of the Kleene [1952] fixed point theorem formulated by Cousot and Cousot [1979] we can also give a constructive procedure of how we can compute the minimum and maximum fixed points. This is indispensable for applying our method to data. This result is formulated in the following:

Theorem 2. *Starting from* $\boldsymbol{\delta} = \mathbf{0}$*, an iteration of the map* $\varphi \circ q$ *converges to the least fire sales equilibrium:* $(\varphi \circ q)^n(\mathbf{0}) \to \boldsymbol{\delta}^*_{\min}$*. Starting from* $\boldsymbol{\delta} = \boldsymbol{\delta}_{\max}$*, an iteration of the map* $\varphi \circ q$ *converges to the greatest fire sales equilibrium:* $(\varphi \circ q)^n(\boldsymbol{\delta}_{\max}) \to \boldsymbol{\delta}^*_{\max}$.

Proof. A proof is given in the appendix. □

We call $\boldsymbol{\delta}^*_{\min}$ the least and $\boldsymbol{\delta}^*_{\max}$ the most severe fire sales equilibrium. Note that in many cases we will have $\boldsymbol{\delta}^*_{\min} = \boldsymbol{\delta}^*_{\max}$ and the fire sale equilibrium is unique.

3.5 The state of the banking system after fire sales

Denote the fixed point of $\varphi \circ q$ by $\boldsymbol{\delta}^*$ with $\boldsymbol{\delta}^* \in D$. In the fire sales equilibrium the selling decision of bank b is $\theta_b(\boldsymbol{\delta}^*)$. Denote these selling decisions by θ^*_b.

The balance sheet parameters of the banks after fire sales are

$$\begin{aligned} S_b^{1+\tau} &= S_b^1(\mathbf{1}_J - \boldsymbol{\delta}^*)(1 - \theta_b^*) && (13) \\ L_b^{1+\tau} &= L_b^1 && (14) \\ e_b^{1+\tau} &= e_b^1 - S_b^1 \boldsymbol{\delta}^* && (15) \\ \lambda_b^{1+\tau} &= \frac{S_b^1(\mathbf{1} - \boldsymbol{\delta}^*)(1 - \theta_b^*) + L_b^1 \mathbf{1}_J}{e_b^1 - S_b^1 \cdot \boldsymbol{\delta}^*}. && (16) \end{aligned}$$

After fire sales no bank sells any more securities—either because its leverage is smaller or equal to the required λ^*, or because it has already sold its entire securities portfolio. If we choose $\boldsymbol{\delta}^*_{\min}$ we get the least and if we choose $\boldsymbol{\delta}^*_{\max}$ we get the most severe state of the banking system after fire sales.

4 An application of systemic loss evaluation to public EBA data

We now analyze a data-set published by the European Banking Authority (EBA). It is from the 2016 EBA stress testing exercise. It contains exposure as well as impairment data and was the basis of the pan European bank solvency stress test of 2016. This data-set allows for comparing the risk assessment that would result from the standard EBA methodology and from our loss evaluation method, which takes into account potential deleveraging effects.

From this analysis we can see both how our ideas can be applied to a practical stress testing situation and it simultaneously gives us some first ideas of the quantitative importance of such an extended evaluation of potential losses. It gives us only a first idea, because the data allow us considering only a limited set of marketable securities, which are held on the bank balance sheet. We cannot look beyond government bonds and beyond the on balance sheet items. There is nothing, however, in our framework that would exclude in principle a wider consideration if data were available. It also gives us only a first idea because the loss evaluation is derived from an assumption about bank behavior which is not yet empirically validated. We are confident, however, that our framework is flexible enough to accommodate more elaborate and more realistic behavioral models of the fire sale process.

4.1 Organizing the EBA data into stylized bank balance sheets

We now give a brief high level description of our data. A more detailed description is given in the appendix. Readers interested in every detail of the data compilation can consult our GitHub repository cited in the introduction.

In the annual transparency exercise EBA discloses detailed bank-by-bank data for given reference dates, usually June and December. Information is published for a wide set of banks across 26 countries at the highest level of consolidation in the European Union (EU-27) and the European Economic Area (EEA) as well as for some banks from UK. The data are made available on the EBA web-page and provide disclosure on banks' assets and liabilities, capital positions, risk exposure amounts, leverage exposures and asset quality as well as information on sovereign exposures.

Biannually the EBA also conducts a bank solvency stress test for the largest banks in the European Union and the European Economic Area. The sample of banks is smaller than in the transparency exercise. The selection threshold is at a value of total assets larger than 30 billion Euro.

Under some assumptions on the aggregation of data, detailed in the appendix, and using our theoretical computational framework of fire-sale equilibrium, we can construct stylized balance sheets for each bank at $t = 0$, $t = 1$ and $t = 1 + \tau$ described in our respective Tables (1), (2) and (3).

Assets	Liabilities
Central banks and central governments: Loans	
Central banks and central governments: Bonds	
Institutions	
Corporates	
Retail	Debt
Equity	
Other non credit obligations	
Residual Position	Core Tier 1 equity

Table 4: Balance sheet of bank $b \in \mathcal{B}$ at $t = 0$.

The stylized balance sheet we get in this way for each bank for the 2016 data is given in 4. This scheme uses the asset classification of the reporting standard according to the internal rating based approach (IRB) at the highest level of consolidation.

We need to explain a few features of this scheme in more detail: Not all banks report to the EBA according to the IRB standard. Some banks report assets partially also according to the standard approach (STA). To organize the data into a unique scheme like in Table (4) we have to make an assumption about how we map STA into IRB classifications, where necessary. The detailed assumption how we do this is described in the appendix.

Observe that as for the position "central banks and central government", we split the position into loans and (sovereign) bonds. Thus when we bring our model to the EBA data, sovereign bonds are the only on balance sheet marketable assets for which we have exposure information. The price impact effects of distressed deleveraging can thus only be partially described given our data.

Finally observe that in our organization of the data we use an asset class called "Residual Position". This position is constructed as the difference between the sum of the value of all asset positions reported as exposed to credit risk in the stress test and the value of total assets reported by banks in their annual reports. That such a gap can be not only negative (total value of EBA assets smaller than total assets) but also positive (total assets smaller than total value of EBA assets) is a consequence of the regulatory reporting framework. A more detailed analysis of these (sometimes substantial) gaps is given in the appendix.

4.2 How do the data look like? Some summary statistics

We now give a brief descriptive overview of our data. Table 5 displays some descriptive statistics for the distribution of total assets, the (unweighted) ratio of Core Tier 1 equity over total assets, the leverage ratio λ as well as the share of the value of sovereign bonds in the value of total assets of the 2016 EBA stress test exposure data. We can see that all of the 51 banks in this sample have total assets of at least 30 billion euro. The average capital ratio is at about 5% with a standard deviation of 2 percentage points. The equity base, if computed without the usual Basel II risk weighting, shows a relatively thin equity base overall. The leverage ratio shows the same information (just expressed as the inverse of the tier 1 capital ratio). We display it here because it is a critical ratio in our behavioral model. From the table we see that even without any shock or stress there is at least one bank with a leverage ratio way above the critical threshold. The average value of sovereign bond holdings in this sample is about 8%.[6]

Statistics	Total assets	CET1 ratio	Leverage ratio	Bond ratio
Min	33.70	0.02	7.70	0.01
Q25	154.08	0.04	17.55	0.05
Median	234.57	0.05	20.47	0.07
Q75	744.83	0.06	23.31	0.11
Max	2218.57	0.13	47.35	0.30
Mean	526.53	0.05	20.87	0.08
StDev	548.06	0.02	6.55	0.06

Table 5: Summary statistics of the data from the 2016 EBA stress testing exercise. There are 51 banks in the sample. The table shows the minimum value the 25% quantile, the median, the 75% quantile, the mean and the standard deviation for total assets, the ratio of equity to unweighted total assets (CET1 ratio), the ratio of (unweigthed) total assets over Core Tier 1 equity (Leverage ratio) as well as the share of the total value of sovereign bond exposures in total assets. All figures are in billion Euro.

The key variable in our analysis is leverage. Leverage is not only critical for the overall resilience to shocks– what is usually studied in traditional stress testing – but it is also critical for potential deleveraging and thus loss amplification processes. Figure 2 shows that already without any stress, in both samples there are banks which already exceed the threshold of $\lambda^* = 33$ even without any shock.[7]

4.3 The EBA 2016 bank solvency stress test

Let us now study our first case, the EBA 2016 stress test. We want to go through the following thought experience: Let us first look at the published stress test data first and observe the assessment which resulted from this

[6]This corresponds to estimates given in the literature. For example Gennaioli et al. [2018] for report a figure of 9% in a sample of 191 big banks around the globe.

[7]The Basel 3 framework sets the minimum required leverage at 3 % which is why we (defining leverage as exposure/capital in contrast to Basel which uses capital/exposure) set λ^* at this particular value.

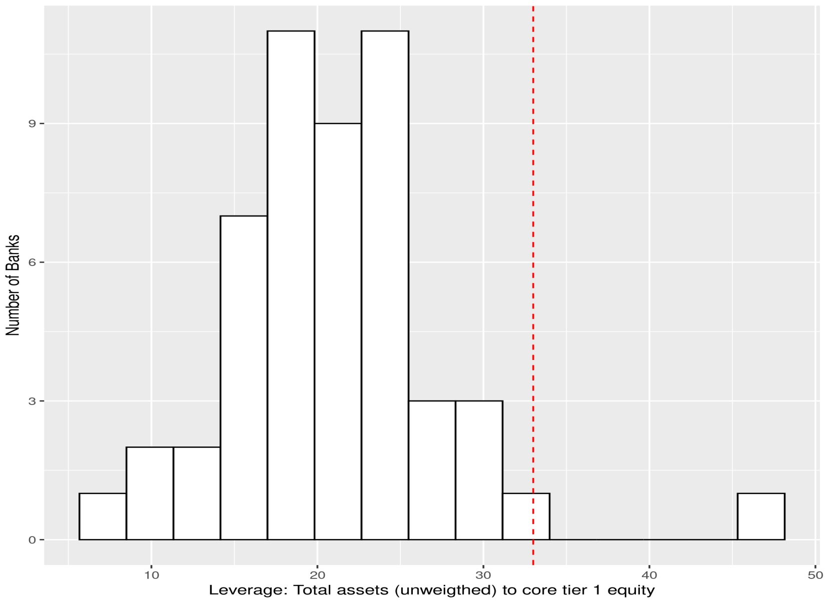

Figure 2: Histogram of leverage in the 2016 EBA stress testing banking sample. The dashed (red) line is the critical leverage threshold above which in our model behavioral reactions are taken into account in the evaluation of losses. In this particular figure the critical threshold is set at $\lambda^* = 33$.

analysis. We then ask the hypothetical question: How would our assessment have looked like if we had factored in the potential deleveraging effects as captured by our framework. In the comparison of these two cases we then can understand how and to which extent both approaches differ.

The sample of banks which participated in the 2016 EBA stress test consisted of 51 banks from 15 EU and EEA countries, 37 from SSM countries and 14 from the Denmark, Hungary, Norway, Poland, Sweden and the UK. The scenario considered in the stress test assumed a deviation of EU GDP from its baseline level by 3.1% in 2016, 6.3% in 2017 and 7.1% in 2018. It furthermore considered a shock in the residential and commercial real estate prices, as well to foreign exchange rates in Central and Eastern Europe under the adverse scenario. The assumption on the advanced economies, including Japan and the US were a cumulative GDP growth between 2.5% and 4.6% lower than under the baseline scenario in 2018. For the main emerging economies the stress test assumed total GDP between 4.5% and 9.7% below the baseline projections in 2018, with a stronger impact for Brazil, Russia and Turkey. Finally the stress test defined an adverse scenario for a number of key prices such as long term interest rates, FX rates, stock prices, inflation and swap rates. These scenarios are processed by the participating banks to "translate" them by their own analytical frameworks into impairments according to the EBA methodology (European Banking Authority [2016a]). The results of the stress test is reported in European Banking Authority [2016b].

Let us note for the following that we do not reproduce the EBA stress test exactly here, since we do not implement the full EBA methodology for this analysis. We do, for instance, not consider risk weighting, we do not consider (exogenous) market risk and operational risks and we do not model income flows but confine ourselves to balance sheets only. The reason why we take so many bold shortcuts here is to focus on the key question of this section: How does a loss assessment based on the EBA data differ between an approach where we use impairments only from one where we factor in additional losses from deleveraging. While the EBA stress test makes a stress assessment focused on 8 different metrics[8] we focus for our purposes on leverage, equity losses and the number and size of affected institutions.

Results under the assumption of no deleveraging Let us first look at the pure credit risk losses implied by the EBA data under the adverse scenario in terms of the leverage ratio λ. This plot may be compared with Figure 13 on page 23 of European Banking Authority [2016b]. The comparison shows that the leverage numbers look very similar, despite of the fact that we do not reproduce the EBA stress test exactly.[9]. From Figure 3 we can see that under the adverse scenario the median leverage as well as the share of banks with significantly higher leverage increases compared to the initial position. The median leverage increases at all horizons above the 75% quantile in the initial state. There are also a number of banks, which would not survive the stress test without help from outside. They are not able to maintain a leverage ratio below the critical boundary of $\lambda^* = 33$. We display this fact graphically in a bar chart in Figure 4. The names of the affected institutions as well as their rank among the 51 banks in terms of total assets are written into the chart for the initial position as well as for the adverse scenario at all horizons.

As we can see from Figure 4, there is one bank, which is above the threshold already in in the initial state. Under the one year ahead adverse scenario there are five additional banks exceeding the threshold. If we go two years ahead an additional bank is going to join the club. Finally in 2018 at the three year horizon we have again only seven banks above the threshold.

[8]Transitional CET1 capital ratio, Fully loaded CET1 capital ratio, Transitional leverage ratio, Transitional CET1 capital, Cumulative credit risk losses (impairment or reversal of impairment on financial assets not measured at fair value through profit or loss), Cumulative gains or losses arising from operational risk, Cumulative market risk losses including CCR, Cumulative profit or loss for the year. See European Banking Authority [2016a] for definitions and details.

[9]Note that in Figure 13 in EBA the leverage ratio is represented as CET1 over total assets whereas we have defined it reciprocally as total assets over CET1.

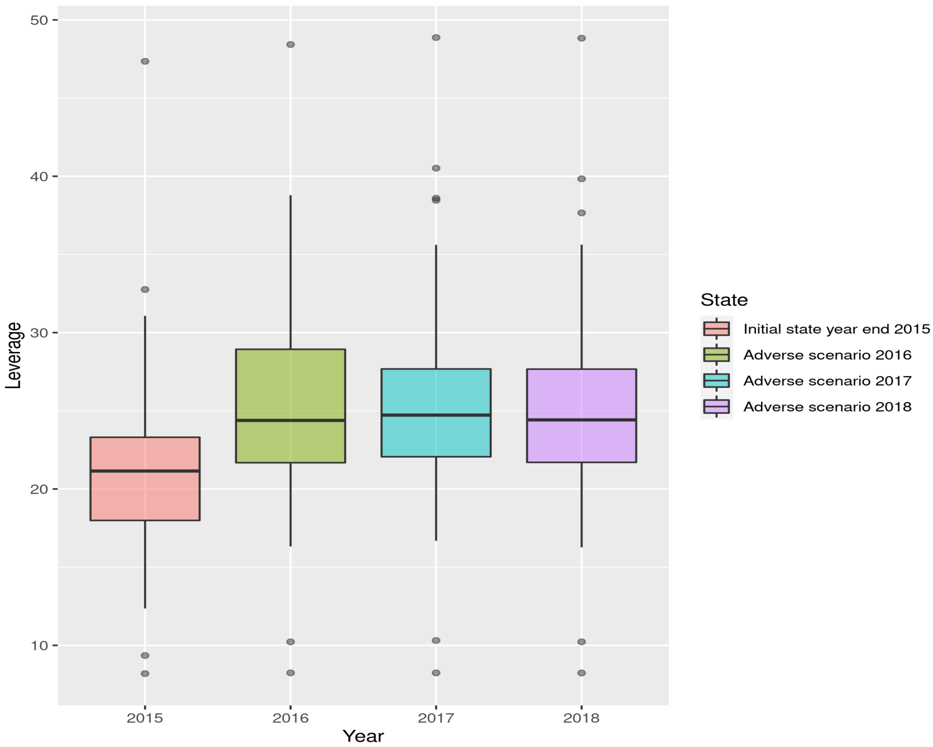

Figure 3: Distribution of leverage λ in the EBA stress test. The left most box-plot shows the initial state of the banking system at year end 2015. This corresponds to $t = 0$ in our model. The next box plots show the leverage distribution under the adverse EBA scenario at different time horizons, 2016 (one year ahead), 2017 (two years ahead) and 2018 (three years ahead). In terms of our model, these horizons would correspond all to different assumptions about $t = 1$. There are a few banks which exceed the critical leverage threshold of $\lambda^* = 33$ under the adverse scenario.

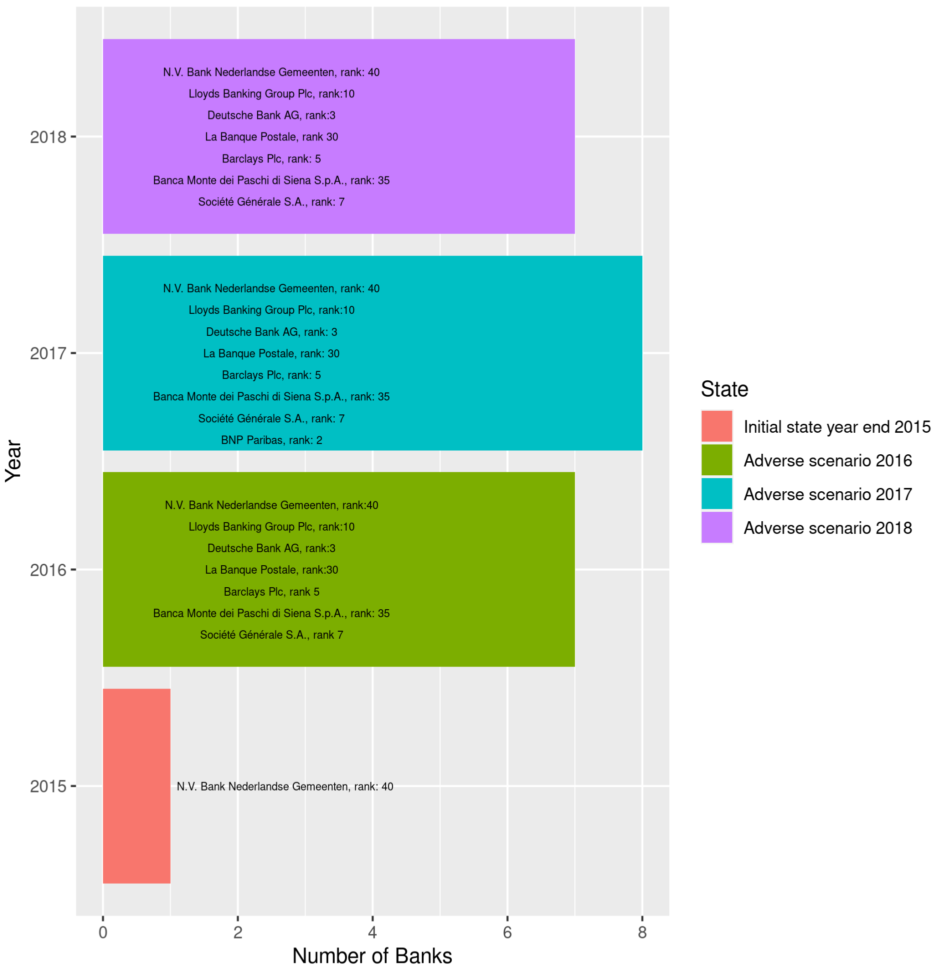

Figure 4: Number of banks with a leverage λ above the threshold $\lambda^* = 33$ in the initial state at year end 2015, in the adverse scenario in 2016, 2017 and 2018. The names of the banks as well as their size rank among the 51 banks in terms of total assets are given as annotations right of the bar (initial state year end 2015) or in the bars (Adverse scenario 2016, 2017, 2018).

Note that the banks who get in trouble in the stress scenario are very large in terms of total assets. These seven or eight banks make up about 6 percent of total assets in 2016 and 2018. The eight banks in 2017 make up a share of 8 percent of the total assets of all banks participating in the stress test. In terms of the GDP of the Eurozone 19 the total assets of distressed banks make up a share of 56, 72 and 52 percent. The institutions which come into trouble are thus really huge and certainly too big to rescue for the national states in which they are residing.

In terms of losses in Core Tier 1 equity relative to the initial position we can say that in aggregate terms the loss of tier 1 equity in the stress scenarios would be at about 16% in 2016, 17% in 2018 and 16% relative to the aggregate tier 1 equity position at the initial date.

Results under the assumption of deleveraging Now let us compare these numbers under the assumption that we also factor in potential fire sales of sovereign bonds. Note that when computing the price impact, according to our impact equation (11) all parameters, except the parameter κ, are pinned down by data. From κ we only know that it is empirically "of order unity", which allows for quite a wide range of values. If we had a time series of observed impact events, we could estimate the value of this parameter. Here we can only make assumptions, which are more of less arbitrary. The order unity constraint is, for example, compatible with values between 1 and 9 but not with 20 or 50. For our simulation we set $\kappa = 5$.

When we take into account the potential for deleveraging we have to compute a fire sale equilibrium for all stress test horizons. Given our data, it turns out that the fixed points are unique. The values of the discount at the fire sale equilibrium is given in Table 6.

Bond	δ^*_{2016}	δ^*_{2017}	δ^*_{2018}
DE	0.0192	0.0213	0.0198
ES	0.0021	0.0021	0.0021
FR	0.0327	0.0363	0.0338
GB	0.0416	0.0439	0.0434
IT	0.0438	0.0558	0.0477
JP	0.0023	0.0027	0.0025
US	0.0169	0.0187	0.0176
Rest_of_the_world	0.0055	0.0063	0.0057

Table 6: Values of the fixed point δ^*. The rows display the different asset classes of marketable securities, the columns display the value of δ^* for the years 2016, 2017 and 2018 in the adverse EBA scenario. The parameter κ is set to a value of 5 in this computation. For the given data the fixed points are unique. We thus only report one value for each security class.

Is this impact large or low? We can get a feeling for the oder of magnitude by bench-marking the price impact against a hypothetical maximum impact which can occur here if all banks would sell their *entire* sovereign bond portfolio. The result of such a hypothetical sovereign bond "meltdown-situation" is shown in Table 7. From the table we can see that the the impact

Bond	$\delta_{\max}$	$\delta^*_{2016}/\delta_{\max}$	$\delta^*_{2017}/\delta_{\max}$	$\delta^*_{2018}/\delta_{\max}$
DE	0.05	0.40	0.44	0.41
ES	0.06	0.03	0.03	0.03
FR	0.07	0.48	0.54	0.50
GB	0.06	0.75	0.79	0.78
IT	0.11	0.41	0.52	0.44
JP	0.00	0.77	0.92	0.84
US	0.03	0.56	0.62	0.58
Rest_of_the_world	0.01	0.39	0.45	0.41

Table 7: Maximum price impact - the impact which would result if all banks sold their entire bond portfolio - and the relative impact in the EBA stress scenario compared to the maximum impact for all adverse scenarios.

in the stress scenario is about half of the impact of a situation in which every bank would sell its entire sovereign bond portfolio. This means that the price impact in a stress scenario can be significant.

In terms of banks, which exceed the threshold of critical leverage $\lambda^* = 33$ under such an evaluation of losses we see that we can observe a "systemic effect". The deleveraging effects push banks beyond the critical threshold which would have stayed below the threshold in the EBA scenario. We have now two additional banks, which get into trouble, as a result of the deleveraging "dynamics'": Banco Popolare - Società Cooperativa and BNP Paribas.[10] One of them is huge: In terms of total assets BNP Paribas is the second largest bank in the sample. Banco Popolare - Società Cooperativa is only the 40th largest bank. Their combined total assets amount to roughly 7% of the entire total assets of all banks combined or about 17% of Euro 19 GDP. This means that the factoring in of deleveraging losses reveal indeed a huge amount of *additional* losses which will be concealed in the traditional EBA approach.

We observe that we can not gauge the entire potential of deleveraging, given our data. Of the 9 banks which are at or above the threshold of $\lambda^* = 33$ N.V. Bank Nederlandse Gemeenten, Lloyds Banking Group Plc, Deutsche Bank AG, Banca Monte dei Paschi di Siena S.p.A. and Société Générale S.A. sell their *entire* bond portfolio but still are unable to restore a stable capital structure. They are not able to restore even the critical leverage of 33. If we had more marketable assets in our data the deleveraging of significant

[10]PNB Paribas is in trouble in the deleveraging scenario in all adverse scenarios, while it is not above the threshold in the adverse EBA scenario 2016 and 2018.

institutions would affect other asset classes and would be bigger. It can not be excluded that we might even run into a major systemic crisis.

Cont and Schaanning [2016] discuss for the EBA 2016 data at which threshold of losses fire selling of marketable assets might cascade into a fully fledged systemic crisis. We refer the interested readers for details of such a threshold analysis to their paper.

Looking at losses in aggregate Core Tier 1 equity taking deleveraging into account, the numbers are a loss of 19%, 21% and 19% in the adverse scenario at the different horizons of 2016, 2017 and 2018. This is significantly more than the 16%, 17% and 16% we observed for the stress scenarios not taking into account potential deleveraging effects. A more detailed picture can be given from looking the the distribution of tier 1 equity losses in the entire sample of banks under the assumption of no deleveraging compared to the case with deleveraging, shown in Figure 5. We see that in the case where we evaluate losses taking into account potential deleveraging the box-plot is stretched in the upper quartiles of the distribution as well as shifted upwards. This means that the entire distribution shifts and the losses become more severe.

The overall conclusion from the analysis of the 2016 EBA stress test is that whether we factor in potential deleveraging processes or not can make a significant difference. We are not in a position, given our data, to pin down more precisely when this difference will be most relevant. We have no precise data on the value of the parameter κ; we have no precise and full picture of marketable securities which can become part of a fire sale but only a small though significant subset: sovereign bonds. We also do not have an empirically validated theory of bank behavior in distress.

We think, however, that our model does not preclude the closure of these gaps in data and modeling in principle. Our results indicate that the significance of indirect losses would be even more pronounced, when these gaps are closed. We therefore think that loss evaluations should take potential deleveraging effects into account to get a more comprehensive picture of the potential fall outs from financial distress.

5 Conclusions

When considering potential impacts of financial distress in a banking stress test, taking into account deleveraging effects in the evaluation of losses is very important. Ignoring these effects leads to a perhaps too benign assessment of risk ignoring important and quantitatively significant indirect loss potentials. We show for the 2016 EBA stress test that a stress test ignoring these effects would overlook important and quantitatively significant losses. It would consider institutions as resilient which—at a closer look—are actually fragile.

A key message of our paper is that we are now able offer a framework to

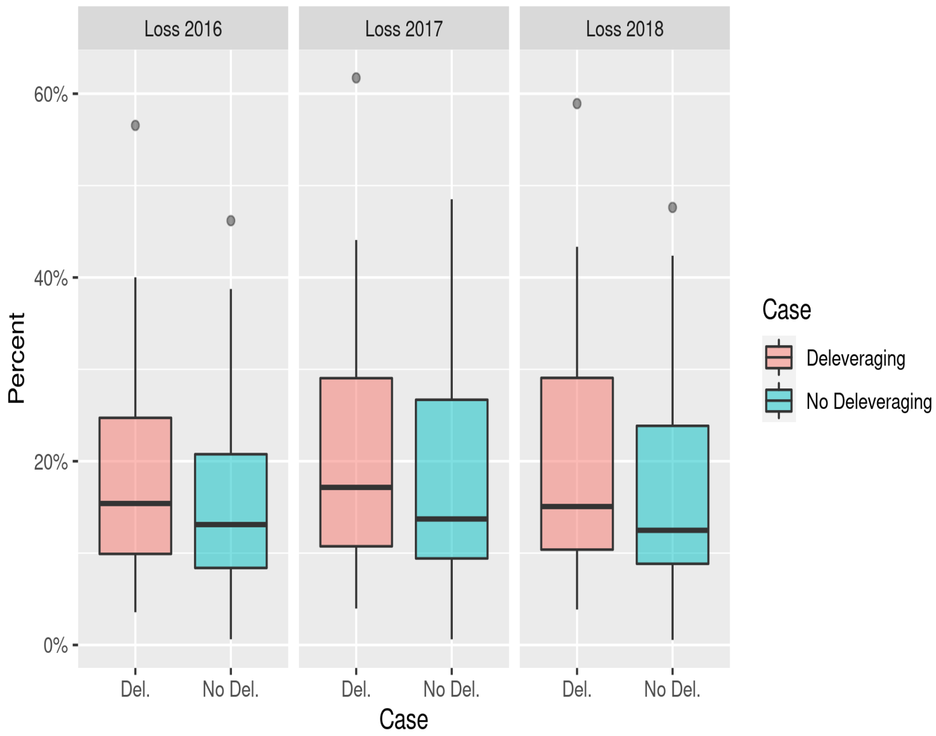

Figure 5: The Figure shows three comparative box-plots of the distribution of CET1 losses relative to the initial position for the adverse scenario in 2016, 2017 and 2018. The left box in each of the three plots (in red) gives a plot of the distribution of these losses for the case where deleveraging is factored in in the evaluation of losses. The right box plot in each of the three plots (the blue box) shows the distribution when potential deleveraging is ignored, as in the EBA stress test.

stress testing practitioners which is practical, simple and rigorous and can be integrated with the standard stress test very easily. The framework uses all familiar concepts of stress testing as practiced today and allows for a modular add on of a deleveraging analysis tool. It is our hope that the results of our paper will encourage stress testing practitioners to include deleveraging analysis in their toolkit and thereby help us to collectively improve and increase our knowledge about this key amplification mechanism of financial distress.

References

C. Aymanns, J. D. Farmer, A. M. Kleinnijenhuis, and T. Wetzer. Handbook of computational economics. volume 24, chapter Models of Financial Stability and Their Application in Stress Tests. North Holland, 2018.

Bank of England. Stress testing the uk banking system: 2017 results. Technical report, Bank of Enland, 2017. Online at `https://www.bankofengland.co.uk/stress-testing/2017/stress-testing-the-uk-banking-system-2017-results`.

J. P. Bouchaud. Price impact. *Working Paper*, 2017.

Y. Braouezec and L. Wagalath. Risk based capital requirements and optimal liquidation in a stress scenario. *Review of Finance*, 2(274):747 –782, 2018.

Y. Braouezec and L. Wagalath. Strategic fire-sales and price mediated contagion in the banking system. *European Journal of Operational Research*, 3(274):1180 – 1197, 2019.

M. Brunnermeier and L. Pedersen. Market liquidity and funding liquidity. *Review of Financial Studies*, 22:2201–2238, 2009.

F. Caccioli, J.-P. Bouchaud, and D. Farmer. A proposal for impact-adjusted valuation: Critical leverage and execution risk. *archivX*, 2012. URL `https://arxiv.org/pdf/1204.0922.pdf`.

R. Cont and E. Schaanning. Fire sales, indirect contagion and systemic stress testing. Technical report, Imperial College London, 2016. URL `ssrn.com/abstract=2541114`.

R. Cont and L. Wagalath. Running for the exit: Distressed selling and endogenous correlation in financial markets. *Mathematical Finance*, 23(4): 718–741, 2013.

R. Cont and L. Wagalath. Fire sales forensics: Measuring endogenous risk. *Mathematical Finance*, 26(4):835–866, 2016.

P. Cousot and R. Cousot. Constructive versions of Tarski's fixed point theorems. *Pacific Journal of Mathematics*, 82(1):43–57, 1979.

B. A. Davey and H. A. Priestley. *Introduction to Lattices and Order*. Cambridge University Press, 2 edition, 2002.

N. Detering, T. Mayert, K. Panagiotou, and D. Ritter. Suffocation fire sales. *Worinking Paper*, 2018. URL `https://www.fm.mathematik.uni-muenchen.de/download/publications/suffocating_fire_sales.pdf`.

N. Detering, T. Meyer-Brandis, K. Panagiotou, and D. Ritter. Suffocating fire sales. *archivX*, 2020. URL https://arxiv.org/abs/2006.08110.

F. Duarte and T. Eisenbach. Fire sales spillovers and systemic risk. *Federal Reserve Bank of New York Staff Report*, (645), 2013.

European Banking Authority. 2016 eu-wide stress test: Methodological note. 2016a. URL https://www.eba.europa.eu/sites/default/documents/files/documents/10180/1259315/e077989b-c5a2-4f1f-a683-da9a53f70704/2016%20EU-wide%20stress%20test-Methodological%20note.pdf.

European Banking Authority. 2016 eu-wide stress test: Results. 2016b. URL https://www.eba.europa.eu/sites/default/documents/files/documents/10180/1532819/e5fe6caf-8a52-4879-a694-d17a45f24c8c/2016-EU-wide-stress-test-Results.pdf?retry=1.

Z. Feinstein and F. El-Masri. The effects of leverage requirements andfire sales on financial contagion via assetliquidation strategies in financial networks. *Stat. Risk Model.*, 3-4(34):113 – 139, 2017.

J. Geanakoplos. NBER Macroeconomics annual. volume 24, chapter The Leverage Cycle, pages 1–65. University of Chicago Press, 2009.

N. Gennaioli, A. Martin, and S. Rossi. Banks,government bonds, and default: What do the data say? *Journal of Monetary Economics*, (98):98–113, 2018.

R. Greenwood, A. Landier, and D. Thesmar. Vulnerable banks. *Journal of Financial Economics*, 115(3):471–485, 2015.

S. C. Kleene. *Introduction to Metamathematics*. D. Van Nostrand, New York, 1952.

A. Kyle and A. Obizhaeva. Market microstructure invariance: Empirical hypothesis. *Econometrica*, 84(4):1345 – 1404, 2016.

H.-S. Shin. *Risk and Liquidity*. Oxford University Press, 2010.

L. Veraart. Distress and default contagion in financial networks. *Mathematical Finance*, 30(3):705–735, 2020.

A Proofs

A.1 Existence of a minimal and maximal fixed point of $\varphi \circ q$

Step 1: $\varphi \circ q$ is a self-map on a complete lattice: The set of possible fire sales discount vectors is

$$D := \{\boldsymbol{\delta} = (\delta_1, \ldots, \delta_I) : 0 \leq \delta_i \leq \delta_{i,max}\}. \tag{17}$$

The composite map $\varphi \circ q$ is a map from D onto itself. D is a compact subset of $\mathbb{R}^I$. Introduce a partial order relation $\leq$ on D by $\boldsymbol{\delta} \leq \boldsymbol{\delta}'$ if $\boldsymbol{\delta}_i \leq \boldsymbol{\delta}'_i$ for all $i \leq I$. With this order D is a lattice with least element $\mathbf{0} = (0, \ldots, 0)$. Each subset $X \subseteq D$ has a supremum $(\sup_{\boldsymbol{\delta} \in X} \delta_1, \ldots, \sup_{\boldsymbol{\delta} \in X} \delta_I)$, which is in D since D is compact in $\mathbb{R}^I$. Also, each subset $X \subseteq D$ has an infimum $(\inf_{\boldsymbol{\delta} \in X} \delta_1, \ldots, \inf_{\boldsymbol{\delta} \in X} \delta_I)$. So D is a complete lattice.

Step 2: If $\lambda^* > 1$, $\varphi \circ q$ is non-decreasing: The map $\phi \circ q$ is non-decreasing on D if the leverage threshold $\lambda^* > 1$: If $\boldsymbol{\delta} \leq \boldsymbol{\delta}'$ then $\phi \circ q(\boldsymbol{\delta}) \leq \phi \circ q(\boldsymbol{\delta}')$. $\boldsymbol{\delta} \leq \boldsymbol{\delta}'$ implies $S_b^1 \cdot \boldsymbol{\delta} \leq S_b^1 \cdot \boldsymbol{\delta}'$ by our choice of partial order $\leq$ and the fact that $S_b^1 \geq 0$. Now consider the following cases:

1. If $\boldsymbol{\delta} \leq \boldsymbol{\delta}'$ then $\lambda_{b,\max}(\boldsymbol{\delta}) \leq \lambda_{b,\max}(\boldsymbol{\delta}')$ and $\lambda_{b,\min}(\boldsymbol{\delta}) \leq \lambda_{b,\min}(\boldsymbol{\delta}')$ by (8).
2. For $\lambda^* \leq \lambda_{b,\min}(\boldsymbol{\delta})$, $\theta_b(\boldsymbol{\delta}') = 1$ and $\theta_b(\boldsymbol{\delta}) = 1$.
3. For $\lambda^* \geq \lambda_{b,\max}(\boldsymbol{\delta})$, $\theta_b(\boldsymbol{\delta}') = 0$ and $\theta_b(\boldsymbol{\delta}) = 0$.
4. For $\lambda_{b,\min}(\boldsymbol{\delta}) \leq \lambda^* \leq \lambda_{b,\min}(\boldsymbol{\delta}')$, $\theta_b(\boldsymbol{\delta}') = 1$ and $\theta_b(\boldsymbol{\delta}) < 1$.
5. For $\lambda_{b,\max}(\boldsymbol{\delta}) \leq \lambda^* \leq \lambda_{b,\max}(\boldsymbol{\delta}')$, $\theta_b(\boldsymbol{\delta}) = 0$ and $\theta_b(\boldsymbol{\delta}') > 0$.
6. For $\lambda_{b,\min}(\boldsymbol{\delta}') \leq \lambda^* \leq \lambda_{b,\max}(\boldsymbol{\delta})$, $0 < \theta_b(\boldsymbol{\delta}) < \theta_b(\boldsymbol{\delta}') < 1$.

In all cases $\theta_b(\boldsymbol{\delta}) \leq \theta_b(\boldsymbol{\delta}')$.

By (9) this implies $q_i(\boldsymbol{\delta}) \leq q_i(\boldsymbol{\delta}')$ for all security types i. Since by Assumption 3 the price impact functions φ_i are strictly increasing we get $\varphi(q(\boldsymbol{\delta})) \leq \varphi(q(\boldsymbol{\delta}'))$.

Step 3: Existence of a minial and maximal fixed point: We have argued already why the set D of possible fire sale discount factors is a complete lattice. By steps 1 and 2 the map $\varphi \circ q$ is an order preserving self-map from D to D. Hence by the Knaster-Tarski Fixed point theorem (see Davey and Priestley [2002], theorem 2.35, p. 50) the set of fixed points contains a maximum and a minimum element under the order $\leq$ on D.

A.2 $\boldsymbol{\delta} = \mathbf{0}$ as a fixed point of $\phi \circ q$

$\boldsymbol{\delta} = \mathbf{0}$ is a fixed point of $\phi \circ q$ if and only if all banks have $\lambda^* > \lambda_{b,\max}(\mathbf{0})$ in (7). To see this, first assume that $\lambda^* > \lambda_{b,\max}(\mathbf{0})$ for all banks. By (7) this implies that $\theta_b(\mathbf{0}) = 0$ for all banks b. Therefore $q_i(\mathbf{0}) = 0$ by (9) and $\phi(q(\mathbf{0})) = \mathbf{0}$. So $\mathbf{0}$ is a fixed point of $\phi \circ q$.

On the other hand, if $\mathbf{0}$ is a fixed point of $\phi \circ q$, $\phi(q(\mathbf{0})) = \mathbf{0}$, then $q(\mathbf{0}) = \mathbf{0}$, since by Assumption 3 φ is strictly increasing and $\varphi(\mathbf{0}) = \mathbf{0}$. By (9) q can be zero only if for all banks $\theta_b = 0$. By (12) $\theta_b(\mathbf{0}) = 0$ for all banks only if $\lambda^* > \lambda_{b,\max}(\mathbf{0})$ for all banks.

A.3 An example of multiple fire sale equilibria

That uniqueness can not be expected in general is demonstrated by the following example for a system of three banks, one security type, one loan type.

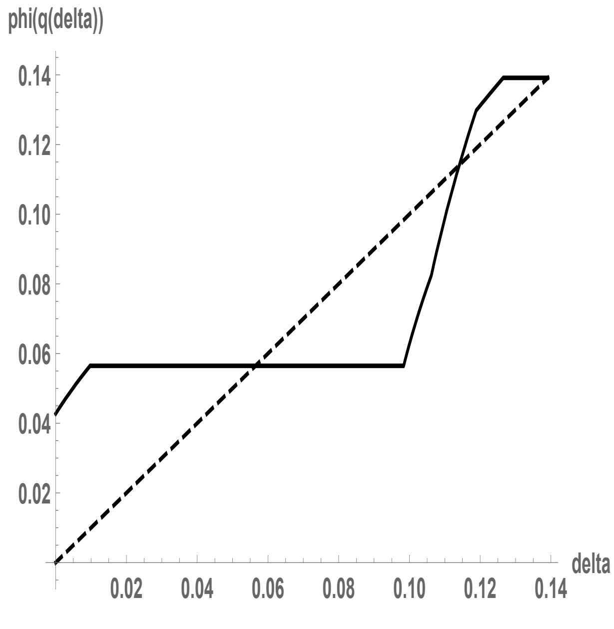

Figure 6: Plot of the map $\phi \circ q$: $[0, \delta_{\max}] \to [0, \delta_{\max}]$ in the example with three banks. It has three fixed points. The fixed points $\delta^* = 5.65\%$ and $\delta^* = 13.9\% = \delta_{\max}$ are stable. The fixed point $\delta^* = 11.4\%$ is unstable.

Under the map $\phi \circ q$, $[0, 11.4\%)$ is the domain of attraction for the first equilibrium $\delta^* = 5.65\%$. $(11.4\%, \delta_{\max}]$ is the is the domain of attraction for the third equilibrium $\delta^* = \delta_{\max}$. The second equilibrium, $\delta^* = 11.4\%$ is unstable.

time	balance	Bank 1	Bank 2	Bank 3
t=1	S	145	435	300
	L	1170	1245	1190
	e	28	80	65
	λ	46.9	21	22.9
$t = 1 + \tau$	S	0	410	283
$\delta^* = 5.65\%$	L	1170	1245	1190
	e	20	55	48
	λ	59.0	29.9	30.6
	θ^*	1	0	0
$t = 1 + \tau$	S	0	91	164
$\delta^* = 11.4\%$	L	1170	1245	1190
	e	11	30	30
	λ	102	44	44
	θ^*	1	0.76	0.38
$t = 1 + \tau$	S	0	0	0
$\delta^* = 13.9\%$	L	1170	1245	1190
	e	7.8	19	23
	λ	149	63	51
	θ^*	1	1	1

Table 8: State of the banking system before fire sales ($t = 1$). There are three fire sales equilibria ($t = 1 + \tau$). Bank 1 fails in all three equilibria. Banks 2 and 3 are in danger in the second equilibrium and fail in the third equilibrium. Assumed leverage constraint: $\lambda^* = 44$. Impact function: $\sqrt{0.000022V}$, where V is the total sales volume of all banks. $\delta_{\max} = 13.9\%$. In all equilibria, all banks are in status "red" or "green", depending on whether or not they meet the leverage constraint.

A.4 Computing the least and the most severe fixed point

The proof of theorem 2 is based on a version of the Kleene [1952] fixed point theorem formulated by Cousot and Cousot [1979]: Suppose $(L, \leq)$ is a complete lattice with least element 0 and $f : L \to L$ a monotone function. Consider the transfinite ascending chain $\{f^{\beta}(0)\}$ where β ranges over the ordinals, defined by $f^0(0) = 0, f^{\alpha+1}(0) = f(f^{\alpha}(0))$ for any ordinal α and $f^{\alpha}(0) = \sup_{\beta<\alpha} f^{\beta}(0)$ for any limit ordinal α. Then the least fixed point of f is $f^{\gamma}(0)$ for some ordinal γ less or equal the height of the lattice L.

In our context we take L to be the set D defined in (17). For f we take the map $\varphi \circ q : D \to D$. We have seen in the proof of theorem 1 that $\phi \circ q$ is monotone. Now the Cousot and Cousot [1979] theorem implies that $\phi \circ q$ has a least fixed point in D and that this fixed point is the supremum of $\{(\phi \circ q)^n(0) : n = 0, 1, 2, \ldots\}$.

The proof for the approximation of the most severe fixed point is also based on Cousot and Cousot [1979]. Starting from the greatest element $\boldsymbol{\delta}_{\max}$ of the lattice D the chain $\{(\varphi \circ q)^n(\boldsymbol{\delta}_{\max}) : n = 0, 1, 2, \ldots\}$ is a descending chain. The greatest fixed point $\boldsymbol{\delta}^*_{\max}$ is the infimum of this descending chain.

B Data: Sources and compilation

B.1 EBA - Exposures and Impairment Data:

The exposure data are composed from raw data provided via the web-page of the European banking authority.[11]

Exposure and impairment data We first retrieve the IRB credit risk exposures from the file `TRA_CR.csv` and filter the data-file according to Table 9

The exposure values for F-IRB and A-IRB positions as well as for defaulted and non defaulted assets are added up for each bank and each country to which the banks are exposed for each of the different exposures. [12]

For the impairment data, which report impairment rates[13] we retrieve from the file `TRA_CR.csv` as in table 10.

[11] `https://eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2016`. Readers who are interested in a line by line documentation of how the exposure data are constructed precisely are welcome to study the R-scripts `make_balance_sheets_2016.R` which is contained in the `data-raw` subfolder in the github repository `https://github.com/Martin-Summer-1090/syslosseval` which hosts the code used for all data compilations and computations used in this paper. We describe the filters used for the 2016 data here in detail.

[12] This aggregation step is requires necessary because the EBA data leave the respective field for the aggregate IRB exposure empty in the raw data file.

[13] The impairment rate is a ratio of the impairment flow which contains the probability of default as well as the loss given default, and the exposure. The EBA file contains only the ratio but not the nominator and the denominator of this ratio separately.

Variable	Value	Meaning
Period	201512	December 31 2015
Portfolio	3,4	Foundation IRB (F-IRB), Advances IRB (A-IRB)
Item	1690201	Exposure values (IRB)
Scenario	1	Actual Figures
Status	1,2	Non defaulted assets, defaulted assets
Exposure	1100, 2000,	Central banks and government, Institutions
	3000, 4000,	Corporates, Retail
	6100, 6300	Equity, Other
Perf_status	0	No-breakdown by performance status

Table 9: Query scheme for the IRB exposures from the file `TRA_CR.csv`.

Variable	Value	Meaning
Period	201612, 201712, 201812	December 31 2016, 2017, 2018
Portfolio	2	IRB
Item	1690205	Impairment rate (IRB)
Scenario	2,3	Baseline, Adverse
Status	0	No break down by status
Exposure	1100, 2000,	Central banks and government, Institutions
	3000, 4000,	Corporates, Retail
	6100, 6300	Equity, Other
Perf_status	0	No-breakdown by performance status

Table 10: Query scheme for the IRB impairments from the file `TRA_CR.csv`.

These impairment rates are reported for one year, two year and three years into the future for a baseline as well as for an adverse scenario.

The next step is to retrieve all the exposures reported according to the STA approach. Here the query scheme is as in table 11

Variable	Value	Meaning
Period	201512	December 31 2015
Portfolio	1	Standard Approach (STA)
Item	1690301	Exposure values (STA)
Scenario	1	Actual Figures
Status	1,2	Non defaulted assets, defaulted assets
Exposure	1100, 1200	Central banks and government, regional government
	1300, 1400	Public sector entities, multilateral development banks
	1500, 1600	International organisations, central banks
	1700, 2000	General governments, institutions
	3000, 4000	Corporates, retail
	5000, 6400	Secured by mortgages, Items with particularly high risk
	6500, 6600	Covered bonds, Claims on inst. and corp. with a ST credit assessment
	6700, 6100	Collective investments undertakings (CIU), equity
	6200, 6300	Securitistaion, Other non-credit obligation assets
Perf_status	0	No-breakdown by performance status

Table 11: Query scheme for the STA exposures from the file `TRA_CR.csv.` text.

For the impairment data on the STA positions we use the query described in Table 12. As with the IRB case we organise these data in the same format in one long-format data table with the same variables.

Data on bank equity We also retrieve data which are independent of the accounting framework (IRB, STA) and which are stored in the data file `TRA_OTH.csv` on the EBA website. These data are the common tier 1 equity, tier 1 equity and the leverage ratio. The data are retrieved using the following query summarized in Table 13

From the Table 9 and Table 11 it can be seen that the IRB and STA data do not use the same classification of assets. To organize the data in a coherent and uniform balance sheet we have to make some assumptions. We map the STA positions to the IRB scheme. We make our assumption on the mapping precise here:

Assumption 4. *Our mapping uses the following rules:*

1. *Exposures 1100, 1200, 1300, 1400, 1500, 1600 and 1700 STA are mapped into Exposure 1100 IRB and then added with the IRB values into an overall position for central banks and central government.*

Variable	Value	Meaning
Period	201612, 201712, 201812	December 31 2016, 2017, 2018
Portfolio	1	Standard Approach (STA)
Item	1690305	Impairment rate (STA)
Scenario	2, 3	Baseline scenario, adverse scenario
Status	0	No break down by status
Exposure	1100, 1200	Central banks and government, regional government
	1300, 1400	Public sector entities, multilateral development banks
	1500, 1600	International organisations, central banks
	1700, 2000	General governments, institutions
	3000, 4000	Corporates, retail
	5000, 6400	Secured by mortgages, Items with particularly high risk
	6500, 6600	Covered bonds, Cl. on inst. and corp. with a ST c.a.
	6700, 6100	Collective investments undertakings (CIU), equity
	6200, 6300	Securitistaion, Other non-credit obligation assets
Perf_status	0	No-breakdown by performance status

Table 12: Query scheme for the STA impairments from the file `TRA_CR.csv`

Variable	Value	Meaning
Period	201512	December 31 2015
Item	1690106	Common Tier 1 equity
Scenario	1	Actual Figures

Table 13: Query scheme for the equity and leverage ratio figures in `TRA_OTH.csv`.

2. *Exposure 2000 in IRB and Exposure 2000 STA are added to one position Institutions.*

3. *Exposure 3000 in IRB and Exposure 3000 STA are added to one position Corporates.*

4. *Exposure 4000 in IRB and Exposure 4000 and 5000 in STA are added to one position Retail.*

5. *Exposure 6100 in IRB and Exposure 6100 in STA are added to one position Equity.*

6. *Exposure 6200 in IRB and Exposure 6200 in STA are added to one position Securitisation.*

7. *Exposure 6300 in IRB and Exposure 6300 in STA are added to one position Other.*

When we have to add up impairment rates across STA categories we use the exposure weighted averages across the subcategories for aggregation of impairment rates.

The biggest exposures are held against, governments, corporates and households. The positions which are classified as equity, and other obligations are significantly smaller. Exposures towards institutions are in between. A more detailed picture of the exposure distribution in the cross section of banks is given in Figure 7.

B.2 Total assets and residual position.

The EBA exposures reports positions which are subject to credit risk according to the supervisory rules. Thus when we add up the assets of each bank, reported in the `TRA_CR.csv` file, we will not get the total assets of the bank but most of the times less than that and in rare cases more than that. These gaps can be quantitatively substantial. The reported sum of assets subject to credit risk is smaller than the total assets reported in the published balance sheet of a bank, if the regulatory reporting framework allows the bank to exclude certain exposures from reporting because they have no credit risk (according to the reporting requirements). Sometimes an actual exposure is considered as not revealing the actual risk and the regulatory framework forces banks to apply certian multipliers to these positions. In that case the total value of credit risky exposures may even exceed the value of total assets reported in the balance-sheet.

How do we deal with this? We thus introduce the residual position as an additional artificial asset class, if the value of the total EBA exposures is less than the total assets reported in the balance sheet. In the case the EBA position is larger we take this value as the total asset figure.

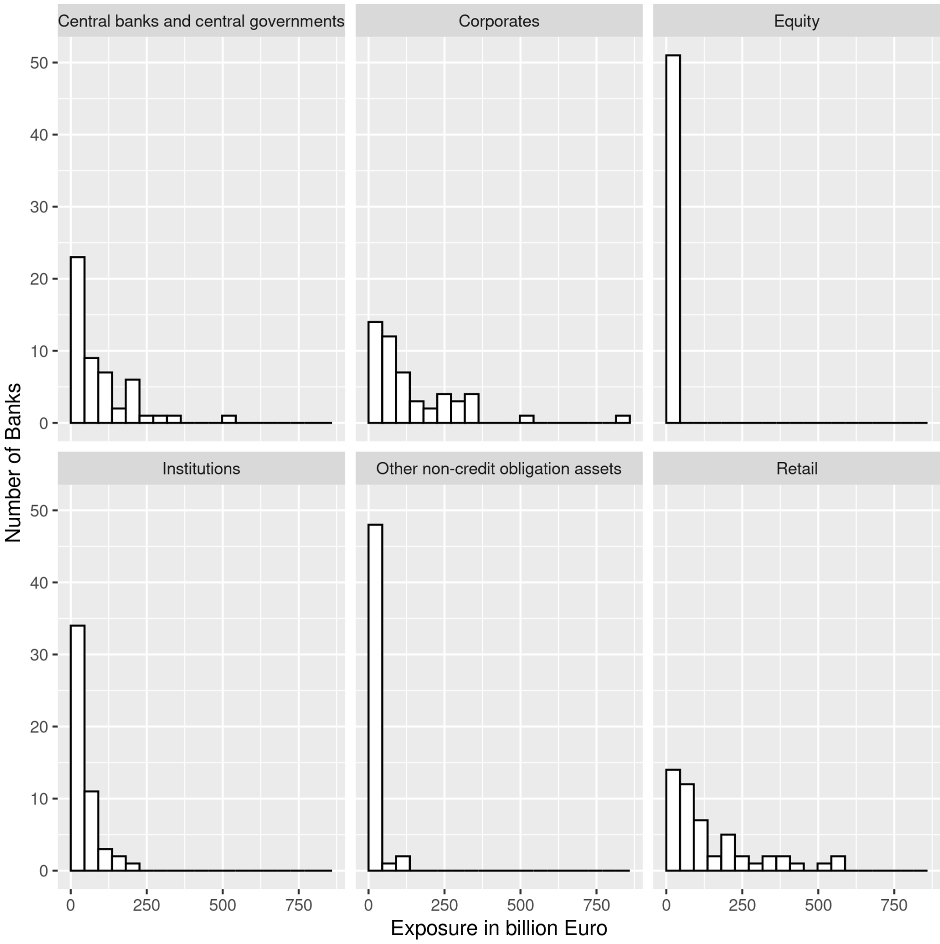

Figure 7: Histograms of exposure size in the different IRB exposure categories in the cross section of banks in the EBA 2016 stress test.

Unfortunately these residuals can be fairly large and can go in either direction. They also show no clear systematic pattern over time. In the 2016 sample the negative gaps dominate the value gap. We can not fully clarify these discrepancies which must have its deeper roots in the financial regulatory reporting framework. To get a rough quantitative impression about the magnitude of these discrepancies we show two histograms displaying the distribution of the gap between the total value of reported EBA assets and the value of total assets as reported in the balance sheet as a percentage of total assets.

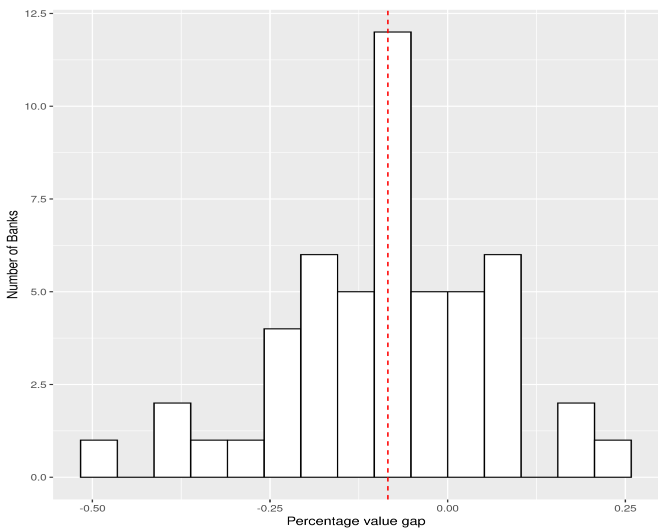

Figure 8: Histogram of the value gap between the total value of reported EBA exposures and total assets as reported in the bank balance sheet as a percentage of reported total assets. The gap is on average slightly negative in the 2016 sample.

B.3 Attributing sovereign bond exposures

The EBA data contain information about the exposures of each bank in government bonds. This information is stored in the `TRA_SOV.csv` file on the EBA website. Sovereign exposures contain subcategories of securities available for sale (AFS), positions designated at fair value through profit and loss (FVO) and securities held for trading (HTF). This allows a split of the overall position into loans and securities. This allows the application of our framework to a limited but very important security class held on the bank balance sheet.

The precise query for these data is given in Table 15

We subtract the sum of the exposure values of 1690503, 1690506, 1690507 and 1690508 from the total position 1100 governments and central banks. This difference is the value we attribute as a sovereign bond position for each

Variable	Value	Meaning
Period	201512	December 31 2015
Item	1690503, 1690506	
	1690507, 1690508	Net direct exposures AFS, FVO, HFT
SOV_maturity	8	All maturities

Table 14: Query scheme for sovereign bond figures from the file `TRA_SOV.csv` AFS means available for sale, FVO means fair value through profit and loss, and HFT means held fro trading.

of the 51 banks in our sample. Though the order of magnitude of sovereign bond exposures in the total assets of the bank look right on average in the data there are some problems we can not fully explain. It is for instance not always the case that the sum of 1690503, 1690506, 1690507 and 1690508 is strictly smaller than the total position 1100. If 1100 reports the entire exposure to central banks and central governments including all loans and securities this should in theory be the case. It is the case for most banks but not all of them. In the case where this sum exceeds the value repored under 1100 we assume that the entire exposure is held in government bonds.

Variable	Value	Meaning
Period	201912	December 31 2019
Item	2020811	Total carrying amount of non-der. financial assets
SOV_maturity	8	all maturities
Accounting Portfolio	0	No breakdown by accounting portfolio

Table 15: Query scheme for sovereign bond figures from the file `TRA_SOV.csv` AFS means available for sale, FVO means fair value through profit and loss, and HFT means held fro trading.

Table 16 shows the geographical distribution of sovereign bond exposures. About half of the exposure is in countries for which we can access public data on average daily volume and the volatility of sovereign bond prices.

We finally show a histogram displaying the distribution of bond exposures in the cross section of banks towards every of the individual regions in Figure 9.

B.4 Market data for bonds and the residual position

Bond prices Bond prices are retrieved from `http://us.spindices.com`. Table 17 gives the data we get from this site. We retrieve the data for Germany, Spain, Great Britain, France, Italy, Japan and the United states. All other countries are aggregated in a position rest of the world.

We show the time series of the indices in Figure 10

Country	Share
DE	0.11
ES	0.08
FR	0.09
GB	0.10
IT	0.09
JP	0.01
US	0.11
Rest of the world	41.00

Table 16: Geographical distribution of sovereign bonds across regions for the aggregate banking system in 2016.

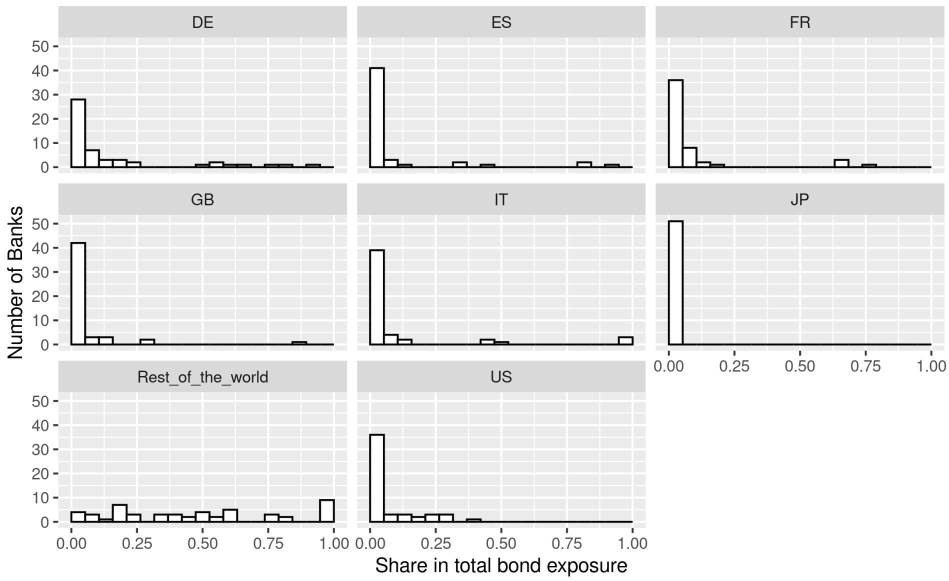

Figure 9: Histograms of exposure size in the different IRB exposure categories in the cross section of banks in the EBA 2016 stress test.

Average daily volume for sovereign bonds We collect the data for average daily volumes from various public sources from the internet. This collection process is rather messy, because the data are only partially available. They are stored in different formats and are often only available as graphics. We give a table describing the sources for our volumes data, for the countries we can actually use in our analysis.

To compute an average daily volume figure for the rest of the world we use an idea from Cont and Schaanning [2016]. They observe a high correlation between the nominal debt outstanding and the average daily volume. Figures about the nominal debt outstanding can be retrieved from

Country	Index
Germany	Germany Sovereign Bond Index
Spain	Spain Sovereign Bond Index
France	France Sovereign Bond Index
Great Britain	U.K. Gilt Bond Index
Italy	Italy Sovereign Bond Index
Japan	Japan Sovereign Bond Index
United States	U.S. Treasury Bond Index
Rest of the World	S&P Global Developed Aggregate Ex-Collateralized Bond Index

Table 17: Description and sources of sovereign bond indices used in the paper.

Figure 10: Time series of the different bond indices of Table 10. The graph shows that the bonds of GB and Italy are the most volatile while US, JP and DE show the least volatility.

Country	Link
Germany	https://www.deutsche-finanzagentur.de/en/institutional-investors/secondary-market/
Spain	https://www.tesoro.es/sites/default/files/estadisticas/15I.xlsx
France	https://www.afme.eu/reports/data/details//Government-Bond-Data-Report-Q2-2019
Great Britain	ttps://www.dmo.gov.uk/data/gilt-market/turnover-data/
Italy	https://infostat.bancaditalia.it/
Japan	https://asianbondsonline.adb.org/data-portal/
US	https://www.sifma.org/resources/research/us-treasury-trading-volume/
Rest of the world	computed

Table 18: Sources of average daily volumes data of sovereign bonds.

the BIS international debt statistics (`https://www.bis.org/statistics/secstats.htm`). Denote the nominal debt outstanding in country i by N_i and using the ADV data we have, following Cont and Schaanning [2016], we run the regression:

$$\log ADV_i = c_1 \log(N_i) + c_0 + \epsilon_i$$

Then we use the values of the estimated parameters c_1 and c_0 and the relation to assign an expected average daily volume for the rest of the world by adding all nominal values outstanding except for the countries where we have direct observations.

Readers who are interested in all details are referred to the R-script `make_price_volume_data.R` in the folder `data-raw` in the GitHub repository for the `syslosseval` package. Here we show the numbers in Table 19.

Country	Volume	Unit	Currency
DE	17039.68	Million	Euro
ES	8288.12	Million	Euro
FR	8500.00	Million	Euro
IT	5164.63	Million	Euro
JP	36736.51	Million	Euro
GB	34853.66	Million	Euro
US	467657.66	Million	Euro
Rest_of_the_world	97852.20	Million	Euro

Table 19: Average daily volumes of different sovereign bond classes for the year 2016. The individual country values are from public data sources listed in table 18. The figure for the rest of the world is based on an estimation.

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