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Increasing Life Expectancy and Pay-As-You-Go Pension Systems

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Editorial

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Increasing Life Expectancy and Pay-As-You-Go Pension Systems

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Abstract

In this paper I study how PAYG pension systems of the notional defined contribution type can be designed such that they remain financially stable in the presence of increasing life expectancy. For this to happen two crucial parameters must be set in an appropriate way. First, the remaining life expectancy has to be based on a cross-section measure and, second, the notional interest rate has to include a correction for labor force increases that are only due to rises in the retirement age which are necessary to "neutralize" the increase in life expectancy. It is shown that the self-stabilization is effective for various patterns of retirement behavior and also—under certain assumptions—if life expectancy reaches an upper limit.

Keywords: Pension System; Demographic Change; Financial Stability;
JEL-Classification: H55; J1; J18; J26

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1 Introduction

Pension systems around the world have come under severe pressure from the two-sided demographic development: declining fertility rates and increasing life expectancy. The latter aspect is of particular interest, since it represents an ongoing process with considerable and far-reaching budgetary consequences. For the EU-countries, e.g., life expectancy at birth is projected to increase over the next 50 years by about 7.5 years. This increase is one of the main factors behind the projected rise in the old-age dependency ratio from 25.4% in 2008 to 53.5% in 2050 (EPC, 2009). This development is a particular challenge for pay-as-you-go (PAYG) pension systems. In their traditional organization PAYG systems are based on the implicit assumption of a stationary age structure while ideally they should be designed in such a way as to automatically react to the steady increases in longevity. The most recent issue of the OECD's *Pensions at a Glance* deals in detail with the links between life expectancy and retirement and it documents that "around half of OECD countries have elements in their mandatory retirement-income provision that provide an automatic link between pensions and a change in life expectancy" (OECD 2011, p. 81). In fact, the report continues to state that "the rapid spread of such life-expectancy adjustments has a strong claim to be the most important innovation of pension policy in recent years" (OECD 2011, p. 82). In a related paper Edward Whitehouse calls this development a "quiet revolution in pension policy" (Whitehouse 2007, p. 5).

Despite this claim by the OECD that automatic life-expectancy-adjustments are a "quiet revolution" and the "most important innovation" in pension policy there does not exist much empirical and—even less though—theoretical work on the basic functioning, the appropriate design and the main properties of these automatic mechanisms. In this paper I try to fill this gap and focus on the effect of increasing life expectancy in notional defined contribution (NDC) systems (cf. Holzmann & Palmer 2006). NDC systems are of particular interest for a number of reasons. First, they are an increasingly popular variant of the PAYG pension system and—starting with the pioneering work of Sweden—currently around 10 countries have implemented a NDC framework. Second, international institutions like the World Bank, the OECD and the European Commission use the NDC structure as a useful reference point (if not benchmark) to discuss reform proposals and to enhance the transnational portability of pension rights. Third, NDC systems are a natural starting point to analyze the linkage between life expectancy and retirement age since they are explicitly designed in a way such as to react to demographic changes. As will be described in a later section, NDC systems take increases in life expectancy into consideration when the notional capital (i.e. the accumulated contributions) is annuitized. Longer life expectancy will, ceteris paribus, decrease pension benefits, while later retirement will increase them.

While this basic mechanism has been one of the main rationales behind the introduction and propagation of NDC systems, much less is known about the details of its functioning and its optimal design. This, however, is important since self-stabilization will only be achieved if the NDC system is built on accurate construction principles, as will be shown in this paper. In particular, I will demonstrate that a good design involves the determination of two crucial parameters. First, the "notional interest rate" (or the "rate of return") that specifies how past contributions to the pension system are revalued over time.[1] Second, the measure of "remaining life expectancy" that is used to calculate the first pension benefit at the time of retirement.

The prevailing opinion on this topic is that one should use the growth rate of the wage bill (or, to be precise, the sum of total contributions) as the notional interest rate[2] and the cohort (i.e. forecasted) life expectancy in order to determine the size of the pension annuity.[3] I will show that this conventional wisdom has to be corrected along both dimensions. First, in as far as the measure of life expectancy is concerned I demonstrate that it is sufficient to use periodic life expectancy to calculate the annuity. This is an attractive feature since the determination of remaining life expectancy is then only based on known, cross-section data and does not involve a process of (potentially controversial and politicized) forecasting. Second, I also show that the use of the growth rate of the wage bill is not appropriate in the case of increasing life expectancy. The reason for this is straightforward. When average longevity increases then the cohort-specific retirement age has to rise as well just in order to keep the proportion of retired years to working years constant. This "neutralizing" postponement of retirement, however, increases by itself the total size of the labor force even if the size of cohorts is constant. Using the growth rate of the wage bill would thus grant an "excessive" rate of return thereby causing a structural deficit of the pension system. The appropriate notional interest rate has to be corrected for this effect. I show that a combination of this adjusted notional interest

[1] In the generic version of the NDC system the rate at which ongoing pension are adjusted over time is also set equal to the notional interest rate.

[2] "Viewed from a macroeconomic perspective, the 'natural' rate of return for an NDC system is the implicit return of a PAYG system: that is, the growth rate of the contribution bill" (Börsch-Supan 2003, p. 38)

[3] "The generic NDC annuity embodies [...] cohort life expectancy at the time the annuity is claimed" (Palmer 2006, p. 18).

rate and period life expectancy as the concept to calculate the annuity establishes a self-stabilizing social security system. Whether the budget is balanced in every period or only over time depends on the pattern of retirement behavior. I show that the deficit is always zero if each cohort chooses its retirement age in such a way as to fix the dependency ratio. On the other hand, if the retirement age is assumed to be constant over time then the budget is also (almost) balanced in every period. For an arbitrarily fluctuating pattern of retirement ages, however, the balance will only be achieved in the long run.

These results are derived under the assumption that life expectancy increases in a linear fashion. This is in line with the demographic literature. What is more, it has been argued that this linear development is also the best prediction for the behavior of life expectancy for the next 100 years (cf. Oeppen & Vaupel 2002). Nevertheless, I will also discuss the case where life expectancy is assumed to reach a maximum age. I show that the budgetary impact depends again on the development of retirement ages and on the predictability of the halt in life expectancy. Under plausible assumptions about both factors it can be shown that the adjustment process will also be compatible with a balanced budget or even a budgetary surplus.

The related literature includes empirical analyses, simulation studies and also s small number of theoretical papers. Whitehouse (2007), OECD (2011) and OECD (2012) contain information about the links between life expectancy and various parameters in existing pension systems of OECD countries. Alho et al. (2005) and Auerbach & Lee (2009) use stochastic simulation models in order to evaluate and compare the risk-sharing characteristics of alternative public pension schemes. Since these models allow for a stochastic development of mortality rates they also—implicitly—show how different systems react to changing life expectancy. On the other hand, these papers do not include a systematic discussion on the working and the different design features of automatic life expectancy adjustments. Shoven & Goda (2008) and Heeringa & Bovenberg (2009) are related papers that study how "life expectancy indexation" could be used to stabilize the budget of the public pension systems in specific countries (the US and the Netherlands, respectively). The latter work also contains a stylized model of the use of changes in the retirement age in order to balance increases in longevity. The paper, however, does not compare different formulations of such an indexation. Andersen (2008), on the other hand, uses a two-period model to derive that a "social security system cannot be maintained by simply indexing pension ages to longevity". This is in contrast to the results of the present paper and I will discuss later how to explain this discrepancy. Ludwig & Reiter (2010) study the optimal policy response of a social planner in the presence of demographic shocks.

Valdés-Prieto (2000) discusses the ability of NDC system to run on "auto-pilot" but he mainly focuses on the role of changing fertility patterns. Settergren & Mikula (2006) and Palmer (2006) present results that are related to the ones of the present paper although they derive them in a different framework (involving the "turnover duration") and they also lack an explicit treatment of the case with constantly increasing life expectancy. I will come back to some of the related literature in section 4.4.

The paper is structured as follows. In the next section I discuss the assumption about the development of life expectancy and I present the structure of a general PAYG system. In section 3 I then describe and formalize the main features of a NDC system and I discuss the two main parameters: the notional interest rate and the measure of life expectancy. In section 4 I show how one can determine these two parameters in order to design a NDC system that is self-stabilizing in the case of constant cohort sizes and linearly increasing life expectancy. In section 5 I analyze the case where life expectancy reaches an upper limit and section 6 concludes.

2 The Model

2.1 Basic set-up

I work with a model in continuous time (cf. Bommier & Lee 2000). In every instant of time t a generation is born that has size $N(t)$ and lives for $X(t)$ years. All members of generation t start to work as soon as they are born and they remain in the labor market for $B(t)$ periods, earning a wage $W(t+a)$ during each of these working periods ($a \in [0, B(t)]$).[4] Thereafter, they receive a pension benefit $P(t, t+a)$ in each period of retirement ($a \in [B(t), X(t)]$). While working, individuals pay contributions to the PAYG pension system at rate $\tau(t+a)$. The (relative) pension level is defined as $q(t, t+a) = \frac{P(t,t+a)}{W(t+a)}$ and the growth rate of wages is denoted by $g(t)$, i.e. $W(t) = W(0)e^{\int_0^t g(s)\,\mathrm{d}s}$.

As far as the development of life expectancy is concerned I make a number of assumptions that allow for simple and intuitive expressions. First, I focus on a representative member of generation t and I abstract from all *intra*generational differences. In particular, I assume that all members of a generation reach their cohort life expectancy $X(t)$. Second, I assume that the retirement age is non-decreasing over time, i.e. $B(t+dt) \geq B(t)$. This makes it possible to express all aggregate values in a compact form without the use

[4] I abstract here from the existence of an age-earnings profile. At each point in time all workers are assumed to earn the same wage.

of "indicator variables". Third, life expectancy is assumed to increase linearly over time:

$$X(t) = X(0) + \gamma \cdot t, \tag{1}$$

where $0 \leq \gamma < 1$. This assumption is in line with the empirical literature. Oeppen & Vaupel (2002), e.g., analyze "record female life expectancy" (i.e. the highest value for female life expectancy reported in any country for which data are available) from 1840 to 2000 and they show that it follows an almost perfect linear development with a slope parameter of 1/4.[5] This is confirmed by Lee (2003) who refers to a number of studies that have found a linear trend in life expectancy for a large sample of industrial countries with slope parameters between 0.15 and 0.25. There exists a long debate on whether it is reasonable to assume a constant increase in life expectancy or whether it is better to work with a model that assumes some biologically determined maximum age. I will come back to this issue in a later section.

In order to be able to distinguish clearly between the viewpoint of generation t (i.e. the one born in t) and the outlook of the pension system in period t I introduce two further variables. $\widetilde{X}(t)$ stands for period life expectancy, i.e. the highest age observed in period t. $\widetilde{B}(t)$, on the other hand, denotes the number of working years of the generation that retires in period t. In general it will be the case that $\widetilde{X}(t) \neq X(t)$ and $\widetilde{B}(t) \neq B(t)$.

Period life expectancy in period t can be calculated from cohort life expectancy by the following relation: $X(t - \widetilde{X}(t)) = \widetilde{X}(t)$. It comes out as:

$$\widetilde{X}(t) = \frac{1}{1+\gamma} X(t). \tag{2}$$

The increase in period life expectancy is given by $\frac{d\widetilde{X}(t)}{dt} = \frac{\gamma}{1+\gamma}$. A value of $\frac{\gamma}{1+\gamma} = 1/4$ thus implies $\gamma = 1/3$, while $\frac{\gamma}{1+\gamma} = 1/5$ corresponds to $\gamma = 1/4$. For the following numerical examples I will use the latter value which is about the mid-point of the estimates reported in Lee (2003).[6]

[5] Their data refer to the "mean age at death under current mortality conditions", i.e. to *period* life expectancy. Equation (1), however, refers to the development of *cohort* life expectancy. I will show later that the estimated slope in Oeppen & Vaupel (2002) implies a value of $\gamma = 1/3$.

[6] In the demographic literature there exist a number of papers that have studied the relation between different life expectancy concepts in standard mortality models. Goldstein (2006) and Missov & Lenart (2011) show, e.g., that under special assumptions (like a "linear shift model" or a Gompertz mortality model with constant yearly improvements at all ages) period and cohort life expectancy increase in a linear fashion. The formulation in (1) can thus also be understood as a short-cut for a fully-fledged model with mortality rates.

2.2 Budget of the pension system

The total size of the active population $L(t)$, of the retired population $R(t)$ and the resulting dependency ratio $z(t)$ are given by:

$$L(t) = \int_0^{\widetilde{B}(t)} N(t-a)\,\mathrm{d}a, \tag{3}$$

$$R(t) = \int_{\widetilde{B}(t)}^{\widetilde{X}(t)} N(t-a)\,\mathrm{d}a, \tag{4}$$

$$z(t) = \frac{R(t)}{L(t)}. \tag{5}$$

The income $I(t)$ and the total expenditures $E(t)$ of the pension system in a certain period t are:

$$I(t) = \int_0^{\widetilde{B}(t)} \tau(t)W(t)N(t-a)\,\mathrm{d}a, \tag{6}$$

$$E(t) = \int_{\widetilde{B}(t)}^{\widetilde{X}(t)} P(t-a,t)N(t-a)\,\mathrm{d}a. \tag{7}$$

The total deficit (or surplus) is denoted by:

$$D(t) = E(t) - I(t), \tag{8}$$

while the deficit ratio d_t—defined as the size of total expenditures $E(t)$ in relation to the income of the pension system $I(t)$—is written as:[7]

$$d(t) = \frac{E(t)}{I(t)}. \tag{9}$$

There are various ways to specify the notion of a "balanced budget" in this framework. On the one hand one can talk about a budget that is balanced in every period:

$$D(t) = 0, \forall t. \tag{10}$$

[7] Alternatively, one could also use the deficit relative to "national income" (i.e. $\frac{D(t)}{W(t)L(t)}$) or simply the deficit-income ratio ($\frac{D(t)}{I(t)} = d(t) - 1$). In general, it does not matter which concept is used since they all lead to qualitatively similar patterns.

It follows immediately that the fulfillment of (10) implies that $d_t = 1, \forall t$. The second definition of a budgetary balance refers to the intertemporal budget constraint and requires that the budget is balanced over time (in present value terms):

$$PVD(t) = \int_0^{\infty} D(s) e^{-\int_t^s \xi(x)\,\mathrm{d}x}\,\mathrm{d}s = 0, \tag{11}$$

where $\xi(t)$ is the discount factor that is used to calculate the present value. There exists a long discussion on what is the correct intergenerational discount factor. In the following I will mostly refer to the case where $\xi(t) = g(t)$, i.e. where the actual growth rate of wages is used as the discount factor. This assumptions allows a clearer perspective on the underlying mechanics of different pension systems without being distracted by assumptions about the relative magnitudes of the discount rate, the growth rate or some relevant interest rate. In particular, in this case the long-term solidity of the budget does not depend on the temporal sequence of deficits and surpluses (cf. Settergren & Mikula 2006).

2.3 Demographic Steady State

For the following analysis it is helpful to use a steady state (or rather a "balanced growth path") as a reference point. It is a "triple stationary state" that involves the demography, the economy and the pension system. First, the demographic situation is assumed to be constant over time, i.e. $N(t) = \overline{N}$ and $X(t) = \overline{X}$. Second, the economy grows at a constant rate ($g(t) = \overline{g}$). Third, also the main parameters of the pension system do not change and the retirement age, the contribution rate and the average pension level are constant, i.e.: $B(t) = \overline{B}$, $\tau(t) = \overline{\tau}$ and $\overline{q}(t) = \frac{\overline{P}(t)}{W(t)} = \frac{\int_{\overline{B}}^{\overline{X}} P(t-a,t)\,\mathrm{d}a.}{W(t)} = \overline{q}$. From (3) and (4) it then follows that also $z(t) = \frac{R(t)}{L(t)} = \frac{\overline{X}-\overline{B}}{\overline{B}} \equiv \overline{z}$ is constant over time, where $\overline{z}$ is the steady state dependency ratio and also the ratio of the average years a person stays in pension to the average years he or she is in work. A permanently balanced budget then requires that in the steady state the following relation must hold:

$$\bar{\tau} = \bar{q}\bar{z}. \tag{12}$$

Each society may choose its favorite values for the steady state contribution rate, pension level and retirement age, as long as condition (12) is fulfilled and as long as $\overline{\tau} < 1$. Otherwise the pension system would be unsolid from the beginning. The specific choice for $\overline{\tau}$, $\overline{q}$ and $\overline{z}$ will depend on social and political preferences, historic circumstances etc.

3 A Notional Defined Contribution Pension System

3.1 Main features

Thus far I have left open how the pension levels $P(t-a,t)$ of the various retired cohorts at a certain period t are determined. It is in this area that one observes the biggest cross-national differences and also the main rift between defined benefit and defined contribution systems. There are various ways to design a system that is sustainable. An immediate way, for example, would be to stipulate an identical pension payment for the entire retired population and to adjust the pension level in reaction to changes in the dependency ratio, i.e.: $q(t-a,t) = q(t) = \overline{q}\frac{\overline{z}}{z(t)}$ for $a \in [\widetilde{B}(t), \widetilde{X}(t)]$. This is similar to the sustainability factor introduced in Germany in 2004 (Börsch-Supan & Wilke 2003, Knell 2010). The drawback of such a formulation, however, is that the identical pension levels for all retired workers do not take the individual retirement ages into consideration and might thus lead to problems with inter- and intragenerational fairness. It is also a rather intransparent mechanism that mixes changes in the average retirement age and in the development of cohort sizes (that both influence changes in the dependency ratio).

There exists, however, one alternative type of PAYG systems that is often viewed to be a superior approach since it is transparent, easily understandable and able to deal with intra- and intergenerational differences in a straightforward, incentive compatible and equitable manner: the notional defined contribution (NDC) system. This system has been implemented first in Sweden and has later been also adapted by a number of additional countries like Italy, Poland, Latvia, Mongolia, Turkmenistan etc. It is now also often used as a benchmark PAYG model by international institutions like the World Bank (Holzmann & Hinz 2005), the OECD (2011) or the European commission (EPC, 2009) and I therefore focus on this system.

The basic design of the Swedish notional defined contribution (NDC) system can be described as follows.[8] Each insured person pays 18.5% of its earnings (up to a ceiling) as contributions from which 16% are credited to the (PAYG financed) notional account while 2.5% are used for the built up of a funded pillar.[9] The value of the "deposits" on the PAYG account grow with the "notional interest rate" which in Sweden is defined as the average growth rate of wages. At the time of retirement the notional capital is transformed into an annuity. In the simplest version this annuization is done by dividing the capital

[8]Detailed descriptions can be found in Palmer (2000), Disney (1999) Williamson & Williams (2003) and Holzmann & Palmer (2006).

[9]Details of this funded pillar can be found in Sundén (2004).

by the (expected) remaining life expectancy. Each increase in life expectancy will thus automatically lead to a decrease in the pension level. In Sweden, however, a different approach for the annuitization is used (called the "G-factor") where a real growth rate of 1.6% is used to "frontload" part of the expected pension adjustments thereby increasing the initial pension. Existing pensions are then only adjusted with the difference between the actual growth rate and this stipulated growth rate of 1.6%.

3.2 Formal expression of a NDC system

The Swedish model can be formalized in the following way. The first crucial feature of every NDC system is that the contribution rate is fixed at $\tau(t) = \overline{\tau}$ for all periods. The determination of the pension payments depends on the notional capital $K(t, t+a)$ that generation t accumulates over the working periods $a \in [0, B(t)]$. The final amount of this notional capital (before it is turned into a continuous pension stream) is given by:

$$K(t, t + B(t)) = \int_0^{B(t)} \overline{\tau} W(t + a) e^{\int_{t+a}^{t+B(t)} \rho(s)\,\mathrm{d}s}\,\mathrm{d}a, \tag{13}$$

where $\rho(s)$ stands for the notional interest rate in period s and the cumulative factor $e^{\int_{t+a}^{t+B(t)} \rho(s)\,\mathrm{d}s}$ indicates how the contribution $\overline{\tau} W(t+a)$ that is paid into the pension system in period $t + a$ is revalued when calculating the final amount of the notional capital in period $t+B(t)$ (the period of retirement). The specification of the notional interest rate is one of the crucial parameters in a NDC system and I will later discuss various possibilities how it can (and should) be defined.

The first pension that is received by generation t in period $t + B(t)$ is given by:

$$P(t, t + B(t)) = \frac{K(t, t + B(t))}{\Gamma(t, t + B(t))}, \tag{14}$$

where the notional capital for generation t is transformed into an annuity by using the remaining life expectancy $\Gamma(t, t + B(t))$ of generation t at the moment of retirement $t+B(t)$.[10] The conceptual measure that is used to calculate the remaining life expectancy (period vs. cohort life expectancy) is another crucial factor for the specification of a NDC system. I will come back to this issue below.

[10]In order to keep the analysis simple I abstract here from front-loading (the "G-factor") as described above.

Existing pensions are adjusted according to:

$$P(t,t+a) = P(t,t+B(t))e^{\int_{t+B(t)}^{t+a}\vartheta(s)\,\mathrm{d}s}, \tag{15}$$

for $a \in [B(t), X(t)]$ and where $\vartheta(s)$ stands for the adjustment rate in period s and the cumulative adjustment factor $e^{\int_{t+B(t)}^{t+a}\vartheta(s)\,\mathrm{d}s}$ indicates how the first pension $P(t,t+B(t))$ received by generation t in period $t+B(t)$ is adjusted to give the pension payment in period $t+a$.

Inserting equations (13), (14) and (15) into (7) and assuming $\vartheta(t) = \rho(t)$ leads to the following expression of expenditures (see appendix A):

$$E(t) = \overline{\tau}W(t)\int_{\widetilde{B}(t)}^{\widetilde{X}(t)} \frac{\int_0^{B(t-a)}\left[e^{\int_{t-a+b}^{t}(\rho(s)-g(s))\,\mathrm{d}s}\right]\mathrm{d}b}{\Gamma(t-a,t-a+B(t-a))}N(t-a)\,\mathrm{d}a. \tag{16}$$

The question is, whether one can find definitions for $\rho(t)$ and $\Gamma(t,t+B(t))$ such that (16) develops in line with the income of the pension system given by (6), i.e. $I(t) = \overline{\tau}W(t)L(t)$.

3.3 Crucial choices in NDC systems

The crucial factors that have to be defined at the outset for a functioning NDC system are thus the notional interest rate $\rho(t)$, the adjustment factor $\vartheta(t)$ and the measure of life expectancy that is used to specify the remaining life expectancy $\Gamma(t,t+B(t))$.

To start with the latter there exist two variants. One can either use period (or cross-section) life expectancy $\widetilde{X}(t+B(t))$ or cohort (or forecasted) life expectancy $X(t)$ to calculate the annuity payment for generation t. This means either:

$$\Gamma(t,t+B(t)) = \widetilde{X}(t+B(t)) - B(t) \tag{17a}$$

or

$$\Gamma(t,t+B(t)) = X(t) - B(t). \tag{17b}$$

In as far as the notional interest rate is concerned there are two possible methods of indexation that are often discussed in the literature and that are used in real-world pension systems: an indexation with the growth rate of average wages and one with the growth rate of the wage bill (or rather the growth rate of the sum of contributions). I will only discuss methods where the notional interest rate and the adjustment rate are identical, i.e. where $\vartheta(t) = \rho(t)$. The formulas are given by:

$$\rho(t) = g^W(t) \tag{18a}$$

or

$$\rho(t) = g^W(t) + g^L(t), \tag{18b}$$

where $g^W(t) \equiv \frac{\dot{W}(t)}{W(t)}$, $g^L(t) \equiv \frac{\dot{L}(t)}{L(t)}$, $\dot{W}(t) \equiv \frac{dW(t)}{dt}$ and $\dot{L}(t) \equiv \frac{dL(t)}{dt}$. Using the definitions for $W(t) = W(0)e^{\int_0^t g(s)\,\mathrm{d}s}$, $N(t) = N(0)e^{\int_0^t n(s)\,\mathrm{d}s}$ and $L(t) = \int_0^{\widetilde{B}(t)} N(t-a)\,\mathrm{d}a$ one can calculate (see appendix A) that:

$$g^W(t) = g(t), \tag{19}$$

$$g^L(t) = \frac{N(t-\widetilde{B}(t))}{L(t)}\frac{d\widetilde{B}(t)}{dt} + \frac{\int_0^{\widetilde{B}(t)} N(t-a)n(t-a)\,\mathrm{d}a}{L(t)}. \tag{20}$$

The benchmark case is the situation where cohort sizes are constant (i.e. $N(t) = \overline{N}, \forall t$) and life expectancy increases in a linear fashion according to (1). One can now ask how the retirement age has to change over time such that the dependency ratio $z(t)$ remains constant at the reference value $\overline{z}$. The solution can be derived from setting $z(t) = \frac{\widetilde{X}(t)-\widetilde{B}(t)}{\widetilde{B}(t)} = \overline{z}$ and solving for $\widetilde{B}(t)$. It comes out as:

$$\widetilde{B}(t) = \frac{\widetilde{X}(t)}{1+\overline{z}} = \frac{X(t)}{(1+\gamma)(1+\overline{z})}, \tag{21}$$

where the latter equality follows from (2). Therefore $\frac{d\widetilde{B}(t)}{dt} = \frac{\gamma}{(1+\gamma)(1+\overline{z})}$ and thus $g^L(t) = \frac{\overline{N}}{\overline{N}\widetilde{B}(t)}\frac{\gamma}{(1+\gamma)(1+\overline{z})} = \frac{\gamma}{X(t)} = \frac{\gamma}{(1+\gamma)\widetilde{X}(t)}$. This result delivers an important insight. If life expectancy increases and if every cohort prolongs its working life in such a manner as to counter this increase and to keep the dependency ratio constant then there will be a continuous increase in the size of the labor force even if the size of the cohorts remains constant. This is simply a consequence of the fact that each cohorts postpones its retirement by a little bit, as specified in (21). In fact, this behavior seems like the "natural" and most appropriate reaction to the increase in life expectancy. Taking this into consideration it also appears unjustified that this "necessary" and "appropriate" increase in the labor force should lead to a higher notional interest rate as would be the case if one uses the growth rate of the wage bill (cf. (18b)) as the relevant concept. It seems more reasonable to propose a new concept that defines the notional interest rate as the growth rate of the wage bill *adjusted* for the necessary increase in retirement age due to increasing life expectancy. This leads to the third concept for the determination of the notional interest rate and the adjustment factor that I will study in the following:

$$\rho(t) = g^W(t) + g^L(t) - \frac{\gamma}{(1+\gamma)\widetilde{X}(t)}. \tag{18c}$$

4 A self-stabilizing NDC system

Looking at the expression for total pension expenditures $E(t)$ in (16) it does not seem obvious whether it is at all possible to find parameters for the notional interest rate and remaining life expectancy such as to implement a NDC system that has a budget that is balanced (at least over the long run). The main challenge is to guarantee financial sustainability for a large (or even an *arbitrary*) pattern of retirement ages. In the following I am therefore abstaining from endogenizing the retirement age (e.g. by assuming optimal intertemporal behavior) and I will focus on a NDC design that leads to a stabilizing budget for various patterns of retirement choices.

In order to focus clearly on the issue of increasing life expectancy I will primarily deal with the situation where cohort sizes are constant:

$$N(t) = \overline{N}. \tag{22}$$

The following proposition specifies how to design a NDC system such that it is compatible with long-run sustainability for a wide variety of retirement behavior.

Proposition 1 *For linearly increasing life expectancy and constant cohort sizes a notional defined contribution system leads to a balanced budget if the following two conditions are fulfilled: (i) The notional interest rate and the adjustment factor are equal to the adjusted growth rate of the wage bill as given in* (18c) *and (ii) the remaining life expectancy is calculated by using period life expectancy as in* (17a)*.*

The use of (18c) and (17a) leads to a situation where the budget is exactly balanced in every period, almost balanced or balanced over the long-run—depending on the assumption about the retirement behavior. Proposition 1 is surprising since it contradicts claims about the most reasonable set-up of a NDC system that can be found in the related literature.[11] There it is stated that the most appropriate design would involve a combination

[11] Various examples for this will be quoted in section 4.4.

Table 1: The deficit-ratio for a linearly increasing retirement age

	(1)	(2)	(3)
	Notional Interest Rate — Growth Rate of:		
	Average Wages	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	1	$\approx 1 + \frac{\gamma}{2}$	1
Cohort Life Expectancy	$\frac{1}{1+\gamma}$	$\approx 1 - \frac{\gamma}{2}$	$\frac{1}{1+\gamma}$

Note: The table shows the period deficit ratio $d(t) = \frac{E(t)}{I(t)}$ for various assumption about the notional interest rate and the measure of remaining life expectancy. Period [cohort] life expectancy is defined as in (17a) [(17b)]. The notional interest rate is given by one of the expressions in (18a), (18b) and (18c), respectively. Furthermore, it is assumed that retirement age increases in a linear fashion according to $B(t) = \frac{X(t)}{1+\overline{z}(1+\gamma)}$.

of cohort life expectancy (cf. (17b)) and the unadjusted growth rate of the wage bill (cf. (18b)).

The proof of proposition 1 is given in the following subsections where I focus on three different assumptions about the development of retirement age. Tables 1 and 2 summarize various cases that are discussed in the following sections 4.1 and 4.2, respectively.

4.1 Retirement age increases linearly

As a starting point I look at the case where every generation chooses a retirement age such as to stabilize the dependency ration at $z(t) = \overline{z}$. From (21) and the fact that $\widetilde{B}(t - B(t)) = B(t)$ this implies that:

$$B(t) = \frac{X(t)}{1 + \overline{z}(1 + \gamma)}. \tag{23a}$$

As said above, this seems to be a "natural" and intergenerational equitable reaction to the continuous increase in life expectancy where the proportion of retirement years to working years stays constant at $\frac{X(t)-B(t)}{B(t)} = \overline{z}(1 + \gamma)$.

As previously mentioned, in this case $g^L(t) = \frac{\gamma}{(1+\gamma)\widetilde{X}(t)}$ and thus the notional interest rate (18c) is given by $\rho(t) = g(t)$ which is the same as using average wage growth (cf. (18a)). In appendix A I show that if one uses this notional interest rate together with (17a) then the budget of the pension system is always balanced, i.e. $D(t) = 0$ and $d(t) = 1$

Table 2: The deficit-ratio for a constant retirement age

	(1)	(2)	(3)
	Notional Interest Rate — Growth Rate of:		
	Average Wages	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$\approx 1 + \frac{\gamma}{2}$	$\approx 1 + \frac{\gamma}{2}$	≈ 1
Cohort Life Expectancy	$\approx 1 - \frac{\gamma}{2}$	$\approx 1 - \frac{\gamma}{2}$	$\approx 1 - \gamma$

Note: The table shows the period deficit ratio $d(t) = \frac{E(t)}{I(t)}$ for various assumption about the notional interest rate and the measure of remaining life expectancy. Period [cohort] life expectancy is defined as in (17a) [(17b)]. The notional interest rate is given by one of the expressions in (18a), (18b) and (18c), respectively. Furthermore, it is assumed that retirement age is constant at $B(t) = \overline{B}$.

for every period.

For the stability of the system it is crucial that one uses the adjusted growth rate of the wage bill (18c) and not the unadjusted wage bill growth rate (18b) as is often suggested in the literature. For $\gamma > 0$ the latter concept would prescribe a higher notional interest rate ($\rho(t) = g(t) + \frac{\gamma}{(1+\gamma)\widetilde{X}(t)}$) which would cause a permanent deficit of the pension system. In particular, in appendix A I show that the deficit ratio in this case is approximately equal to $d(t) = 1 + \frac{\gamma}{2}$. This is a non-trivial amount. For a realistic value of γ it would amount to a permanent deficit ratio of about 12.5%. The reason for this imbalance is that such a system would grant an extra rate of return due to the permanent increase in the labor force which is, however, just a necessary reaction to the increases in life expectancy and should be neglected when determining the rate of return.

On the other hand, if one uses adjusted wage bill growth (18c) but cohort life expectancy (17b) instead of period life expectancy (17a) then the deficit ratio is given by $d(t) = \frac{1}{1+\gamma}$ and the pension system runs a permanent surplus. The use of cohort life expectancy (as frequently recommended for NDC systems) is thus "overambitious" as it will lead to excessively small annuities that cause a permanent surplus in the budget. It is sufficient to use period life expectancy if this is combined with the appropriate notional interest rate (18c).[12]

[12] Since the use of (17b) instead of (17a) leads to a permanent surplus while the use of (18b) instead of (18c) to a permanent deficit one might want to know what happens if one were to combine (17b) and (18b). The answer is that in this case the deficit ratio is approximately $1 - \frac{\gamma}{2}$ (see appendix A) and one

4.2 Retirement age stays constant

It is certainly an optimistic scenario to assume that retirement age always adjusts in lockstep to the increases in life expectancy. As the opposite (very "pessimistic") extreme one could also assume that the retirement is constant despite the advances in longevity:

$$B(t) = \overline{B}. \tag{23b}$$

Note that then the labor force is constant ($g^L(t) = 0$) and thus the notional interest rate (18c) is given by $\rho(t) = g(t) - \frac{\gamma}{(1+\gamma)\widetilde{X}(t)}$.

As shown in appendix A, the combination of a notional interest rate set according to (18c) and a remaining life expectancy that is based on period life expectancy as in (17a) leads to a deficit ratio given by:

$$d(t) = \frac{\overline{B}}{\widetilde{X}(t)} \left(\frac{(2+\gamma)\ln(1+\gamma)}{2\gamma} - 1 \right) + 1 \approx 1,$$

where the approximation is around $\gamma = 0$. So in this case the combination of (18c) and (17a) leads to a budget that is almost balanced in every period. This can also be seen by looking at the non-approximated version and by setting $\gamma = 1/4$, $\overline{B} = 45$ and $X(0) = 60$ (such that $z(0) = 1/3$). For $t = 0$ one gets that $d(0) = 1.00389$ (the expenditures exceed income by only 0.4%) while $\lim_{t\to\infty} d(t) = 1$.

It is interesting to study here what would happen if one uses a "conventional" notional interest rate with the growth rates of either average wages (18a) or the wage bill (18b). These two cases are now identical (since $g^L(t) = 0$) implying a notional interest rate of $\rho(t) = g(t)$. Using period life expectancy one can calculate that:

$$d(t) = \frac{(1+\gamma)\ln(1+\gamma)}{\gamma} \approx 1 + \frac{\gamma}{2}.$$

This seems to confirm the belief that the use of period life expectancy is not enough to keep a NDC system in balance. The use of cohort life expectancy (17b), however, as is often suggested as the better alternative is also not appropriate as it leads to a permanent *surplus*.

$$d(t) = \frac{\ln(1+\gamma)}{\gamma} \approx 1 - \frac{\gamma}{2}.$$

The punchline of this consideration is that in the case of constant retirement ages and

faces a permanent surplus.

a notional interest rate $\rho(t) = g(t)$ both methods of calculating the remaining life expectancy for annuitization at the time of retirement lead to an unbalanced budget. The first method is too "generous" causing persistent deficits while the second method is too "harsh" leading to ongoing surpluses. In fact, one could think about using a mixture of both life expectancy concepts in order to stabilize the system:

$$\Gamma(t, t + B(t)) = \left[\mu X(t) + (1 - \mu)(\widetilde{X}(t + B(t)))\right] - B(t), \tag{17c}$$

where μ gives the relative weight of cohort life expectancy. One can calculate the weight that leads to a permanently balanced budget with $d(t) = 1, \forall t$. It comes out as:

$$\mu^* = \frac{1}{\gamma^2}\left((1 + \gamma)\ln(1 + \gamma) - \gamma\right) \approx \frac{1}{2}\left(1 - \frac{\gamma}{3}\right). \tag{24}$$

A mixture of both life expectancy concepts will thus lead to a balanced NDC system in the case where the retirement age does not react to increasing life expectancy. For $\gamma = 1/4$, e.g., the optimal value is $\mu^* = 0.46$ that puts slightly more weight on period life expectancy. I want to stress, however, that the same result (a continuously balanced budget) can also be guaranteed if one uses the adjusted growth rate of the wage bill (18c) and period life expectancy (17a). In fact, the latter option is preferable since it does work for many different patterns of retirement behavior and not only for the special case where $B(t) = \overline{B}$.

In closing this section I want to stress that the calculations abstract from the existence of a minimum pension. If social security legislation prevents a fall of the relative pension level $q(t)$ below some minimum level q^{min} then a fixed retirement age will lead to an increasing deficit even in a NDC system.

4.3 Retirement age is random

The assumptions about the behavior of retirement age made so far are certainly rather special. As a first step towards a more general behavioral pattern one can look at two variants of the examples in sections 4.1 and 4.2. First, one can assume that $B(t) = \lambda X(t)$ and $0 < \lambda < 1$. In this case the combination of (18c) and (17a) also leads to a permanently balanced budget as in section 4.1 (which is a special case with $\lambda = \frac{1}{1+\overline{z}}$). On the other hand, if retirement age is a linear combination between (23a) and (23b), i.e. $B(t) = \varsigma \frac{X(t)}{1+\overline{z}(1+\gamma)} + (1 - \varsigma)\overline{B}$ then one gets a similar result as in section 4.2. The budget is almost balanced for all values of ς. These extensions, however, are only slightly more general than the ones considered in the previous sections since they still assume that all cohorts behave in the same systematic manner.

In a next step I have therefore used numerical simulations to study the consequences of a completely arbitrary pattern of retirement age behavior.[13] I only want to present the results of one stylized case where I assume that retirement age is a uniformally random variable between the values given in (23a) and (23b):

$$B(t) = \text{Uniform}\left(\overline{B}, \frac{X(t)}{1+\overline{z}(1+\gamma)}\right). \tag{23c}$$

This is certainly an implausible and extreme scenario that is only chosen to illustrate the self-stabilizing possibilities of a NDC system that uses adjusted wage bill growth and period life expectancy as its crucial parameters. Figure 1 shows one path for $B(t)$ under assumption (23c) while Figure 2 reports the pattern of $d(t)$ that emerges in this scenario.

As expected the pension system is unbalanced in almost every period and the deficit ratio fluctuates with a minimum and maximum of 0.96 and 1.04, respectively, and a standard deviation of 0.018. Over time, however, the surpluses and the deficits counteract each other and the average deficit ratio over the 120 year period is 0.999.[14] This outcome is not due to the specific numerical example reported in figure 1 but it seems to be true in general. For any of the many simulations I have run the average deficit ratio has been very close to 1.

4.4 Comparison to the existing literature

The results challenge some of the "conventional wisdom" about the appropriate design of NDC systems that can be found in the related academic and policy literature.

First, I have shown that if the retirement age is a choice variable then it is fairly simple to achieve a permanently balanced budget in the presence of increasing life expectancy. It is sufficient to determine the period-specific retirement age according to (21) (or the cohort-specific retirement age according to (23a)) in order to stabilize the dependency

[13] Details about the simulations can be found in appendix B. In particular, the simulations are based on a discrete-time version of the model and therefore the variables $B(t)$, $d(t)$ etc. should be understood as B_t, d_t etc. I do not make these notational substitutions here in order to keep the formulas and figures in line with the rest of the text.

[14] This has been calculated under the assumption that the discount factor $\xi(t)$ is equal to the growth rate $g(t)$. For alternative assumptions the long-run balance will depend on the size of the discount (or interest) rate and on the exact sequence of surpluses and deficits. In general, however, the deficit will not be huge and it will have a tendency to balance over time.

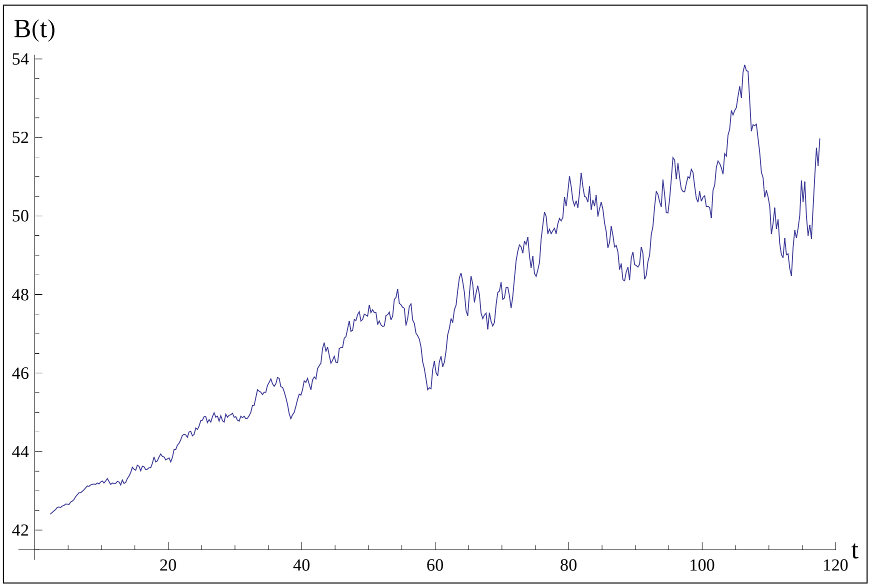

Figure 1: The figure shows a random path of the cohort-specific retirement age when $B(t)$ is a uniformally distributed variable between $\overline{B}$ and $B(t) = X(t)/(1+\overline{z}(1+\gamma))$. The simulation has been run on a quarterly basis and was transformed into annual values. To smooth out short-run fluctuations the graph shows a 5-year moving average. The parameters were chosen as follows: $\overline{\tau} = 1/4$, $\overline{z} = 1/3$, $\gamma = 1/4$, $X(0) = 60$ and $\overline{B} = X(0)/(1+\overline{z}(1+\gamma)) = 42.4$.

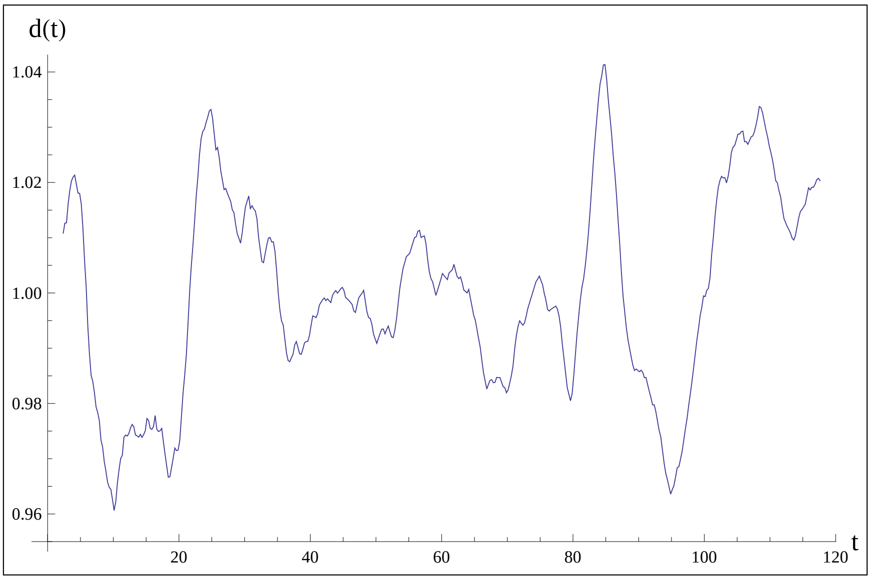

Figure 2: The figure report the development of $d(t)$ when the cohort-specific retirement age is given by the pattern shown in figure 1. The simulation has been run on a quarterly basis and was transformed into annual values. To smooth out short-run fluctuations the graph shows a 5-year moving average. The parameters were chosen as follows: $\overline{\tau} = 1/4$, $\overline{z} = 1/3$, $\gamma = 1/4$, $X(0) = 60$ and $\overline{B} = 42.4$.

ratio at $z(t) = \overline{z}$ and the budget at $D(t) = 0$. This stands in contrast to the findings of Andersen (2008): "An indexation of pension ages to longevity may seem a simple and fair solution. This would imply that the relative amount of time spent as contributor to and beneficiary of a social security scheme would be the same across generations with different longevity. [...] However, as is shown in this paper, this solution is not in the feasibility set." (p. 634f.). The difference between the results in Andersen (2008) and in this paper stems from the use of different definitions of "proportionality" which might itself be the consequence of different modeling frameworks. While I work in a continuous time framework, Torben Andersen uses a "two life phases" model where the first life phase ("youth") has a given length normalized to unity while the second phase has length β (≤ 1). The labor period in the second life phase is, however, flexible and individuals can choose to retire at an age $\alpha \leq \beta$. The retirement period is thus $(\beta - \alpha)$ and the dependency ratio in time t is given by $z_t = \frac{\beta_t - \alpha_t}{1+\alpha_t}$. Andersen defines the relative retirement age in period t as $\frac{\alpha_t}{\beta_t}$. He concludes that "when the retirement age relative to longevity $(\frac{\alpha}{\beta})$ is constant, we have that the economic dependency ratio is increasing in longevity, and this explains why the simple indexation solution is not economically feasible." (ibd., p. 639). This follows from the fact that $\left.\frac{\partial z_t}{\partial \beta_t}\right|_{\frac{\alpha_t}{\beta_t}=\text{const.}} > 0$. This, however, is only true for the rather unconventional definition of the relative retirement age as $\frac{\alpha_t}{\beta_t}$ where α_t is not the total length of working time but rather only the years of work since the "middle age". In order to make the total number of working years $(1+\alpha_t)$ proportional to the complete life span $(1+\beta_t)$ would require to set $\alpha_t = \lambda(1+\beta_t) - 1$ for some constant λ. This then leads to a constant dependency ratio of $z_t = \frac{1-\lambda}{\lambda}$. This is the same result as in my model and confirms—contrary to Andersen's claim—that the social security system can be stabilized if the retirement age is indexed to life expectancy.

For the (more realistic) case where the actual retirement age is not completely under control of the policymaker, I have furthermore shown that a NDC system that uses adjusted wage bill growth (18c) and period life expectancy (17a) will stabilize the budget in the long run. This result also differs from the widely held beliefs about the functioning of NDC systems. Broadly speaking, the general opinion holds that the best NDC design involves the use of the growth rate of the (unadjusted) wage bill as the notional interest rate and of cohort life expectancy as the measure to calculate remaining life expectancy. The following quotations illustrate this dominant view.[15]

[15] I want to stress, however, that most of the following quotes are taken from papers that are more "policy-oriented" and less "model-based". The claims are often made in the context of a general assessment of the workings and the appropriate design of NDC systems. The results of my model are thus

For example, Palmer (2000) argues: “Wage sum indexation always moves the system in the direction of financial stability. Per capita wage growth reflects the growth in productivity, but not changes in the size of the labor force. For this reason, per-capita wage indexation does not yield financial stability if wage growth is greater than the growth of the contribution wage sum” (ibd., p. 18). Concerning life expectancy one can read: “Swedish politicians have made another decision that runs counter to long-term stability. If the system were 100 percent consistent with its underlying principles, it would be necessary to successively recalculate the pensions of all pensioners as life expectancy increases, or to attempt to perform a sound actuarial forecast. [...] Instead, the life expectancy calculation will be based on an average of observed outcomes prior to retirement. The factor will be biased towards to produce benefit levels that are persistently too high, thus creating an additional source of financial stress” (ibd., p. 28).

Similarly, Börsch-Supan (2003), writes that: “Viewed from a macroeconomic perspective, the ‘natural’ rate of return for an NDC system is the implicit return of a PAYG system: that is, the growth rate of the contribution bill. [...] Using up-to-date cohort-specific life tables guarantees actuarial sustainability”(ibd., p. 38f.).

Finally, Holzmann (2006) includes among the “crucial elements for design and implementation” of NDC systems: “The choice of a notional interest rate consistent with internal rate of return of a PAYG scheme: that is, growth rate of aggregate (covered) wage sum. [...] The choice of remaining life expectancy. Politically determined underestimation (for example, by taking the cross-section life expectancies instead of estimated cohort life expectancies) to deliver higher annuities will also jeopardize financial sustainability” (ibd., p. 244).

A smaller part of the literature is more in line with my findings. As far as life expectancy is concerned, N. Barr and P. Diamond write, e.g.: “A process of automatic adjustment that relies heavily on projected mortality rates could easily become politicized. Thus a system may function better if it adjusts benefits on the basis of realized mortality information” (Barr & Diamond 2009, p. 89).

Closely related to my approach is the analysis and the results in Settergren & Mikula (2006) that are also summarized in Palmer (2006). These authors note that the use of the wage bill as the notional interest rate is “not sufficient to maintain financial equilibrium under various circumstances. [...] Different distributions of contribution and benefit payment flows will affect the NDC scheme’s ability to maintain temporal balance between

better understood as a refinement than as a criticism of this complementary literature.

assets and liabilities. In insurance terms, the time money remains in the system before it has to be paid out affects the instantaneous liquidity of the system" (Palmer 2006, p. 24). This "time money remains in the system" is termed the "turnover duration". Changes in the turnover duration have to be reflected in the notional interest rate in order to maintain the financial stability of the system. Increasing life expectancy is one important factor why the turnover duration is likely to change over time. In fact, it can be shown (see appendix A.5) that for the case where the retirement age increases in a linear fashion according to equation (23a) the growth rate of the turnover duration is exactly given by $\frac{\gamma}{X(t)}$. Correcting for this development will then lead to the same expression as in (18c). The framework with turnover durations can thus be regarded as an alternative way to look at the issue of a self-stabilizing NDC system with increasing life expectancy.

5 Results when life expectancy has an upper limit

So far I have assumed that life expectancy is increasing in a linear fashion. As argued above, this assumption is in line with the historic development over the last century. Nevertheless, this is a controversial assumption and there exists a heated debate among demographers on his issue. On the one hand there is group of researchers arguing that there exists a maximum human life span that cannot be extended. James Fries, e.g., has argued in the 1980ies that humans are born with a maximum potential life expectancy that he predicted to be normally distributed among the population with a mean of 85 and a standard deviation of 7 years. Carnes et al. (2003) have further strengthened this point and they argued that "the biological evidence suggests that organisms operate under warranty periods that limit the duration of life of individuals and the life expectancy of populations" (ibd., p. 31). They conclude that: "Although it is likely that anticipated advances in biomedical technology and lifestyle modification will permit life expectancy to continue its slow rise over the short-term, a repetition of the large and rapid gains in life expectancy observed during the 20th century is extremely unlikely" (ibd. p. 43).

On the other hand, however, Oeppen & Vaupel (2002) have strongly argued against the proponents of the view that there exists a fixed life-span that we are already approaching. Their reasons are: "First, experts have repeatedly asserted that life expectancy is approaching a ceiling: these experts have repeatedly been proven wrong. Second, the apparent leveling off of life expectancy in various countries is an artifact of laggards catching up and leaders falling behind. Third, if life expectancy were close to a maximum, then the increase in the record expectation of life should be slowing. It is not. For 160 years, best-performance life expectancy has steadily increased by a quarter of a year per year, an extraordinary constancy of human achievement" (ibd., p. 1031). In a similar vein, Robert Fogel has argued that the *technophysio evolution* (i.e. the co-evolution of technological and physiological improvements) give humans an increasing control over future medical progress and further reduction in mortality rates (cf. Fogel & Costa 1997). So even if everybody agrees that there exists some limit to life expectancy short of immortality, this limit is so uncertain that the assumption of a pattern like (1) is quite reasonable in order to study developments and appropriate policy reactions in the foreseeable future. This also seems to be the position of Oeppen & Vaupel (2002) who conclude: "Because best-practice life expectancy has increased by 2.5 years per decade for a century and a half, one reasonable scenario would be that this trend will continue in coming decades. If so, record life expectancy will reach 100 in about six decades" (ibd., p. 1030).

Nevertheless, it is interesting to see how a NDC system that uses a combination of period life expectancy (17a) and an adjusted wage bill growth (18c) will react if life expectancy hits an upper limit. In particular, I assume now that:

$$\begin{aligned} X(t) &= X(0) + \gamma \cdot t, \text{ for } t < \hat{t} \\ X(t) &= X(\hat{t}) = X^{max}, \text{ for } t \geq \hat{t}. \end{aligned} \tag{25}$$

This is an unrealistic assumption that is only made to fix ideas. It would certainly be more plausible to assume a constant slow-down of the trend increase before the upper limit is reached. In order to study the impact of this alternative life expectancy development one has to first consider what it implies for the pattern of period life expectancy. For $t < \hat{t}$ it still holds (cf. (1)) that $\widetilde{X}(t) = \frac{X(t)}{1+\gamma}$. For $\hat{t} \leq t < \hat{t} + X^{max}$ period life expectancy continues to grow at speed $\frac{\gamma}{1+\gamma}$, i.e. $\widetilde{X}(t) = \widetilde{X}(\hat{t}) + \frac{\gamma}{1+\gamma}(t - \hat{t})$. Finally, for $t \geq \hat{t} + X^{max}$ one has that period life expectancy is constant at $\widetilde{X}(t) = X^{max}$.

The budgetary development of the pension system clearly depends on the way how people deal with this slow-down in life expectancy improvements. I simulate the impact of such a situation for the case where the maximum age is reached in period $\hat{t} = 0$ and where $X^{max} = 70$. This is illustrated in Figure 3 for three assumptions about the reaction of retirement behavior. There are two important messages from this picture. First, the combination of (18c) and (17a) can lead to a budget that stabilizes over time. Second, whether this automatic stabilization takes place (and also the exact pattern of deficits and surpluses) depends, however, crucially on the retirement behavior before and after the

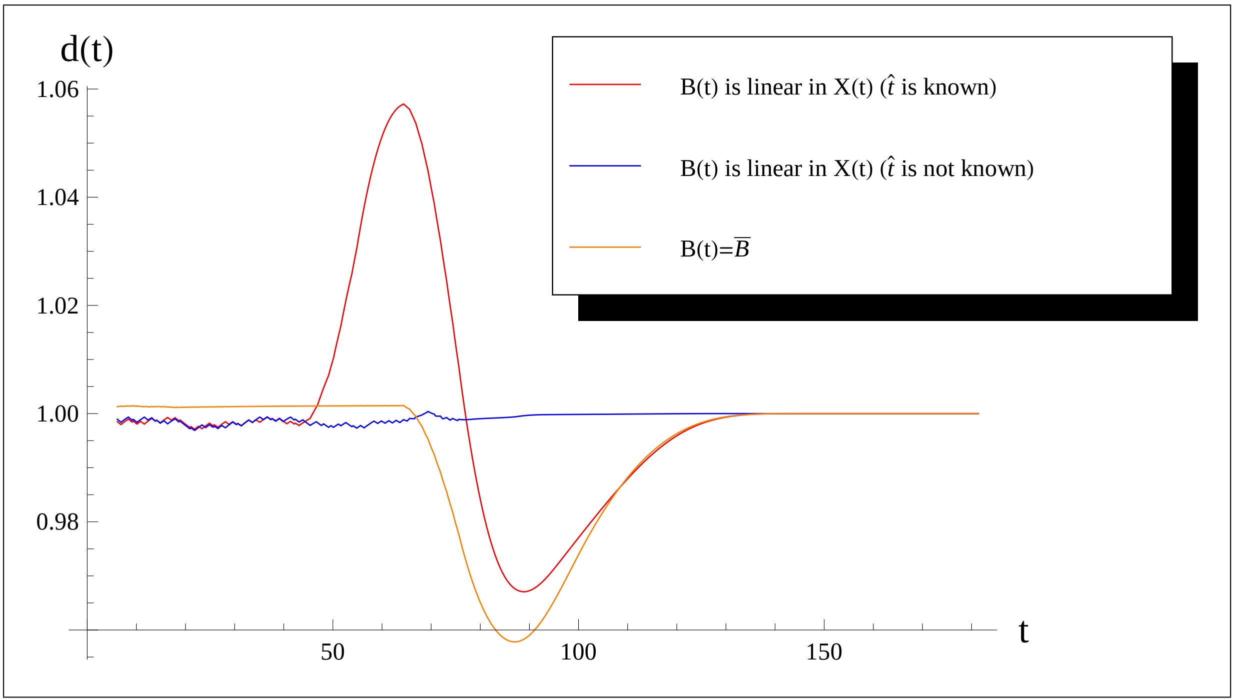

Figure 3: The figure report the development of $d(t)$ when cohort life expectancy has grown linearly with $\gamma = 1/4$ up to generation $\hat{t} = 0$ and from then on has reached a maximum with $X(0) = X^{max} = 70$. The rest of the parameters is as in figure 2. The simulation is based on quarters and was transformed into annual values and smoothed by a 15-year moving average.

transition. In particular, it is important to consider whether the drop in γ is an expected event or whether it comes as a surprise.

Let's start with the case where every generation is assumed to know its exact hour of death in advance and so generation $\hat{t}$ and every succeeding generation sees the break in life expectancy coming. As a benchmark assumption about retirement one can now look at the case where all cohorts follow the rule given by (23a) and therefore choose $B(t) = \frac{X(t)}{1+\overline{z}(1+\gamma)}$. Up to the period where cohort $\hat{t}$ has to decide about its retirement age nothing changes vis-à-vis the benchmark case and one will observe a balanced budget with $d(t) = 1$. For the parameters of the model this happens in period $\hat{t} + B(\hat{t}) = 49.4$.[16] Cohort $\hat{t} + dt$ sees that it will not get older than X^{max} and so it stops prolonging the working period and it will retire at the age of 49.4 as all other succeeding generations will do as well. This, however, has two consequences. First, the notional interest rate will turn negative (assuming $g = 0$) since $\widetilde{X}(t)$ is still increasing until $t = \hat{t} + X^{max} = 70$. Second,

[16]The small wiggles around $d(t) = 1$ in figure 3 before $t = 49.4$ are due to the fact that I use a simulation in discrete time while the results in section 4 are based on a continuous-time framework. See appendix B.

this increase in the notional interest rate is not enough to counterbalance the financial effects of the advanced retirement of cohort $\hat{t}$ that has the double impact of decreasing the number of contributors and increasing the number of beneficiaries. This “extra burden” increases with every additional cohort that retires at an age of $B(t) = 49.4$ while period life expectancy is still increasing. This only changes in period $t = 70$ when both cohort and period life expectancy are constant and from when on the notional interest rate is again zero. The deficit, however, stays negative for a number of periods. The reason for this is that in the old equilibrium the pension level has been given by: $q^{old}(t) = \overline{q}$ while in the new steady state it reduces to $q^{new}(t) = \frac{\overline{q}}{1+\gamma}$. For the parameters of the simulation this means a drop from 0.75 to 0.6. The pension calculation of the first generations that retired at the age of 49.4 has, however, still implemented a pension level of 0.75. On the other hand, however, there are also a number of generations (i.e. the ones that had worked during the periods between 49.4 and 70) that had to accept lower-than-normal notional interest rates which has depressed their pension claims even below the new steady state values. This reduces the total pension expenditures for some time and between the periods 77 and 140 they even decline below the income of the system thus resulting in a financial surplus. Only if there is no cohort left in the pension system that has experienced a single one of those periods with a reduced notional interest rate the system will again settle down on the new steady state values with a permanently balanced budget ($d(t) = 1$). The mean deficit ratio across all the adjustment periods is, however, equal to 1 (with a standard deviation of 0.02). The system will thus again be able to stabilize itself and guarantee a budget that is balanced in the long-run. The complete stabilization will, however, take a rather long time (around 140 years).

The picture is very different if one looks at the case where the end in the increases in life expectancy comes completely unexpected. This is also not an entirely plausible assumption but probably a more realistic case than the situation where the structural break is perfectly foreseen. Until the stop in life expectancy people will again set their retirement age according to (23a) which is (written in terms of $\widetilde{X}(t)$): $B(t) = \frac{(1+\gamma)\widetilde{X}(t)}{1+\overline{z}(1+\gamma)}$. Then all in the sudden in period $t = \hat{t}+X^{max} = 70$ period life expectancy does not increase anymore and settles at $\widetilde{X}(\hat{t} + X^{max}) = 70$. The retirement age in this period is given by: $\widetilde{B}(\hat{t} + X^{max}) = \frac{\widetilde{X}(\hat{t}+X^{max})}{1+\overline{z}}$ and from this one can calculate that the generation that retires in this moment is generation $\hat{t} + X^{max} - \widetilde{B}(\hat{t} + X^{max}) = \hat{t} + \frac{\overline{z}}{1+\overline{z}}X^{max}$. The cohort-specific retirement ages are thus given by: $B(t) = \frac{\widetilde{X}(t)(1+\gamma)}{1+\overline{z}(1+\gamma)}$ for $t < \hat{t} + \frac{\overline{z}}{1+\overline{z}}X^{max}$, and $B(t) = \frac{X^{max}}{1+\overline{z}}$ for larger t. The notional interest rate, however, is always equal to zero in this case since the actual retirement age increases in lockstep with the increase in period life expectancy.

Therefore, all pension claims are the same both between and within cohorts and the system runs a permanently balanced budget. In reality, the stopping in increases in life expectancy will probably be a slow process and it will take some time to notice it. It will then, however, be possible to make estimates about $\gamma(t)$ and this will affect the retirement behavior and the budgetary situation. All in all one can expect a rather smooth and self-stabilizing adjustment also in the case of a break in the development of life expectancy as long as retirement age was moving in line with life expectancy before the limit has been reached.

The third case that is documented in figure 3 is the one where the retirement age stays always constant. In this case one sees a different reaction of the system. The first thing to note in this context is that due to the fixed retirement age the only variable that changes at time $\hat{t} + X^{max}$ is the notional interest rate. While before this period it has been given by $\rho(t) = -\frac{\gamma}{(1+\gamma)\widetilde{X}(t)}$, afterwards it increases to $\rho(t) = 0$, which is the new equilibrium rate. For a considerable number of periods, however, there are still cohorts in the system that have pension claims that are based on periods where the lower, pre-shock notional interest rate has been valid. This depresses aggregate expenditures and leads to a surplus in the system between periods 70 and 140. In this situation there are no counterbalancing periods of deficits and the occurrence of a maximum age has a permanent effect on the balance of the PAYG pension system.[17]

6 Conclusion

In this paper I have studied how to design a NDC pension system that is able to stabilize its budget in the presence of increasing life expectancy. I have shown that the financial sustainability depends on the appropriate determination of two parameters: the notional interest rate and the measure that is used to calculate remaining life expectancy. A combination of the growth rate of an adjusted wage bill as notional interest rate and period life expectancy will lead to a balanced budget for a large variety of possible retirement behavior. These findings are a challenge to the conventional wisdom on the appropriate design of NDC systems and none of the countries that are currently organized in such a way uses the combination of parameters that suggests itself in the modeling framework of this paper. These findings might thus be useful for the refinement of existing or the

[17] I have also studied an intermediate case between (1) and (25) where $X(t)$ grows with speed γ_1 for $t < \hat{t}$ and with speed γ_2 for $t \geq \hat{t}$ and where $\gamma_2 < \gamma_1$. The results in this scenario are qualitatively identical to the one illustrated in figure 3 (which is a special case with $\gamma_2 = 0$).

construction of future NDC systems.

The focus of this paper has been to analyze the impact of increasing life expectancy on the stability of PAYG pension systems. Therefore, I have abstracted from all other economic and demographic factors that might also be potential sources of instability for the system. First and foremost this concerns changes along the second demographic dimension: the size of the birth or working cohorts $N(t)$. Different fertility scenarios have already been studied in the related literature (cf. in particular Valdés-Prieto 2000). The main finding is that non-monotonic shifts in the development of cohort size can lead to short-run and/or long-run financial instability of the pension system. Irregular developments of fertility are, however, only one reason why a NDC pension system might not be capable of achieving a balanced budget, neither in the short nor in the long run. There exists a large number of other factors that might change in an erratic fashion. Examples for such events include sudden changes in the average fertility age, in the average age of labor market entry, in the age-earnings profiles or in age-specific mortality. It is an interesting area for future research to study and systematize the effects of these changes and to analyze their interaction with increasing life expectancy.

Given that there are many sources for unpredictable shocks it seems inevitable that a NDC system includes some additional mechanism that adjusts for unforeseen imbalances like the Swedish "automatic balance mechanism" (Settergren 2001, Auerbach & Lee 2009). Independent of the design of such an additional balance mechanism it is important to note, however, that the appropriate definition of the notional interest rate and remaining life expectancy will in any case lead to a more stable system and will make the activation of the automatic balance mechanism a less frequent event.

References

Alho, J. M., Hougaard Jensen, S. E., Lassila, J. & Valkonen, T. (2005), 'Controlling the Effects of Demographic Risks: The Role of Pension Indexation Schemes', *Journal of Pension Economics and Finance* **4**(02), 139–153.

Andersen, T. M. (2008), 'Increasing Longevity and Social Security Reforms–A Legislative Procedure Approach', *Journal of Public Economics* **92**(3-4), 633–646.

Auerbach, A. J. & Lee, R. (2009), Notional Defined Contribution Pension Systems in a Stochastic Context: Design and Stability, *in* 'Social Security Policy in a Changing Environment', NBER Chapters, NBER, pp. 43–68.

Barr, N. & Diamond, P. (2009), *Pension Reform: A Short Guide*, Oxford University Press, Oxford, New York.

Bommier, A. & Lee, R. (2000), 'Overlapping Generations Models with Realistic Demography', *Journal of Population Economics* **16**, 135–160.

Börsch-Supan, A. (2003), What are NDC Pension Systems? What Do They Bring to Reform Strategies?, Mannheim Institute for the Economics of Aging (MEA), Discussion Paper 42/2003.

Börsch-Supan, A. & Wilke, C. B. (2003), The German Public Pension System: How it Was, How it Will Be, Mannheim Institute for the Economics of Aging (MEA), Discussion Paper 34/2003.

Carnes, B. A., Olshansky, S. J. & Grahn, D. (2003), 'Biological Evidence for Limits to the Duration of Life', *Biogerontology* **4**(1), 31–45.

Disney, R. (1999), Notional Account-Based Pension Reform Strategies: An Evaluation, University of Nottingham and The World Bank.

EPC (Economic Policy Committee and European Commission) (2009), The 2009 Ageing Report: Economic and Budgetary Projections for the EU-27 Member States Public (2008-2060), European Economy, Special Report, No. 2/2009.

Fogel, R. W. & Costa, D. L. (1997), 'A Theory of Technophysio Evolution, with Some Implications for Forecasting Population, Health Care Costs, and Pension Costs', *Demography* **34**(1), 49–66.

Goldstein, J. (2006), 'Found in Translation?: A Cohort Perspective on Tempo-Adjusted Life Expectancy', *Demographic Research* **14**(5), 71–84.

Heeringa, W. & Bovenberg, A. L. (2009), Stabilizing Pay-as-You-Go Pension Schemes in the Face of Rising Longevity and Falling Fertility: An Application to the Netherlands, Netspar Discussion Paper 08/2009-025.

Holzmann, R. (2006), Toward a Coordinated Pension System in Europe: Rationale and Potential Structure, *in* R. Holzmann & E. Palmer, eds, 'Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes', Washington, DC: World Bank, pp. 225–265.

Holzmann, R. & Hinz, R. (2005), *Old-Age Income Support in the 21st Century. An International Perspective on Pension Systems and Reform*, The World Bank, Washington, D.C.

Holzmann, R. & Palmer, E. (2006), *Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes*, Washington, DC: World Bank.

Knell, M. (2010), 'How Automatic Adjustment Factors Affect the Internal Rate of Return of PAYG Pension Systems', *Journal of Pension Economics and Finance* **9**(01), 1–23.

Lee, R. (2003), Mortality Forecasts and Linear Life Expectancy Trend, CEDA Paper 2003-0003.

Ludwig, A. & Reiter, M. (2010), 'Sharing Demographic Risk–Who Is Afraid of the Baby Bust?', *American Economic Journal: Economic Policy* **2**(4), 83–118.

Missov, T. I. & Lenart, A. (2011), 'Linking Period and Cohort Life-Expectancy Linear Increases in Gompertz Proportional Hazards Models', *Demographic Research* **24**(19), 455–468.

OECD (2011), *Pensions at a Glance 2011 — Retirement-Income Systems in OECD and G20 Countries*, OECD, Paris.

OECD (2012), *Pensions Outlook 2012*, OECD, Paris.

Oeppen, J. & Vaupel, J. (2002), 'Demography — Broken Limits to Life Expectancy', *Science* **296**(5570), 1029–1031.

Palmer, E. (2000), The Swedish Pension Reform Model: Framework and Issues, The World Bank, Social Protection Discussion Paper No. 0012.

Palmer, E. (2006), What is NDC?, *in* R. Holzmann & E. Palmer, eds, 'Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes', Washington, DC: World Bank, pp. 17–34.

Settergren, O. (2001), 'The Automatic Balance Meachanism of the Swedish Pension System. A Non Technical Introduction', *Wirtschaftspolitische Blätter* **38**, 339–349.

Settergren, O. & Mikula, B. D. (2006), The Rate of Return of Pay As You Go Pension Systems: A More Exact Consumption-Loan Model of Interest, *in* R. Holzmann & E. Palmer, eds, 'Pension Reform: Issues and Prospects for Non-Financial Defined Contribution (NDC) Schemes', Washington, DC: The World Bank, pp. 117–142.

Shoven, J. B. & Goda, G. S. (2008), Adjusting Government Policies for Age Inflation, NBER Working Papers, National Bureau of Economic Research.

Sundén, A. (2004), How Do Individual Accounts Work in the Swedish Pension System, manuscript, Swedish National Social Insurance Board.

Valdés-Prieto, S. (2000), 'The Financial Stability of Notional Account Pensions', *Scandinavian Journal of Economics* **102**, 395–417.

Whitehouse, E. R. (2007), Life-Expectancy Risk and Pensions: Who Bears the Burden?, OECD Social, Employment and Migration Working Papers 60, OECD, Directorate for Employment, Labour and Social Affairs.

Williamson, J. B. & Williams, M. (2003), The Notional Defined Contribution Model: An Assessment of the Strengths and Limitations of a New Approach to the Provision of Old Age Security, Center for Retirement Research at Boston College, WP 2003-18.

Appendices

A Derivations and proofs

A.1 Derivation of $g^W(t)$ and $g^L(t)$

Note that $\frac{d\int_0^x f(\omega)\,\mathrm{d}\omega}{dx} = f(x)$. Therefore:

$$\frac{dW(t)}{dt} = W(0)e^{\int_0^t g(s)\,\mathrm{d}s}\frac{d\int_0^t g(s)\,\mathrm{d}s}{dt} = W(t)g(t)$$

and so $g^W(t) = \frac{\dot{W}(t)}{W(t)} = g(t)$ as stated in (19).

$$\frac{d\int_{u(x)}^{v(x)} f(x,t)\,\mathrm{d}t}{dx} = f(x, v(x))v'(x) - f(x, u(x))u'(x) + \int_{u(x)}^{v(x)} f'_x(x,t)\,\mathrm{d}t.$$

Therefore:

$$\frac{dL(t)}{dt} = N(t - \widetilde{B}(t))\frac{d\widetilde{B}(t)}{dt} - N(t)\cdot 0 + \int_0^{\widetilde{B}(t)} \frac{dN(t-a)}{dt}\,\mathrm{d}a.$$

Furthermore, if $N(t) = N(0)e^{\int_0^t n(s)\,\mathrm{d}s}$ then $\frac{dN(t-a)}{dt} = N(t-a)n(t-a)$ and thus $g^L(t) = \frac{\dot{L}(t)}{L(t)} = \frac{N(t-\widetilde{B}(t))}{L(t)}\frac{d\widetilde{B}(t)}{dt} + \frac{\int_0^{\widetilde{B}(t)} N(t-a)n(t-a)\,\mathrm{d}a}{L(t)}$.

A.2 Derivations for section 3.2

Using the expressions (13), (14) and (15) in (7) one can write the expenditures of the NDC pension system as:

$$E(t) = \overline{\tau}\int_{\widetilde{B}(t)}^{\widetilde{X}(t)} \frac{\int_0^{B(t-a)}\left[W(t)e^{-\int_{t-a+b}^t g(s)\,\mathrm{d}s}e^{\int_{t-a+b}^{t-a+B(t-a)}\rho(s)\,\mathrm{d}s}\right]\mathrm{d}b}{\Gamma(t-a, t-a+B(t-a))} e^{\int_{t-a+B(t-a)}^t \vartheta(s)\,\mathrm{d}s} N(t-a)\,\mathrm{d}a,$$

where I have used the fact that $W(t-a+b) = W(t)e^{-}\int_{t-a+b}^t g(s)\,\mathrm{d}s$. For the case where $\vartheta(t) = \rho(t)$ one can use the fact that:

$$e^{\int_{t-a+b}^{t-a+B(t-a)}\rho(s)\,\mathrm{d}s}e^{\int_{t-a+B(t-a)}^t \rho(s)\,\mathrm{d}s} = e^{\int_{t-a+b}^t \rho(s)\,\mathrm{d}s}$$

to write $E(t)$ in the form as shown in (16).

A.3 Derivations for increasing retirement age (section 4.1)

It is assumed here that retirement age follows (23a).

A.3.1 Adjusted wage bill growth and period life expectancy

When the notional interest rate is set according to (18c) and one uses period life expectancy (17a) the following relation holds:

$$\int_0^{B(t-a)} e^{\int_{t-a+b}^{t}(\rho(s)-g(s))\,\mathrm{d}s}\,\mathrm{d}b = B(t-a) \tag{26}$$

and expression (16) simplifies to:

$$E(t) = \overline{\tau}W(t)\overline{N}\int_{\widetilde{B}(t)}^{\widetilde{X}(t)} \frac{B(t-a)}{\widetilde{X}(t-a+B(t-a))-B(t-a)}\,\mathrm{d}a.$$

Using $B(t) = \frac{X(t)}{1+\overline{z}(1+\gamma)}$ and $\widetilde{X}(t) = \frac{X(t)}{1+\gamma}$ one can calculate that $\frac{B(t-a)}{\widetilde{X}(t-a+B(t-a))-B(t-a)} = \frac{1}{\overline{z}}$ and thus $E(t) = \overline{\tau}W(t)\overline{N}\frac{\widetilde{X}(t)-\widetilde{B}(t)}{\overline{z}} = \overline{\tau}W(t)\overline{N}\widetilde{B}(t)$. Given that the income of the pension system is $I(t) = \overline{\tau}W(t)L(t) = \overline{\tau}W(t)\overline{N}\widetilde{B}(t)$ it follows that $D(t) = 0$ for every period. The NDC system is in this case permanently balanced.

A.3.2 Unadjusted wage bill growth and period life expectancy

Using (18b) and (17a) leads to the following relations (for (16)):

$$\int_{t-a+b}^{t}(\rho(s)-g(s))\,\mathrm{d}s = \int_{t-a+b}^{t}\frac{\gamma}{X(s)}\,\mathrm{d}s = \ln\left(\frac{X(t)}{X(t-a+b)}\right),$$

$$\int_0^{B(t-a)} e^{\int_{t-a+b}^{t}(\rho(s)-g(s))\,\mathrm{d}s}\,\mathrm{d}b = \frac{X(t)}{\gamma}\ln\left(\frac{(1+\overline{z})(1+\gamma)}{1+\overline{z}(1+\gamma)}\right),$$

$$\int_{\widetilde{B}(t)}^{\widetilde{X}(t)} \frac{\frac{X(t)}{\gamma}\ln\left(\frac{(1+\overline{z})(1+\gamma)}{1+\overline{z}(1+\gamma)}\right)}{\widetilde{X}(t-a+B(t-a))-B(t-a)}\,\mathrm{d}a = \\ \widetilde{B}(t)\frac{(1+\overline{z})(1+\gamma)(1+\overline{z})}{\gamma^2\hat{z}}\ln\left(\frac{(1+\overline{z})(1+\gamma)}{1+\overline{z}(1+\gamma)}\right)\ln\left(\frac{1+\overline{z}(1+\gamma)}{1+\overline{z}}\right).$$

The deficit ratio is therefore given as:

$$d(t) = \frac{(1+\overline{z})(1+\gamma)(1+\overline{z})}{\gamma^2 \hat{z}} \ln\left(\frac{(1+\overline{z})(1+\gamma)}{1+\overline{z}(1+\gamma)}\right) \ln\left(\frac{1+\overline{z}(1+\gamma)}{1+\overline{z}}\right) \approx 1+\frac{\gamma}{2},$$

where the approximation follows from a first-order Taylor expansion around $\gamma = 0$. The use of unadjusted wage bill growth thus leads to a permanent deficit. This has also been confirmed by numerical simulations without using the approximation.

A.3.3 Adjusted wage bill growth and cohort life expectancy

This combines (18c) and (17b). Similar steps as in A.3.1 lead to $\frac{B(t-a)}{X(t-a)-B(t-a)} = \frac{1}{\overline{z}(1+\gamma)}$ and thus $d(t) = \frac{1}{1+\gamma}$.

A.3.4 Unadjusted wage bill growth and cohort life expectancy

Combining (17b) and (18b) and using similar steps as in A.3.2 leads to:

$$d(t) = \frac{(1+\overline{z})(1+\overline{z})}{\gamma^2 \hat{z}} \ln\left(\frac{(1+\overline{z})(1+\gamma)}{1+\overline{z}(1+\gamma)}\right) \ln\left(\frac{1+\overline{z}(1+\gamma)}{1+\overline{z}}\right) \approx 1-\frac{\gamma}{2}. \tag{27}$$

In this case one faces a permanent surplus as has also been confirmed by numerical simulations.

A.4 Derivations for constant retirement age (section 4.2)

For this case it is assumed that retirement age is constant as specified in equation (23b).

A.4.1 Adjusted wage bill growth and period life expectancy

When the notional interest rate is set according to (18c) and one uses period life expectancy (17a) the following relations hold:

$$\int_{t-a+b}^{t} (\rho(s) - g(s))\, \mathrm{d}s = \int_{t-a+b}^{t} -\frac{\gamma}{X(s)}\, \mathrm{d}s = \ln\left(\frac{X(t-a+b)}{X(t)}\right),$$

$$\int_0^{\overline{B}} e^{\int_{t-a+b}^{t} -\frac{\gamma}{X(s)}\, \mathrm{d}s}\, \mathrm{d}b = \frac{\overline{B}}{X(t)} \left(X(t-a) + \frac{\gamma}{2}\overline{B}\right),$$

$$\int_{\overline{B}}^{\widetilde{X}(t)} \frac{\frac{\overline{B}}{X(t)}\left(X(t-a)+\frac{\gamma}{2}\overline{B}\right)}{\widetilde{X}(t-a+\overline{B})-\overline{B}}\,\mathrm{d}a = \frac{\overline{B}(1+\gamma)}{X(t)}\int_{\overline{B}}^{\widetilde{X}(t)} \frac{X(t-a)+\frac{\gamma}{2}\overline{B}}{X(t-a)-\overline{B}}\,\mathrm{d}a$$

$$= \frac{\overline{B}(1+\gamma)}{X(t)}\left(\overline{B}\left(\frac{2+\gamma}{2\gamma}\ln(1+\gamma)\right)+\frac{X(t)}{1+\gamma}-\overline{B}\right).$$

Since the income of the system is given by $I(t)=\hat{\tau}W(t)\overline{N}\bar{B}$ the deficit ratio boils down to:

$$d(t)=\frac{\overline{B}(1+\gamma)}{X(t)}\left(\frac{(2+\gamma)\ln(1+\gamma)}{2\gamma}-1\right)+1, \tag{28}$$

which, using a first-order Taylor expansion around $\gamma=0$, is approximately 1.

A.4.2 Adjusted wage bill growth and cohort life expectancy

Combining (17b) and (18c) and using similar steps as before one gets that:

$$d(t)=\frac{\overline{B}}{X(t)}\left(\frac{(2+\gamma)\ln(1+\gamma)}{2\gamma}-1\right)+\frac{1}{1+\gamma}\approx 1-\gamma. \tag{29}$$

A.4.3 Average wage growth or wage bill growth and period or cohort life expectancy

For a notional interest rate according to (18c) or (18a) and the "hybrid" life expectancy concept (17c) with $\Gamma(t,t+B(t))=\left[\mu X(t)+(1-\mu)(\widetilde{X}(t+\overline{B}))\right]-\overline{B}$ it holds that:

$$\int_{t-a+b}^{t} 0\,\mathrm{d}s=0,$$

$$\int_{0}^{\overline{B}} e^{\int_{t-a+b}^{t}0\,\mathrm{d}s}\,\mathrm{d}b=\overline{B},$$

$$\int_{\overline{B}}^{\widetilde{X}(t)} \frac{\overline{B}}{(\mu X(t-a)+(1-\mu)(\widetilde{X}(t-a+\overline{B})))-\overline{B}}\,\mathrm{d}a = \int_{\overline{B}}^{\widetilde{X}(t)} \frac{\overline{B}(1+\gamma)}{(1+\mu\gamma)(X(t-a)-\overline{B})}\,\mathrm{d}a$$

$$= \frac{\overline{B}(1+\gamma)}{1+\mu\gamma}\frac{\ln(1+\gamma)}{\gamma}.$$

The deficits ratios for the cases with cohort and period life expectancy follow for $\mu=1$ and $\mu=0$, respectively. The "optimal" μ can be calculated by setting $d(t)=1$ and solving for μ. The resulting μ^* is stated in equation (24).

A.5 Derivations for the turnover duration (section 4.4)

The turnover duration is defined as the average time a unit of money is in the pension system, i.e. the average time that passes between the incoming and the outgoing payment. Settergren & Mikula (2006, p. 120) show that it can be written as: $TD(t) = A_R(t) - A_C(t)$, where $A_R(t)$ is the "money weighted average age of retiree" and $A_C(t)$ is the "money weighted average age of contributor". Expressed in formal terms one can thus write:

$$\begin{aligned} A_C(t) &= \frac{\int_0^{\widetilde{B}(t)} aW(t)N(t-a)\,\mathrm{d}a}{\int_0^{\widetilde{B}(t)} W(t)N(t-a)\,\mathrm{d}a,} = \frac{\int_0^{\widetilde{B}(t)} aN(t-a)\,\mathrm{d}a}{L(t)}, \\ A_R(t) &= \frac{\int_{\widetilde{B}(t)}^{\widetilde{X}(t)} aP(t-a,t)N(t-a)\,\mathrm{d}a}{\int_{\widetilde{B}(t)}^{\widetilde{X}(t)} P(t-a,t)N(t-a)\,\mathrm{d}a}. \end{aligned}$$

For the case with $N(t) = \overline{N}$ one has that $A_C(t) = \frac{\widetilde{B}(t)}{2}$. If the notional interest rate is defined as $\rho(t) = g^W(t) + g^L(t) - g^{TD}(t)$, where $g^{TD}(t) \equiv \frac{\dot{TD}(t)}{TD(t)}$ and $\dot{TD}(t) \equiv \frac{dTD(t)}{dt}$ then the calculation of $A_R(t)$ is not straightforward. In order to calculate the stream of $P(t-a,t)$ one has to know $g^{TD}(t)$ while one has to know the various $P(t-a,t)$ in order to calculate $TD(t)$ and thus $g^{TD}(t)$. For the case where retirement age is determined according to (23a) one can, however, make a guess. In particular, let's assume that $g^{TD}(t) = \frac{\gamma}{X(t)}$ and the notional interest rate is thus the same as given in (18c). It is then easy to show that $A_R(t) = \frac{\widetilde{X}(t)+\widetilde{B}(t)}{2}$ and therefore $TD(t) = \frac{\widetilde{X}(t)}{2}$. From this it follows that $g^{TD}(t) = \frac{\gamma}{X(t)}$ thus confirming the conjecture. For other assumptions about the retirement age, however, this procedure does not work and one would have to use simulations in order to study the self-stabilizing properties of the framework based on turnover duration and to compare it to the approach of this paper.

B Simulations

In order to study the behavior of the NDC system for arbitrary patterns of retirement one has to rely on numerical simulations. For this purpose one also has to use a discrete-time version of the continuous-time set-up presented in section 3.2 of the paper. While allowing for a wide range of assumption, this discrete-time framework has the disadvantage that many of the precise results derived in the paper are only valid in an approximate sense. The main problem is that the discrete-time version only allows for integer values of life expectancy and retirement age while the development of life expectancy and the assumptions about parallel retirement adjustments involve non-integer values. The following equations are used for the simulation:

$$X_t = \text{Round}\left(X_0 + \gamma{\cdot}t\right),$$

$$L_t = \sum_{a=1}^{\widetilde{X}_t} I_{t-a+1,t} N_{t-a+1},$$

where $\widetilde{X}_t$ is the maximum age observed in period t and $I_{t,s}$ is an indicator variable with $I_{t,s} = 1$ if generation t works in period s and $I_{t,s} = 0$ otherwise.

$$R_t = \sum_{a=1}^{\widetilde{X}_t} \left(1 - I_{t-a+1,t}\right) N_{t-a+1},$$

$$z_t = \frac{R_t}{L_t},$$

$$I_t = \overline{\tau} \sum_{a=1}^{\widetilde{X}_t} I_{t-a+1,t} W_t N_{t-a+1},$$

$$E_t = \sum_{a=1}^{\widetilde{X}_t} \left(1 - I_{t-a+1,t}\right) P_{t-a+1,t} N_{t-a+1},$$

$$K_{t,t+B_t} = \sum_{a=1}^{B_t} \overline{\tau} W_{t+a-1} \Psi_{t+a-1,t+B_t},$$

where $\Psi_{t+a-1,t+B_t}$ is the cumulative notional interest rate factor given by:

$$\Psi_{t+a-1,t+B_t} = \prod_{s=t+a}^{t+B_t} (1 + \rho_s),$$

for $1 \leq a \leq B_t$ and ρ_t is the notional interest rate from period $t-1$ to t. The first pension that is received by generation t in period $t + B_t$ is given by:

$$P_{t,t+B_t} = \frac{K_{t,t+B_t}}{\Gamma_{t,t+B_t}},$$

Existing pensions are adjusted according to:

$$P_{t,t+a-1} = P_{t,t+B_t}\Theta_{t+B_t,t+a-1},$$

where $\Theta_{t+B_t,t+a-1}$ is the cumulative adjustment rate factor given by:

$$\Theta_{t+B_t,t+a-1} = \prod_{s=t+B_t+1}^{t+a-1} (1 + \vartheta_s),$$

for $B_t + 2 \leq a \leq X_t$ and ϑ_t is the adjustment rate from period $t-1$ to t. The two possibilities for specifying the remaining life expectancy (equations (17a) and (17b)) are now given by:

$$\Gamma_{t,t+B_t} = \widetilde{X}_{t+B_t} - B_t,$$

$$\Gamma_{t,t+B_t} = X_t - B_t.$$

The different types of notional interest rate are again given by:

$$\rho_t = g_t^W,$$

$$\rho_t = g_t^W + g_t^L,$$

$$\rho_t = g_t^W + g_t^L - \frac{\gamma}{X_{t-1}},$$

where now $g_t^W = \frac{W_t - W_{t-1}}{W_{t-1}}$, $g_t^L = \frac{L_t - L_{t-1}}{L_{t-1}}$ and where $g_t^L = \frac{\gamma}{X_{t-1}}$ if the retirement age is chosen according to $\widetilde{B}_t = \frac{\widetilde{X}_t}{1+\bar{z}}$ (or the respective integer values thereof).

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Call for Applications: Visiting Research Program

The Oesterreichische Nationalbank (OeNB) invites applications from external researchers for participation in a Visiting Research Program established by the OeNB's Economic Analysis and Research Department. The purpose of this program is to enhance cooperation with members of academic and research institutions (preferably post-doc) who work in the fields of macroeconomics, international economics or financial economics and/or with a regional focus on Central, Eastern and Southeastern Europe.

The OeNB offers a stimulating and professional research environment in close proximity to the policymaking process. Visiting researchers are expected to collaborate with the OeNB's research staff on a prespecified topic and to participate actively in the department's internal seminars and other research activities. They will be provided with accommodation on demand and will, as a rule, have access to the department's computer resources. Their research output may be published in one of the department's publication outlets or as an OeNB Working Paper. Research visits should ideally last between 3 and 6 months, but timing is flexible.

Applications (in English) should include

- a curriculum vitae,
- a research proposal that motivates and clearly describes the envisaged research project,
- an indication of the period envisaged for the research visit, and
- information on previous scientific work.

Applications for 2013 should be e-mailed to *eva.gehringer-wasserbauer@oenb.at* by November 1, 2012.

Applicants will be notified of the jury's decision by mid-December. The following round of applications will close on May 1, 2013.