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Take it and leave it: Banks' balance sheet optimization and targeted longer-term refinancing operations

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Abstract

We develop and solve a dynamic optimization model of a bank's balance sheet, highlighting the critical factors influencing banks' optimization dynamics: balance sheet adjustment costs and the spreads between bank-specific lending and deposit rates and the interbank rate. We apply the model to evaluate the impact of the European Central Bank's (ECB) targeted longer-term refinancing operations (TLTROs) on banks in a simulation exercise, and we estimate the policy functions with monthly data from 200 large euro area banks spanning from 2007 to 2021. The estimation results confirm the theoretical prediction and simulations that the TLTRO programs did not stimulate lending to the private sector, but banks mainly increased their central bank assets and liabilities, especially with TLTRO III in 2020, in which the ECB implemented a reversal of its policy rates by setting the TLTRO rate below the deposit facility rate.

JEL classification: E43, E44, E58, F42, G20

Keywords: Monetary policy transmission; dynamic programming; unconventional monetary policy; panel vector autoregression.

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Non-Technical Summary

Since the 2007-2008 financial crisis, many central banks have used unconventional monetary policies to meet inflation targets. In the euro area, the ECB combined asset purchase programs with new refinancing facilities, especially in response to COVID-19. These measures have significantly impacted central banks' balance sheets and monetary policy. This paper focuses on how these measures affected the balance sheets of banks, highlighting the need to understand the responses of banks, which is crucial for policymakers but methodologically challenging due to the need to unravel various effects and channels.

We propose a dynamic optimization model of a bank's balance sheet to analyze how banks respond to monetary policy measures, specifically the ECB's TLTRO program. The interaction with other instruments and mitigation elements within TLTROs means that their impact on lending volumes, rates, and bank profitability can only be understood through a structural model.

The adjustment cost approach has already been used to model optimal bank behavior in Cosimano (1988); Elyasiani et al. (1995); Kopecky and Van Hoose (2012), without including central bank assets or liabilities. We analytically solve the dynamic programming model of the bank's balance sheet optimization by constructing policy functions that prescribe optimal adjustments to future balance sheet positions based on current positions and expected and current interest rate changes. In the model, we examine three cases: Case 1, which excludes adjustment costs; Case 2, incorporating both adjustment costs and portfolio separation; and Case 3, featuring adjustment costs but no portfolio separation. For each case, we derive optimal policy functions. The policy functions are empirically testable. For Case 3, we show theoretically that the decision to increase or decrease lending to non-financial corporations depends on the volumes and interest rates of all other balance sheet items. This finding has important implications for the evaluation of monetary policy measures.

In the econometric approach, we derive two testable hypotheses based on the estimated coefficients of a panel vector autoregression model, whether adjustment costs and portfolio separation are present. Our empirical results suggest the presence of adjustment costs and no portfolio separation (Case 3). We can determine which optimal strategies the average bank follows after the introduction of TLTRO. We go beyond the ideas of Castillo Lozoya et al. (2022), who identify four strategies: lending to non-financial corporations (NFCs) and households (HH), holding reserves in the central bank, buying government securities, and substituting for market funding. We identify a fifth strategy: granting interbank market loans.

Most importantly, we can explain why during the pandemic, banks that opted for TLTROs held these funds as central bank assets. This "take it and leave it" strategy follows directly from the optimization model, since the ECB set the TLTRO rate at -1% (lending costs for banks) while the deposit rate ranged from 0% to -0.5% due to the two-tier system on bank reserves. Consequently, the ECB provided a risk-free carry trade without capital requirements until the end of 2024.

1. Introduction

Since the global financial crisis of 2007–2008, many central banks have implemented a variety of unconventional monetary policy measures to meet their inflation targets. In the euro area, the ECB combined different asset purchase programs with new refinancing facilities, in particular, in response to the COVID-19 pandemic. The footprint of these measures on central banks' balance sheets and the implications for monetary policy is ongoing (e.g., Fricke et al., 2024; De Grauwe and Ji, 2023; Borio, 2023). Here, we focus on the initial transmission of the measures and their subsequent effects on banks' balance sheets. A clear understanding of how banks respond to such measures is essential for policymakers and is also challenging from a methodological point of view, because it requires the disentanglement of the effects of the different measures and of the channels involved.

Banks seek to optimize their balance sheet structure in response to monetary policy measures. We refer to this as the bank balance sheet channel of monetary policy, which has been extensively explored in empirical research (Jiménez et al., 2012; Igan et al., 2017; Bittner et al., 2022). This bank balance sheet channel of monetary policy includes the famous "bank lending channel" (e.g., Bernanke and Blinder, 1988; Kashyap and Stein, 1994; Bernanke and Gertler, 1995) and the recently introduced "bank deposit channel" (Drechsler et al., 2017). These channels operate simultaneously, interact with each other, and collectively shape the bank's balance sheet.

We propose a dynamic optimization model of a bank's balance sheet to study how a bank responds to monetary policy measures. We apply the model to study the effects of the ECB's targeted longer-term refinancing operation (TLTRO) program. Due to the interaction with other instruments and the elements to mitigate their possible side effects, embedded in the design of the TLTROs, the impact on lending volumes, lending rates or bank profitability can be better understood with a structural model. It is not surprising that the existing literature evaluating the TLTRO programs shows that the net effect on bank lending or profitability is ambiguous. It depends, among other things, on credit demand, bank capitalization, the bank's business model, and its risk aversion (Gambacorta and Shin, 2018; Abadi et al., 2023), which are all variables beyond the control of the central bank. The TLTROs illustrate the limitations of simpler, non-structural models.

Nonstructural models often neglect banks' optimization behavior and the interaction between the channels. The focus of such models is on individual balance sheet components, without considering the joint dynamics of the balance sheet items. Furthermore, due to confounding factors, studying only a single balance sheet component or a single transmission mechanism tends to overestimate or underestimate the effects of monetary policy measures. Hence, such models likely fail to capture the effects of monetary policy measures, and a comprehensive approach is required to understand the mechanics of the measures and the responses by banks.

TLTROs were implemented together with the asset purchase program (APP). The result was an increase in the balance sheet not only of the Eurosystem but also of the banking system. In this environment, an increase in bank lending to the real economy is easily found, as some papers have done (Altavilla et al., 2020, 2023). We propose a different angle and analyze how banks adjusted their balance sheets and how they used the funds provided by the program. Focusing solely on the increase in lending provides an incomplete picture and does not capture how TLTROs have fundamentally reshaped banks' balance sheets.

Our theoretical model captures a bank's behavior over many periods. It allows measuring balance sheet rebalancing and expansion under the constraint that total assets equal total liabilities, and to predict how a bank adapts its balance sheet in response to changing interest rates. An important feature of the model is adjustment costs, which have their origins in the investment theory developed by Eisner (1969); Lucas (1967); Gould (1968). Without adjustment costs, there would be an instantaneous jump to the new optimal balance sheet structure when needed. With adjustment costs, one would not observe an instantaneous jump, but instead a gradual adjustment to the optimal balance sheet structure.

The adjustment cost approach has already been used to model the optimal bank behavior in Cosimano (1988); Elyasiani et al. (1995); Kopecky and Van Hoose (2012) but without including central bank assets or liabilities. We solve the dynamic programming model of the bank's balance sheet optimization by constructing policy functions that prescribe how to optimally adjust the future balance sheet positions given the current balance sheet positions. In contrast to Elyasiani et al. (1995), we correctly solve the model when portfolio separation does not hold, that is, when the bank's profit function is not separable in all assets and liabilities (Sealey, 1980, 1985).

Our contribution to the literature is as follows. First, we model the behavior of banks in a dynamic optimization model with adjustment costs that also include central bank assets and liabilities in a monopolistically competitive banking system. We study three different cases in the model: Case 1 without adjustment costs and with portfolio separation, Case 2 with both adjustment costs and portfolio separation, and Case 3 with adjustment costs and no portfolio separation. For each case, we theoretically derive the exact optimal policy functions that are empirically testable. In the second and third cases, the policy functions are analytically derived using the Blanchard-Kahn method (Blanchard and Kahn, 1980).

In the first case, the optimal policy function for each asset and liability position depends only on the corresponding interest rate (e.g., the NFC lending rate for NFC loans), the interbank rate, and the holding costs of the balance sheet position. In the second case, the optimal policy is a function of the lagged balance sheet position, the corresponding interest rate, and the interbank rate (for example, the optimal NFC loan volume depends on the lagged NFC loan volume, the expected NFC lending, and the expected 3-month Euribor). In the third case, the optimal balance sheet position depends on the lags of the other balance sheet positions, all interest rates, and the interbank rate.

Second, we simulate the model for the three cases and estimate the resulting policy functions. In the simulation, we also incorporate how the TLTRO conditions have improved over time. On the one hand, our simulation suggests that the impact of TLTROs on lending to non-financial corporations and households would be greatest if there were no adjustment costs and portfolio separation (Case 1). On the other hand, the growth in lending would be the lowest if there were adjustment costs but no portfolio separation (Case 3).

Third, we use a confidential, harmonized, and granular monthly bank-level dataset on lending and deposit rates and individual balance sheet information covering a representative sample of around 200 large euro area banks from July 2007 to December 2021 available at the ECB to test the theoretically derived policy functions for the presence of adjustment costs and no portfolio separation.

Our empirical findings provide robust evidence of adjustment costs (Case 2 and Case 3) and most probably also of no portfolio separation (Case 3). This offers a compelling explanation, compared to previous studies, for the limited effects of TLTROs in boosting lending to non-financial corporations. Banks mainly take up TLTROs and leave them as central bank assets.[4] Second, we find that there was portfolio rebalancing on the liability side because banks can replace expensive funding (mostly securities and, to a lesser extent, deposits) with TLTROs under very favorable conditions. In combination with rising policy rates, this led to a massive increase in bank profitability, especially for banks that held central bank assets funded by TLTROs. Since the ECB DFR has been positive, the exemption scheme limit has had no impact on the volume of the carry trade. The TLTROs have therefore improved the impaired profitability of banks following the introduction of negative interest rates, but they only lead to a limited increase in lending to the real economy.

The paper is structured as follows. After describing the ECB's TLTRO program in Section 2, we introduce the theoretical model in Section 3, and present the solutions as policy functions. In Section 4, we provide an overview of the euro area banking dataset used for the empirical analysis. Section 5 outlines the empirical approach used to estimate the policy functions. In Section 6, we simulate data for Case 1, Case 2, and Case 3 to see if the estimated policy functions are in line with our theoretical predictions. The results of the panel estimation with real data are summarized and discussed in Section 7. Section 8 summarizes and concludes.

2. The ECB's TLTRO program

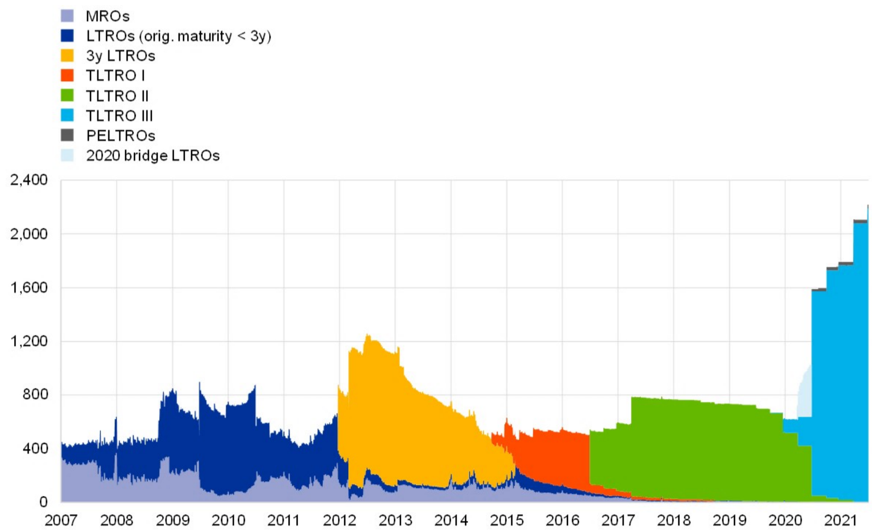
The TLTRO program replaced longer-term refinancing operations, which had been introduced in 2008 and already offered fixed rates and full allotment. Before 2008, the ECB conducted weekly auctions in which banks could bid for mostly short-term central bank liabilities. In 2008, the list of eligible collateral accepted for Eurosystem refinancing operations was expanded, allowing banks to refinance larger shares of their balance sheets with the Eurosystem.

With TLTRO I, the ECB incorporated the idea of "funding for lending" into fixed rate longer-term refinancing operations. It reduced the rate for fixed-rate tenders for main refinancing operations to be closer to the ECB DFR, provided that banks fulfilled the lending criteria of around 0% for most banks (i.e., no deleveraging of loan portfolios). As the demand for loans for house purchases was already very high at that time, this type of loan was excluded from the calculation of the interest rate that banks had to pay on the funds borrowed under the TLTROs. Over the years, the refinancing operations became more

[4] For a discussion on the implications of high excess reserves for the transmission of monetary policy, see Fricke et al. (2024) and also De Grauwe and Ji (2023).

favorable for banks in an attempt to stimulate their lending and to reduce their refinancing costs.[5]

Figure 1: Borrowing from the Eurosystem



Source: ECB. Barbiero et al. (2021); Altavilla et al. (2023). The figure shows developments in borrowing from the Eurosystem broken down into different lending facilities. Volumes are expressed in millions of euros on the y-axis. "MROs" are main refinancing operations. "LTROs (orig. maturity < 3y)" are longer-term refinancing operations with an original maturity below three years. "3y LTROs" are longer-term refinancing operations with a three-year original maturity. "TLTRO I", "TLTRO II" and "TLTRO III" refer to the three programs of targeted longer-term refinancing operations. "PELTROs" are pandemic emergency longer-term refinancing operations. "2020 bridge LTROs" are longer-term refinancing operations introduced to bridge liquidity needs between the announcement of the TLTRO recalibration in March 2020 and the first subsequent operation in June 2020.

The conditions of TLTRO III, set during the COVID-19 crisis, were particularly generous. The intention for this was to reassure banks to keep lending to firms and households for consumption, but also to mitigate the costs of a significant rise in excess liquidity due to the implementation of the pandemic emergency purchases program (PEPP).[6]

[5]See https://www.ecb.europa.eu/mopo/implement/omo/tltro/html/index.en.html and Table 1 for an overview of the TLTROs.

[6]See https://www.ecb.europa.eu/mopo/implement/pepp/html/index.en.html for details of the program.

Table 1: Summary of TLTRO parameters

TLTRO	Date	Allowance	Interest Rate(s)	Growth-Benchmark	Reference Period
I (8 operations)	2014-16	7%	ECB DFR+0.35%	0%	May 14-Apr 16
II (4 operations)	2016-17	30%	ECB DFR/ ECB DFR+0.5%	0% & 2.5%	Feb 16-Jan 18
III (2 operations)	2019	30%	ECB DFR/ ECB DFR+0.5%	0% & 1.15%	Apr 19-Mar 21
III (4 operations)	2020	50%	ECB DFR-0.5%/ ECB DFR	0%	Mar 20-Mar 21
III (4 operations)	2021	55%	ECB DFR-0.5%/ ECB DFR	0%	Oct 20-Dec 21

Source: Da Silva et al. (2021).
ECB DFR refers to the ECB Deposit Facility Rate.
Growth-Benchmark refers to the lending growth benchmark. For a few tranches of the TLTRO programs, different growth benchmarks were set for banks that deleveraged before the program and those who did not (e.g., 0% and 2.5%). Since 2020, the zero-growth benchmark was extended to all participating banks.
For TLTRO III 2019, the indicated pricing does not apply between June 2020 and June 2022. Then, it is the pricing shown for TLTRO III 2020 and 2021 that applies.
For TLTRO III 2020 and TLTRO III 2021, the indicated pricing only applies between June 2020 and June 2022. Outside this period, the pricing of TLTRO III 2019 applies.

Provided that banks were able to meet the lending criteria, TLTRO III marked the first time in the history of the ECB that it was cheaper to borrow from the central bank than to deposit. According to Schnabel (2021), this was a major game-changer. We also argue that TLTRO III represents a new kind of liability on banks' balance sheets.

From an economic perspective, the ECB offered a carry trade, banks borrow at the ECB DFR-0.5% and deposit at the ECB DFR. From June 2020 to June 2022, the ECB implemented an exemption scheme on central bank assets to compensate banks for the negative ECB DFR. The ECB decided on 12 September 2019 to set the initial multiplier for the calculation of the allowance at six times the minimum reserve requirement, and the initial applicable remuneration rate at 0%.[7] During this period, banks borrowed at -1% and deposit a big part of their central bank assets even at 0%.

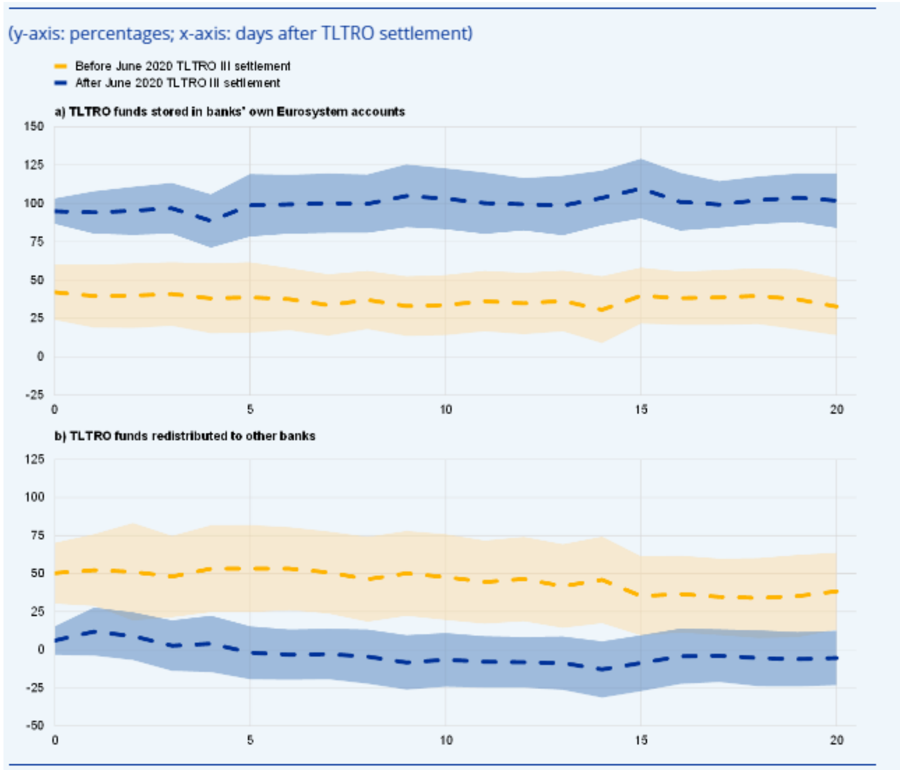
The effect of this carry trade can be seen in Figure 1. The total value of outstanding central bank liabilities skyrocketed from around 700 billion euro under TLTRO II to $2,000$ billion euro under TLTRO III. Clearly, the asset purchase programs (APPs) also had a positive impact on central bank reserves. However, the strategy of selling securities and keeping the transfers from the central bank as central bank assets was only profitable under the TLTRO III funding conditions.

Before TLTRO III, banks pushed central bank assets off their balance sheets by purchasing other securities instead of lending to the real economy. This "hot potato" effect was documented by Ryan and Whelan (2021). Only under TLTRO III conditions, holding central bank assets became attractive, turning them into "cool cucumbers" (Fricke et al., 2024) as can be seen in Table 2.[8]

[7] See https://www.ecb.europa.eu/mopo/two-tier/html/index.en.html for more details of this important measure.

[8] For the Eurosystem balance sheet by component from 2017 to 2021, see https://www.ecb.europa.eu/pub/annual/

Figure 2: Estimated percentage of TLTRO funds that banks stored and redistributed

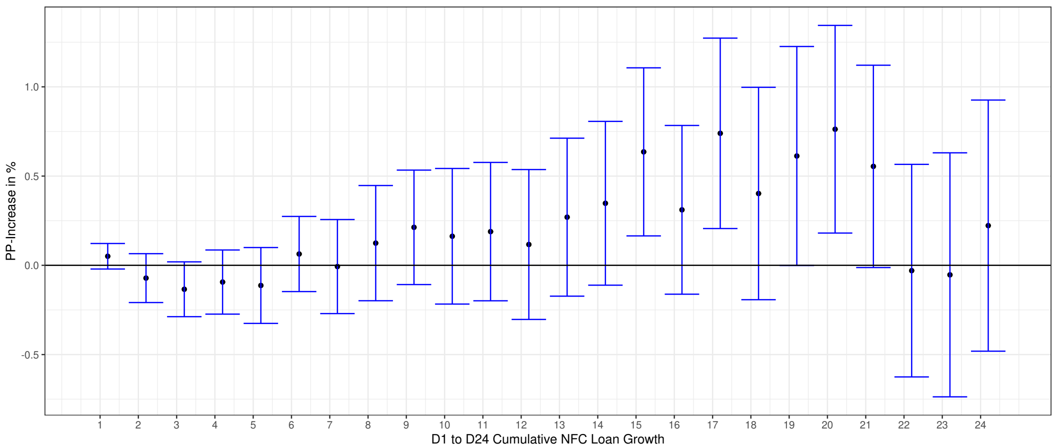


Source: ECB. Barbiero et al. (2021) Box 3 Chart B. The estimated coefficients in the chart are based on a panel data local projections model which relates daily changes in a bank's excess liquidity holdings to: its own TLTRO borrowing, panel (a); TLTRO borrowing by other banks in the Eurosystem, panel (b), controlling for other refinancing operations; the bank's own asset sales and asset sales by other banks to the Eurosystem; redemptions of assets bought by the Eurosystem; and autonomous factors at the euro area level. The first estimation period is August 2014 to 31 May 2020 for settlement before the June 2020 TLTRO, and 1 June 2020 to 1 May 2021 for settlement after the June 2020 TLTRO III. As the results are model-based and due to noise in the data and the inability to fully control for autonomous factors at the bank level (as such data are not available), the sum of funds stored in banks' own Eurosystem accounts and TLTRO funds redistributed to others do not exactly add up to 100% and are subject to uncertainty, as reflected by the shaded areas denoting 90% confidence intervals.

During the COVID-19 crisis, the zero-growth lending benchmark was extended to all banks. Hence, all participating banks were only required to keep their lending volumes constant to achieve the most favorable funding terms. Therefore, TLTRO III had only a limited effect on credit growth but could have prevented a potential collapse of credit supply during the pandemic. In Figure 3, the small impact of TLTROs on the growth of NFC lending can already be seen in a simple reduced-form model.

balance/html/ecb.eurosystembalancesheet202206_commentary~fa1d143aa2.en.html .

Figure 3: TLTRO participation and loan growth



Source: ECB. Altavilla et al. (2020). Own calculations. We replicate the results in Figure 9 of Altavilla et al. (2020) with the same raw data set, which we describe in detail in Section 4. In particular, we estimate the following equation. $\Delta L_{i,t+h} = \alpha_{c,t,h} + \beta_h TLTRO^{Dummy}_{i,t}$. h runs from 1 to 24 months. $\Delta L_{i,t+h}$ refers to the cumulative NFC loan growth up to 24 months. $TLTRO^{Dummy}_{i,t}$ is a dummy variable that takes the 1 if bank i participate takes up central bank liabilities in the last three month and 0 otherwise. $\alpha_{c,t,h}$ are country-time fixed effects.

Empirical studies on the TLTRO effects yield ambiguous results. Albertazzi et al. (2021) compare the transmission of conventional and unconventional monetary policy for banks in the euro area and find different results for bank lending and bank capital channels. They find that business models, and therefore the underlying balance sheet structures, are important with regard to the transmission of unconventional monetary policy. Altavilla et al. (2020, 2023) study the impact of the TLTROs and their interaction with the macroprudential measures in releasing certain capital buffers in the euro area during the COVID-19 crisis. Altavilla et al. (2020) find an increase in NFC loan growth of around 1.7 percentage points in each year.

Da Silva et al. (2021) develop a static theoretical model for understanding credit supply and test it empirically on credit registry data for the euro area. Their main findings are that the TLTRO III had a significant effect on credit supply, especially because the pricing of the TLTROs below market rates boosted banks' profitability. Castillo Lozoya et al. (2022) study the impact of the change in the terms to TLTRO III on the balance sheets of Spanish banks. They describe possible adjustment strategies and find that lending and holding central bank assets are the most important ones. Perdichizzi et al. (2023) examine the impact of TLTROs on the local economic development of Italian provinces and found no beneficial effects for firms, except for a general improvement in banks' funding conditions.

3. Theoretical model

The theoretical model we examine is a dynamic optimization problem that incorporates adjustment costs. Banks adjust their balance sheets when lending or deposit rates change or new instruments like TLTROs emerge. Portfolio rebalancing will generally take place gradually over time and will not be performed within one period. The observation that banks do not immediately change their balance sheet structure to shocks motivates the concept of adjustment costs, which was originally developed in investment theory by Eisner (1969); Lucas (1967); Gould (1968); Treadway (1969); Uzawa (1969). If there were no adjustment costs between periods, there would either be no investments if a bank already held the optimal capital stock, or else there would be an instantaneous jump to the new optimal balance sheet structure.

We introduce the theoretical model and derive the policy functions in Section 3.1. In Section 3.2, we discuss the theoretical effects of reversed ECB policy rates.

3.1. Dynamic optimization of a bank's balance sheet

The model is an extension of the model in Elyasiani et al. (1995), who study the implications of both loan and deposit adjustment costs with and without the assumption of portfolio separation in perfectly competitive banking markets. In our setting, a bank takes market interest rates as given, and bank-specific interest rates as predetermined. The decision variables are balance sheet quantities. Local market power is therefore possible, similar to a monopolistically competitive banking system, but we do not model the strategic interaction between banks.

Consider a representative bank with n_D different types of deposits and n_L different types of loans on its balance sheet with depositors and obligors outside the banking sector, for example, NFC deposits and consumption loans. Let $D_{j,t} \geq 0$ and $L_{i,t} \geq 0$ denote the respective volumes in period t with $t = 0, 1, 2, \ldots,$ $j = 1, \ldots, n_D$ and $i = 1, \ldots, n_L$.

We assume that the only other balance sheet items are deposits from the central bank and excess reserves held with the central bank, D_t^{CB} and L_t^{CB}. For instance, TLTROs would be captured under D_t^{CB}. Note that we define L_t^{CB} as the central bank asset holding above the minimum reserve requirement to keep the parameter ρ in our model. For all $t = 0, 1, 2, \ldots,$ we have

$$D_{j,t}, L_{i,t}, D_t^{CB}, L_t^{CB} \geq 0 \text{ for all j and i,}$$

as a bank cannot hold negative loans or deposits. Let $d_{j,t}$ be the deposit rate for deposit type j, $j = 1, \ldots, n_D$, and let $l_{i,t}$ be the lending rate for loan type i, $i = 1, \ldots, n_L$ in period t. Likewise, let d_t^{CB} denote the rate for deposits from the central bank, and let l_t^{CB} be the remuneration rate for excess reserves held at the central bank, e.g., the ECB DFR. Let F_t be the net difference between interbank assets and interbank liabilities in period t, and let r_t be the interbank rate.

The revenue of the bank in a period is driven by the profits generated by the different loans minus the refinancing costs and minus the holding and adjustment costs. That is, the net portfolio revenue π_t in $t \geq 1$ is

$$
\begin{aligned}
\pi_t = & \sum_{i=1}^{n_L} l_{i,t} L_{i,t} - \sum_{j=1}^{n_D} d_{j,t} D_{j,t} + r_t F_t + l_t^{CB} L_t^{CB} - d_t^{CB} D_t^{CB} - \\
& - \sum_{i=1}^{n_L} \frac{\phi_{1,i}}{2} L_{i,t}^2 - \sum_{i=1}^{n_L} \frac{\phi_{2,i}}{2} \left(L_{i,t} - L_{i,t-1}\right)^2 - \sum_{j=1}^{n_D} \frac{\theta_{1,j}}{2} D_{j,t}^2 - \sum_{j=1}^{n_D} \frac{\theta_{2,j}}{2} \left(D_{j,t} - D_{j,t-1}\right)^2 - \\
& - \frac{\zeta_1}{2} (L_t^{CB})^2 - \frac{\zeta_2}{2} \left(L_t^{CB} - L_{t-1}^{CB}\right)^2 - \frac{\kappa_1}{2} (D_t^{CB})^2 - \frac{\kappa_2}{2} \left(D_t^{CB} - D_{t-1}^{CB}\right)^2 - \frac{\delta}{2} F_t^2,
\end{aligned} \tag{1}
$$

subject to the simplified balance sheet constraint,

$$
\sum_{i=1}^{n_L} L_{i,t} + F_t + L_t^{CB} = (1-\rho) \sum_{j=1}^{n_D} D_{j,t} + D_t^{CB}, \tag{2}
$$

where $\rho \in (0, 1)$ is the minimum reserve requirement. The balance sheet constraint (2) asserts that the amount of assets always equals the amount of liabilities.

The parameters $\phi_{1,i}$, $\theta_{1,j}$, ζ_1, and κ_1 are the parameters of holding costs associated with the service of existing loans, deposits, central bank liabilities, and central bank assets. We assume that $\zeta_1 > 0$ and $\kappa_1 > 0$, e.g., fees for having an account at the ECB and personal costs of managing these accounts. These holding costs are most likely much lower than $\phi_{1,i}$ and $\theta_{1,j}$.

The parameters $\phi_{i,2}$, $\theta_{j,2}$, ζ_2, and κ_2 are the adjustment cost parameters. In the adjustment costs literature, there are usually quadratic adjustment costs assumed for changing the bank's portfolio from one period to the next (Gould, 1968). Flannery (1982) provides strong empirical evidence for deposit adjustment costs. On the loan side, similar costs are likely to exist. If banks want to increase their loans, they will need more staff to process loan applications and to screen the granted loans. If a bank wants to reduce loans, they will need to sell them at a discount to other banks. Based on the solution of the model, we will derive statistically testable hypotheses that provide evidence for the presence of adjustment costs. We assume $\phi_{i,2}, \theta_{j,2}, \zeta_2, \kappa_2 \geq 0$.

Adjustment and holding costs are deep structural parameters of the model that are independent of policy changes by the central bank. The policy changes of the central bank are reflected in the setting of l_t^{CB} and d_t^{CB}. For a given level of interest rates, the holding costs of the bank determine the size of the balance sheet item and, in relation to each other, the balance sheet decomposition. The adjustment costs determine the speed of convergence to a new equilibrium balance sheet structure after a shock to any of the interest rates (see Appendix B).

An important parameter of the profit function is δ, the cost parameter for the quadratic net position on the interbank market F_t. If $\delta > 0$, then it couples the different balance sheet items. If $\delta = 0$, the profit function in equation (1) is separable in all assets and liabilities (Sealey, 1980, 1985).

According to the portfolio separation theory (Sealey, 1985), banks separately maximize their interest income on the asset side of the balance sheet and minimize their expenses on the liability side. Sealey

(1985) derives the following conditions for portfolio separation: (i) shareholder unanimity on optimal portfolio decisions, (ii) separability of the resource cost functions for the assets and liabilities of the banks, and (iii) access by banks to a market for funds with equal ex-post borrowing and lending rates.[9]

If $\delta > 0$, there is no portfolio separation, violating condition (ii) of Sealey (1985). No portfolio separation implies that no market exists where banks can close their funding gap at the same interest rate and without adjustment costs. Some researchers argue that the interbank market could be this market.[10] Also, banks cannot turn to the central bank to meet their short-term liquidity needs either because there is usually a spread between the ECB lending facility and the ECB deposit facility rate.

The bank seeks to maximize the present value of the expected future cash flows by optimally adjusting the balance sheet from one period to another, which leads to a dynamic programming problem. Without loss of generality, we assume $n_L = n_D = 1$ from now to simplify the notation. The mathematical structure that we describe below and, likewise, the argumentation in Appendix A also holds for the case $n_L, n_D > 1$. We write D_t and L_t for the deposit and loan volumes, and denote the corresponding interest rates by d_t and l_t.

By equation (2), the net profit in equation (1) for $n_D = n_L = 1$ is a function of D_t, L_t, D_t^{CB} and L_t^{CB}. Given initial values $D_0, L_0, D_0^{CB}, L_0^{CB} \geq 0$, we write equation (1) as

$$\begin{aligned}
\pi_t &= l_t L_t - d_t D_t + r_t F_t + l_t^{CB} L_t^{CB} - d_t^{CB} D_t^{CB} - \\
&\quad - \frac{\phi_1}{2} L_t^2 - \frac{\phi_2}{2} (L_t - L_{t-1})^2 - \frac{\theta_1}{2} D_t^2 - \frac{\theta_2}{2} (D_t - D_{t-1})^2 - \\
&\quad - \frac{\zeta_1}{2} (L_t^{CB})^2 - \frac{\zeta_2}{2} \left(L_t^{CB} - L_{t-1}^{CB}\right)^2 - \frac{\kappa_1}{2} (D_t^{CB})^2 - \frac{\kappa_2}{2} \left(D_t^{CB} - D_{t-1}^{CB}\right)^2 - \frac{\delta}{2} F_t^2 \\
&= (l_t - r_t) L_t - (d_t - (1-\rho) r_t) D_t + (l_t^{CB} - r_t) L_t^{CB} - (d_t^{CB} - r_t) D_t^{CB} - \\
&\quad - \frac{\phi_1}{2} L_t^2 - \frac{\phi_2}{2} (L_t - L_{t-1})^2 - \frac{\theta_1}{2} D_t^2 - \frac{\theta_2}{2} (D_t - D_{t-1})^2 - \\
&\quad - \frac{\zeta_1}{2} (L_t^{CB})^2 - \frac{\zeta_2}{2} \left(L_t^{CB} - L_{t-1}^{CB}\right)^2 - \frac{\kappa_1}{2} (D_t^{CB})^2 - \frac{\kappa_2}{2} \left(D_t^{CB} - D_{t-1}^{CB}\right)^2 - \frac{\delta}{2} F_t^2,
\end{aligned}$$

with balance sheet constraint

$$L_t + L_t^{CB} + F_t = (1-\rho) D_t + D_t^{CB}. \tag{3}$$

Let $N \colon \mathbb{R}_+^4 \times \mathbb{R}_+^4 \times \mathbb{R}^5 \to \mathbb{R}$ be the net profit function, that is,

$$\pi_t = N(X_{t-1}, X_t, Z_t), \quad t = 1, 2, \ldots,$$

[9] The theory of portfolio separation is also connected to a bank's asset and liability management (Consigli and Dempster, 1998). No portfolio separation might explain the empirical evidence that almost every bank in the euro area has an asset and liability management (Deloitte Central Europe, 2019) in one department.

[10] However, it is well documented that the interbank market in the euro area is an over-the-counter market with frictions (Gabrieli, 2011; Temizsoy et al., 2017) and banks, on average, pay different interbank market lending and deposit rates (Sigmund and Siebenbrunner, 2024).

where we set $X_t := (D_t, L_t, D_t^{CB}, L_t^{CB})$ and $Z_t := \left(d_t, r_t, l_t, d_t^{CB}, l_t^{CB}\right)$. The Hessian of N is negative semi-definite, thus N is concave.

Interest rates are exogenous random variables that we assume to be uniformly bounded.

Assumption 1. *There is a bounded set $B \subset \mathbb{R}^5$ such that*

$$Z_t \in B$$

for all $t \geq 0$.

In this chapter, we do not make any assumptions about the stochastic dynamics of interest rates Z_t. For the simulation in Section 6, we assume that the rates follow an AR(1) process. In addition, d_t and l_t are bank-specific and heterogeneous between banks, implying that banks have local market power.

The dynamic programming problem of the bank is to find a sequence of functions

$$\xi_t \colon B^t \to \mathbb{R}_+^4, \quad t = 1, 2, \dots,$$

such that the total expected future return is maximal if $X_t = \xi_t[Z_1, \dots, Z_t]$ for all $t \geq 1$. The sequence $\{\xi_t\}_{t\geq 1}$ is called a plan: it prescribes how to choose the portfolio composition in each future period t, contingent on the information available in time t. The portfolio composition X_t in period t will thus be chosen as a function of the realizations of exogenous shocks up to t. However, the plan $\{\xi_t\}_{t\geq 1}$, is selected using only the information available in $t = 0$.

The bank considers only plans that maximize the net return for each period. If the portfolio yields a loss in a period, the bank will adjust its balance sheet to minimize the loss in the next period. Thus, a plan is only feasible if it generates returns with $\pi_{t+1} \geq \pi_t$ for $t \geq 0$. As N is concave, and we assume that the interest rates are bounded, there is an upper bound $R = R(X_0)$ such that a plan is only feasible from X_0 if $\xi_t \colon B^t \to \Gamma(X_0)$ for $t = 1, 2, \dots$ and $\Gamma(X_0) := \{X \in \mathbb{R}_+^4 \colon |X| \leq R\}$.

For given $X_0 \in \mathbb{R}_+^4$, let $\Xi(X_0)$ be the set of sequences of measurable functions $\{\xi_t \colon B^t \to \Gamma(X_0)\}_{t\geq 1}$, and let $\mathbb{E}_t$ denote the expectation conditional on the information available at time t. The optimization problem of the bank then reads

$$\sup_{\xi \in \Xi(X_0)} \mathbb{E}_0 \left(\sum_{t=0}^{\infty} \beta^t N\left(X_t, X_{t+1}, Z_{t+1}\right) \right), \tag{4}$$

where $X_t = \xi_t[Z_1, \dots, Z_t]$ for $t = 1, 2, \dots$, and $\beta \in (0, 1)$ is the discount factor.

We will establish a solution of (4) by constructing a time-invariant policy function g that generates an optimal plan ξ,

$$\xi_1 = \max\{g(X_0, Z_1), 0\}, \quad \xi_t[Z_1, \dots, Z_t] = \max\{g(\xi_{t-1}[Z_1, \dots, Z_{t-1}], Z_t), 0\}, \quad t \geq 2.$$

To find the policy function, consider the stochastic Euler equations for (4),

$$0 = \nabla_2 N(X_{t-1}, X_t, Z_t) + \beta \mathbb{E}_t \left[\nabla_1 N(X_t, X_{t+1}, Z_{t+1}) \right], \quad t \geq 1, \tag{5}$$

where $\nabla_1 N$ and $\nabla_2 N$ denote the vectors of partial derivatives of N with respect to the first two arguments, the state variables, and to the third and fourth argument, the control variables. The Euler equations (5) are necessary conditions for a sequence $X_t = (D_t, L_t, D_t^{CB}, L_t^{CB})$ in the interior of $\Gamma(X_0)$ to be optimal.

We study (4) and (5) for three different cases:[11]

Case 1: No adjustment costs and portfolio separation: $\phi_2, \theta_2, \kappa_2, \zeta_2, \delta = 0$

Case 2: Adjustment costs and portfolio separation: $\phi_2, \theta_2, \kappa_2, \zeta_2 > 0$, $\delta = 0$

Case 3: Adjustment costs and no portfolio separation: $\phi_2, \theta_2, \kappa_2, \zeta_2 > 0$, $\delta > 0$.

In Case 1, the Euler equations decouple, and its solution in period t will only depend on the interest rates in t. In Case 2 and Case 3, the Euler equations (5) lead to a rational expectation system of Blanchard-Kahn type. We will show that the system has four explosive eigenvalues,

$$\lambda_1, \dots, \lambda_4 > 1,$$

and four non-explosive eigenvalues,

$$\lambda_5, \dots, \lambda_8 < 1.$$

For the specific ordering of the eigenvalues used, see Appendix A. We will find that the number of explosive eigenvalues equals the number of non-predetermined variables, so that we can apply Proposition 1 in Blanchard and Kahn (1980) to construct a solution of (5).

The main result is the following theorem. The proof is given in Appendix A.

Theorem 1. *Let $\phi_1, \theta_1, \kappa_1, \zeta_1 > 0$, $X_0 \in \mathbb{R}_+^4$, and $\beta \in (0, 1)$ be given. Let Assumption 1 hold, then we obtain the following results:*

(i) Case 1: If $\phi_2, \theta_2, \kappa_2, \zeta_2, \delta = 0$, there is a unique solution of (4), given as

$$D_t = \frac{(1-\rho)r_t - d_t}{\theta_1}, \quad L_t = \frac{l_t - r_t}{\phi_1}, \quad D_t^{CB} = \frac{r_t - d_t^{CB}}{\kappa_1}, \quad L_t^{CB} = \frac{l_t^{CB} - r_t}{\zeta_1} \tag{6}$$

in the interior of $\Gamma(X_0)$ for $t \geq 1$.

[11] In all cases, we assume that holding costs apply, $\phi_1, \theta_1, \kappa_1, \zeta_1 > 0$.

(ii) Case 2: If $\phi_2, \theta_2 > 0$ and $\delta = 0$, there is a unique solution of (4), given as

$$\begin{aligned} D_t &= \sum_{k=0}^{\infty} \frac{\lambda_1^{-(k+1)}}{\beta\theta_2} \mathbb{E}_t \left[(1-\rho) r_{t+k} - d_{t+k}\right] + \lambda_5 D_{t-1} \\ L_t &= \sum_{k=0}^{\infty} \frac{\lambda_2^{-(k+1)}}{\beta\phi_2} \mathbb{E}_t \left[l_{t+k} - r_{t+k}\right] + \lambda_6 L_{t-1} \\ D_t^{CB} &= \sum_{k=0}^{\infty} \frac{\lambda_3^{-(k+1)}}{\beta\kappa_2} \mathbb{E}_t \left[r_{t+k} - d_{t+k}^{CB}\right] + \lambda_7 D_{t-1}^{CB} \\ L_t^{CB} &= \sum_{k=0}^{\infty} \frac{\lambda_4^{-(k+1)}}{\beta\zeta_2} \mathbb{E}_t \left[l_{t+k}^{CB} - r_{t+k}\right] + \lambda_8 L_{t-1}^{CB} \end{aligned} \tag{7}$$

in the interior of $\Gamma(X_0)$ for $t \geq 1$.

(iii) Case 3: If $\theta_2, \phi_2 > 0$ and $\delta > 0$ with $\theta_2 \geq \delta$, there is a unique solution of (4), given as

$$\begin{aligned} D_t &= (\lambda_1 - \lambda_5) U_{1,t} + \sum_{j \neq 1} p_{1j} U_{j,t} + q_{11} D_{t-1} + q_{12} L_{t-1} + q_{13} D_{t-1}^{CB} + q_{14} L_{t-1}^{CB} \\ L_t &= (\lambda_2 - \lambda_6) U_{2,t} + \sum_{j \neq 2} p_{2j} U_{j,t} + q_{21} D_{t-1} + q_{22} L_{t-1} + q_{23} D_{t-1}^{CB} + q_{24} L_{t-1}^{CB} \\ D_t^{CB} &= (\lambda_3 - \lambda_7) U_{3,t} + \sum_{j \neq 3} p_{3j} U_{j,t} + q_{31} D_{t-1} + q_{32} L_{t-1} + q_{33} D_{t-1}^{CB} + q_{34} L_{t-1}^{CB} \\ L_t^{CB} &= (\lambda_4 - \lambda_8) U_{4,t} + \sum_{j \neq 4} p_{4j} U_{j,t} + q_{41} D_{t-1} + q_{42} L_{t-1} + q_{43} D_{t-1}^{CB} + q_{44} L_{t-1}^{CB} \end{aligned} \tag{8}$$

in the interior of $\Gamma(X_0)$ for $t \geq 1$, with

$$\begin{aligned} U_{i,t} = -\sum_{k=0}^{\infty} \lambda_i^{-(k+1)} \bigg\{ &\frac{\hat{h}_{i1}}{\beta\theta_2} \mathbb{E}_t \left[(1-\rho) r_{t+k} - d_{t+k}\right] + \frac{\hat{h}_{i2}}{\beta\phi_2} \mathbb{E}_t \left[l_{t+k} - r_{t+k}\right] + \\ &+ \frac{\hat{h}_{i3}}{\beta\kappa_2} \mathbb{E}_t \left[r_{t+k} - d_{t+k}^{CB}\right] + \frac{\hat{h}_{i4}}{\beta\zeta_2} \mathbb{E}_t \left[l_{t-k}^{CB} - r_{t+k}\right] \bigg\} \end{aligned} \tag{9}$$

and coefficients p_{ij}, q_{ij}, and $\hat{h}_{ij}$, $i, j = 1, \ldots, 4$.

The proof of Theorem 1 will show that the Euler equations (5) are sufficient conditions for an interior plan to be optimal. That is, whenever the plans given by (6) – (8) yield positive D_t, L_t, D_t^{CB} and L_t^{CB}, equations (6) – (8) give policy functions for the optimization problem (4), defining unique solutions for the different cases. In all three cases, the optimal net interbank position is given by

$$F_t = L_t + L_t^{CB} - (1-\rho) D_t - D_t^{CB}.$$

The solution (8) in Case 3 is defined for $\delta \geq 0$, and equation (8) simplifies to equation (7) with $\delta = 0$, see Appendix A.

The solutions are driven by the spreads of the different interest rates to the interbank rate,

$$(1-\rho)r_t - d_t, \quad l_t - r_t, \quad r_t - d_t^{CB}, \quad l_t^{CB} - r_t. \tag{10}$$

If all these spreads are strictly positive for all t, the solutions (6) and (7) give inner solutions of (4). By taking the maximum of zero and the policy function for each item on the balance sheet, these solutions can be easily extended to solutions of (4) also covering situations where the spreads in (10) are zero or turn negative.

In Case 3, Kuhn-Tucker case distinctions are necessary to extend the solution to encompass negative interest rate spreads in equation (10). Each component of (8) is a function of all spreads, where the "off-diagonal" spreads are of order δ. For positive spreads with generic cost parameter settings, (8) gives a solution of (4). This was exactly the setting of TLTRO III, and (8) applies to the environment we analyze in this paper.

All three solutions in Theorem 1 have a clear economic interpretation. In Case 1, the bank adjusts its balance sheet instantaneously to the optimal structure. The adjustment costs in Case 2 require considering not only the current period, but also planning for the entire future and taking into account the structure of the current balance sheet when deciding the new one. If the bank expects a spread to be close to or equal to zero, the second term on the right-hand side of (7) is the relevant driver for the respective position, prescribing to reduce the respective volumes as $\lambda_5, \ldots, \lambda_8 < 1$. If the relevant spread of a balance sheet item is favorable for the bank, i.e., well positive, the first term on the right-hand side of (7) will dominate the dynamics and let the volumes of that item rise.

This is in principle also the underlying mechanics of Case 3, with the additional challenge of a portfolio that is fully coupled by $\delta > 0$. Any additional funding gap resulting from "optimal" loan and deposit decisions cannot be closed on the interbank market without additional costs of order δ. In line with this, the steady-state solutions of Case 1 and Case 2 are identical (see Appendix B). In Case 3, the steady states will depend on the interbank position. For instance, the steady state of L will be below that in Case 1 or Case 2 if there is a funding gap that needs to be closed on the interbank market.

3.2. How do TLTROs influence the bank optimization model

The interest rate spreads in equation (10) are of specific importance. Assuming $(1-\rho)r_t - d_t > 0$ and $l_t - r_t > 0$ is consistent with our empirical data on interest rates presented in Figure 6. If $l_t - r_t < 0$, the bank would not grant any loans L_t, but would instead only provide interbank loans refinanced by deposits D_t. On the other hand, if $(1-\rho)r_t - d_t < 0$, the bank would not supply deposits D_t and would only supply interbank deposits. If $(1-\rho)r_t - d_t < 0$ and $l_t - r_t < 0$, the bank might not operate at all.

Consider now the spreads of $r_t - d_t^{CB}$ and $l_t^{CB} - r_t$. In most periods of our data, the ECB fixed rate in fixed rate tenders is above the ECB DFR: $d_t^{CB} > l_t^{CB}$. Thus, from the bank's perspective, granting a loan to the ECB yields less than borrowing from the ECB, which is, of course, intentionally. For the risk-free interest rate, $d_t^{CB} > r_t > l_t^{CB}$ holds. Hence, in normal times, banks are not interested in holding central bank reserves above the minimum ($L_t^{CB} > 0$) and only turn to central bank liabilities if they have a refinancing problem on the interbank market.

With the TLTRO III program in combination with the exemption scheme until September 2022 (that is, $l_t^{CB} = 0\%$ for six times the minimum reserve requirement and the ECB DFR (-0.5%) for the rest of the CB assets in 2020) and the fixed TLTRO III rate at ECB DFR-0.5% (that is, $d_t^{CB} = -1\%$ in 2020) for the 3-year duration of the TLTRO III program. Hence, the spreads reversed for a three-year period: $d_t^{CB} < r_t < l_t^{CB}$, even under a rising ECB DFR after September 2022. This immediately explains why banks held reserves above the minimum reserve requirements at the central bank. It is optimal for them, and the central bank offers a "carry trade": Take TLTRO III and leave it as central bank reserve. As a side note, there are no capital requirements for central bank assets, which makes CB assets even more attractive.

4. Data and descriptive statistics

The empirical analysis is based on two unique proprietary datasets collected by the ECB, based on Guideline ECB/2016/45 amending Guideline ECB/2014/15 on monetary and financial statistics.[12]

The first dataset, *Individual Balance Sheet Items* (IBSI), collects information on balance sheet items. The IBSI dataset includes monthly observations of approximately 200 banks on balance sheet indicators, both on the asset and liability side between July 2007 and December 2021, yielding a total of $T = 174$ time periods. The panel is unbalanced. It contains unconsolidated data (i.e., outstanding amounts/stocks at the end of the month) for a representative sample from euro area countries. The sample covers, on average, around 70% of euro area banks' main assets.

The IBSI dataset is complemented with monthly bank-level data for new business volumes on deposits and loans as well as borrowing and lending rates to non-financial corporations (NFC) and to households (HH) with different maturities, loan sizes and loan purposes for the same time period. This is the second dataset used, *Individual Monetary and Financial Institution Interest Rates* (IMIR). The calculated bank-level lending and deposit rates to NFCs and HHs correspond to the interest income/expenses weighted by volume outstanding in a given month.

For calculating the bank-specific weighted interest rates, we only consider HH deposits with agreed maturity, but exclude deposits that are overnight and redeemable at notice due to the lack of data on new business volumes at these maturities. For NFC deposits, we exclude overnight deposits for the same reason. Alternatively, for calculating banks' weighted lending rates to HH, we include information on

[12] See https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:32016O0045, and ECB (2019).

consumption loans, loans for house purchases, or loans to HH for other purposes with different maturities, while loans to NFCs include loans other than overdrafts split into those up to 1 million EUR and above 1 million EUR with different maturities.

It is possible that omitting HH deposits, which are redeemable at notice, from the sample leads to higher average deposit rates, especially in the pre-crisis period, compared to those that prevailed in practice. However, in particular, after the global financial crisis, all deposit rates decreased steadily and slowly approached the zero lower bound, while the interest rate difference between the different types of deposits with different maturities dropped. We use the 3-month Euribor (via Macrobond) because it is a good indicator of the ECB's current monetary policy stance, and it is widely used as a reference rate for many financial products.

We use the main refinancing operations (MRO) rate as the central bank refinancing rate until May 2014. Then we switch to the most favorable TLTRO rate. We assume that ex-ante banks expect to fulfill the lending criteria.

To control for the other unconventional monetary policy measures on interest rate spreads and on lending and deposit rates, we add time dummies for all purchase programs. Given that the APP consisted of four different programs, namely the third covered bond purchase program (CBPP3), the asset-backed securities purchase program (ABSPP), the so-called public sector purchase program (PSPP), and the purchase of corporate sector bonds (CSPP), we add four dummies, which take the value 1 after their first implementation and 0 before. A fifth dummy is added for the PEPP from March 2020. Finally, due to the overlap of several programs, we use one dummy variable for the existence of asset purchase programs.[13]

To limit the impact of outliers from distorting the empirical results, we apply a two-stage cleaning algorithm to some selected variables by removing possible reporting errors in the regulatory reporting system. First, all negative values for shares and other equity, total main assets, lending, deposits, and sovereign holdings are eliminated. We also remove zero values of sovereign bond holdings, total main assets, and capital and reserves.[14] Second, we remove extreme values from some constructed variables. An observation is regarded as an outlier if the interquartile range exceeds 2.8 times the distance between the median and the boundaries defined by the 2.5% and 97.5% percentiles.

[13]For details on the APP, see https://www.ecb.europa.eu/mopo/implement/app and https://www.ecb.europa.eu/mopo/implement/pepp for the PEPP. To be precise, CBPP3 and ABSPP were implemented before the official start of PSPP. For a recent discussion, see also Benigno et al. (2022) or Hartmann and Smets (2018).

[14]According to ECB (2019), the item capital and reserves on the liabilities side of the balance sheet includes issued equity, profits and losses, and various provisions.

Table 2: Stylized bank balance sheet

Assets	**Liabilities**
Consumption loans	HH deposits
Mortgage loans	
NFC loans	NFC deposits
Assets securities	Liab. securities
Assets CB	Liabilities CB
	Equity
Interbank assets	*Interbank Liabilities*

This table shows a simplified balance sheet of a bank. Consumption loans refer to loans to households for consumption. Mortgage loans refer to loans used for buying residential properties by households. NFC loans refer to loans granted to non-financial corporations. Assets securities refer to securities on the asset side (e.g., bonds). Assets CB refers to loans to the central bank (e.g., minimum reserve requirements). HH deposits refer to household deposits. NFC deposits refer to deposits from non-financial corporations. Liab. securities refer to securities on the liability side of the balance sheet (e.g., covered bonds, market funding). Liabilities CB refers to liabilities to the central bank (e.g., TLTRO). Equity is defined as capital and reserves on the liabilities side of the balance sheet includes issued equity, profits and losses and various provisions.

The summary statistics of all included variables, as well as a detailed description of each variable, are given in Table 3.

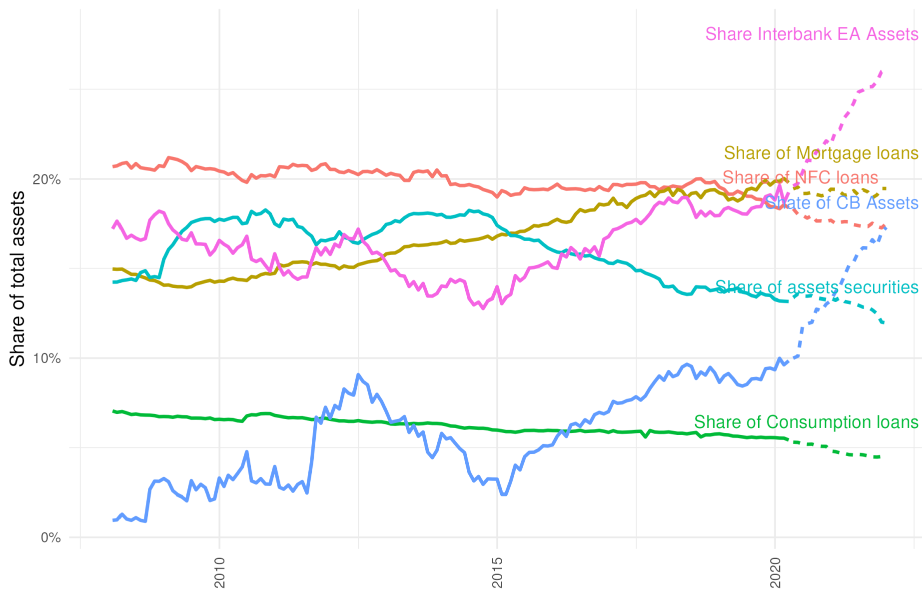
Table 3: Summary statistics

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max	Data Cov.
Balance sheet indicators							
HH deposits	0.00	233	3,850	13,512	13,857	270,804	82.59
NFC deposits	0.00	319	1,247	4,674	3,614	124,509	82.59
Liab. securities	0.00	0.00	839	10,959	8,135	439,415	74.19
Liabilities CB	0.00	30	1,502	4,863	4,862	127,399	32.42
Equity	0.44	589	1,667	5,037	4,858	104,978	82.15
NFC loans	0.00	926	3,915	10,329	10874	173,171	80.83
Mortgage loans	0.00	60	2,326	8,899	9,208	175,073	80.83
Consumption loans	0.00	153	751	2,413	2,706	34,725	80.83
Assets securities	0.00	509	3,491	10,981	12,049	182,600	82.59
Assets CB	0.01	117	489	3,499	2,579	144,782	55.61
Total assets	1.00	9,594	25,036	65,453	60,807	1,276,535	80.81
Interest rate statistics							
DR NFC	-1.67	0.13	0.73	1.25	1.88	7.51	65.26
DR HH	-0.70	0.35	1.17	1.52	2.40	13.24	64.40
LR Mortgage HH	0.00	1.96	2.77	3.02	4.13	8.28	65.80
LR Consumption HH	0.00	2.78	4.08	4.40	5.46	20.77	67.77
LR NFC	-0.25	2.03	2.89	3.14	3.98	17.33	70.34
Further variables							
CB refinancing rate	-1.00	0.10	0.15	0.34	1.00	3.75	89.80
3M-Euribor	-0.58	-0.33	0.08	0.62	0.88	5.11	98.14
2-year gov bond yield	-0.91	-0.44	0.09	0.70	1.38	20.10	67.68
10-year gov bond yield	-0.70	0.59	1.75	2.26	3.66	16.14	69.91
Dummy neg. Euribor	0.00	0.00	0.00	0.46	1.00	1.00	98.14
Dummy neg. Euribor x Euribor	-0.58	-0.33	0.00	-0.16	0.00	0.00	98.14
PSPP CSPP PEPP	0.00	0.00	0.00	0.41	1.00	1.00	98.14

Sources: ECB SDW, IBSI, IMIR, and Macrobond.
This table presents the minimum, first quartile, median, mean, third quartile, maximum, and data coverage for all variables. Data coverage indicates the percentage of available observations, assuming the panel was balanced. The data cover the period from July 2007 to December 2021 and are provided every month for 274 banks in the euro area. HH deposits refer to household deposits. NFC deposits refer to deposits from non-financial corporations. Liab. securities represent securities on the liability side of the balance sheet (e.g., covered bonds, market funding.). Liabilities CB stands for liabilities to the central bank (e.g., TLTRO). Equity refers to capital and reserves that include issued equity, profits and losses and various provisions. Consumption loans refer to loans to households for consumption. Mortgage loans denote loans used for buying residential properties by households. NFC loans stand for loans granted to non-financial corporations. Assets securities represent securities on the asset side (e.g., bonds). Assets CB refers to loans to the central bank (e.g., minimum reserve requirements). Total Assets denote the total assets of a bank. DR NFC represents the average deposit rate of non-financial corporations. DR HH refers to the average deposit rate of households. LR Mortgage HH denotes the average loan rate of mortgage loans to households. LR Consumption HH refers to the average loan rate of consumption loans to households. LR NFC stands for the average loan rate of loans to non-financial corporations. CB refinancing rate refers to the most favorable interest rate paid on TLTROs. 2-year gov bond yield denotes the two-year zero coupon government bond yield, which is used as a proxy for the average yield for the securities on the liability side. 10-year gov bond yield stands for the ten-year zero coupon government bond yield, which is used as a proxy for the average yield for the securities on the asset side. Furthermore, we include dummy variables for negative interest rates and asset purchase programs.

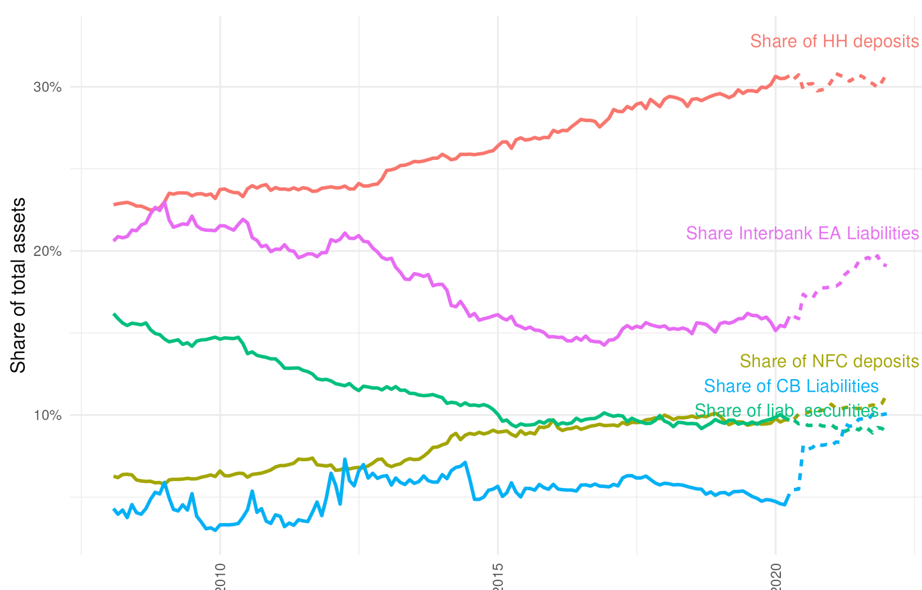
The developments of our dependent variables are shown in Figure 4 for assets and Figure 5 for liabilities. The development of the interest rates is shown in Figure 6.

Figure 4: Dependent Variables: Assets



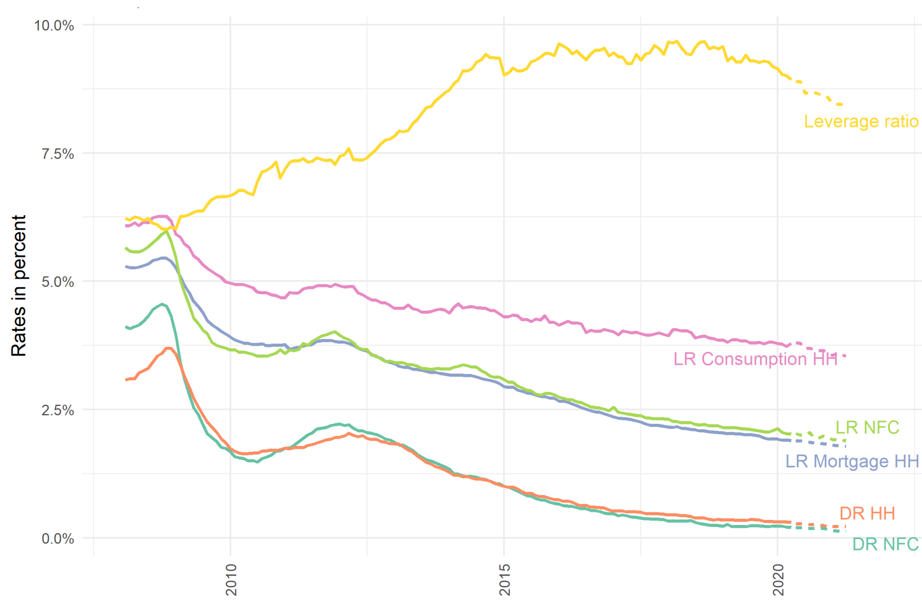
Source: ECB SDW, IBSI, and IMIR. This figure shows the evolution of the monthly averages of all dependent variables. To make them easier to compare, we calculated the ratios for each bank using total assets. The share of NFC loans refers to the share of non-financial corporation loans. Share of Mortgage loans refers to the share of mortgage loans. The share of consumption loans refers to the share of household consumption loans. Share of assets securities refers to the share of securities on the asset side. Share of CB Assets refers to the share of central bank assets. Share Interbank EA Assets refer to the share of interbank loans granted to other euro area banks.

Figure 5: Dependent Variables: Liabilities



Source: ECB SDW, IBSI, and IMIR. This figure shows the evolution of the monthly averages of all dependent variables. To make them easier to compare, we calculated the ratios for each bank using total assets. The share of HH deposits refers to the share of household deposits. Share of NFC deposits refers to the share of non-financial corporation deposits. Share of liab. securities refers to the share of securities on the liability side. Share of CB liabilities refers to the share of central bank liabilities. Share Interbank EA Liabilities refer to the share of interbank market deposits from euro area banks.

Figure 6: Predetermined Variables: Interest Rates



Source: ECB SDW, IBSI, and IMIR. This figure shows the evolution of the monthly averages of all predetermined variables. DR NFC refers to the average deposit rate of non-financial corporations. DR HH refers to the average deposit rate of households. LR Mortgage HH refers to the average lending rate of mortgage loans. LR Consumption HH refers to the average lending rate of household consumption loans. LR NFC refers to the average lending rate of non-financial corporations. No bank in our sample reports its interbank lending or deposit rates.

5. Econometric approach

The analytically derived policy function in Theorem 1 can be estimated by a PVAR model. We must make a few simplifications to estimate the large number of parameters in equation (9). We assume rational expectations for all interest rates and that every interest rate follows a mean reverting AR(1) process. This allows us to summarize the large number of coefficients for each interest rate and its expectation for the future in one coefficient for each interest rate.

With these assumptions, we can estimate equation (8) with a PVAR model:[15]

$$\mathbf{y}_{i,t} = \mu_i + \tilde{\mathbf{A}} y_{i,t-l} + \tilde{\mathbf{B}} \mathbf{x}_{i,t} + \tilde{\mathbf{C}} \mathbf{s}_{i,t} + \epsilon_{i,t}, \tag{11}$$

where $\mathbf{y}_{i,t}$ is a 9×1 vector of endogenous variables from Table 4 for the i^{th} bank at time t. $\mathbf{y}_{i,t-1}$ is a 9×1 vector of lagged endogenous variables of order one. Let $\mathbf{x}_{i,t}$ be a $k \times 1$ vector of predetermined variables

[15]PVAR models were introduced by Holtz-Eakin et al. (1988), who also introduced the first difference GMM estimator. We apply the model of Sigmund and Ferstl (2021), who added the system GMM moment conditions (Blundell and Bond, 1998) to PVAR models and allow exogenous and predetermined variables, similar to Roodman (2009b), who does it for dynamic panel models.

(see Table 4) that are potentially correlated with past errors. Let $\mathbf{s}_{i,t} \in \mathbb{R}^n$ be an $n \times 1$ vector of strictly exogenous variables (see Table 4) that depend neither on ϵ_t nor on ϵ_{t-s} for $s = 1, ..., T$. The cross-section i and the time section t are defined as $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$. Moreover, the disturbances $\epsilon_{i,t}$ are independently and identically distributed (i.i.d.) for all i and t with $\mathbb{E}[\epsilon_{\mathbf{i,t}}] = 0$ and $Var[\epsilon_{i,t}] = \Sigma_\epsilon$. Σ_ϵ is a positive semidefinite matrix. We assume that $\tilde{\mathbf{A}}$ $(m \times m)$, $\tilde{\mathbf{B}}$ $(m \times k)$ and $\tilde{\mathbf{C}}$ $(m \times n)$ are the same for all banks.

Our policy function estimations are not subject to the Lucas critique because holding and adjustment costs are deep structural parameters reflecting the "technology", "preferences", or "costs" of the bank. All reduced form parameters estimated with equation (11) are functions of them. Changes in the economic environment (e.g., the central bank policies) are reflected in changes in the interest rates. Based on the significance of the estimated coefficients, we are then able to test if the relevant structural parameters for the presence of adjustment costs ($\phi_2, \theta_2, \kappa_2, \zeta_2 > 0$) and no portfolio separation ($\delta > 0$) are statistically significantly different from zero. In particular, we have the following two propositions.

Proposition 1. ***Adjustment Costs****: If the diagonal coefficients of* $\tilde{\mathbf{A}}$ *in equation* (11) *are statistically significantly different from* 0*, then there is empirical evidence for adjustment costs (i.e.,* $\phi_2, \theta_2, \kappa_2, \zeta_2 > 0$*).*

Proof. No adjustment costs would imply Case 1. One directly confirms that equation (6) gives the unique solution of equation (A.2), proving claim (ii) of Theorem 1. This implies that the reduced form coefficients in the matrix $\tilde{\mathbf{A}}$ are all zero, i.e., $\tilde{\mathbf{A}} = 0$, no lagged dependent variable influences the current value. □

Proposition 2. ***No Portfolio Separation*** *(*$\delta > 0$*): If at least one of the off-diagonal coefficients of* $\tilde{\mathbf{A}}$ *are statistically significantly different from zero, then there is empirical evidence for no portfolio separation. Additionally, we must check the off-diagonal elements of* $\tilde{\mathbf{B}}$*, e.g., if the NFC lending rate affects any deposit volumes or "other" lending volumes.*

Proof. We test if Case 2 or Case 3 is more likely to be the data generating process. Consider equation (A.9), under $\delta = 0$, H_3 is the identity matrix, H_1 and H_2 are diagonal matrices. This implies that $H_2 H_3^{-1} = H_2$ is diagonal, hence $\tilde{\mathbf{A}}$ in equation (11) should not have any statistically significant coefficients in the off-diagonal. Also, $H_1 - H_2$ is a diagonal matrix, implying that $\tilde{\mathbf{B}}$ in equation (11) should not have any statistically significant coefficients in the off-diagonal. □

Based on the estimated matrices $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$, and $\tilde{\mathbf{C}}$ it is possible to identify the holding and adjustment costs. However, these lengthy calculations would only inform us on the holding and adjustment costs of the average bank. We argue that this is less interesting than the empirical evidence for the presence of adjustment costs and portfolio separation. To proxy the holding costs of each bank's balance sheet items, under the assumption that δ is small and the bank's balance sheet is in long-run equilibrium, is simply a transformation of equation (6).

In theory, there are different methods to estimate equation (11) after removing the fixed effects (μ_i) by applying the first differences or the forward orthogonal transformation,[16]

$$\Delta \mathbf{y}_{i,t} = \tilde{\mathbf{A}} \Delta \mathbf{y}_{i,t-l} + \tilde{\mathbf{B}} \Delta \mathbf{x}_{i,t} + \tilde{\mathbf{C}} \Delta \mathbf{s}_{i,t} + \Delta \epsilon_{i,t} \ .$$

In our estimations, Δ refers to the forward orthogonal transformation.

We use the two-step system GMM estimator, to avoid the Nickell bias, the higher-order GMM bias (Newey and Smith, 2004) and the well-known weak instrument problem (Blundell and Bond, 1998) when only using the first difference GMM moment conditions, in the case of highly autocorrelated time series. We also include predetermined variables to avoid further endogeneity issues, which is a key issue when using bank-specific explanatory variables. At the same time, we reduce the number of possible moment conditions by fixing a maximum lag length with $L_{max} < T$ and collapse the set of remaining instruments (Mehrhoff, 2009; Roodman, 2009a; Sigmund and Ferstl, 2021).

[16] $y^{\perp}_{i,t+1} = c_{i,t}(y_{i,t} - 1/T_{i,t} \sum_{s>t} y_{i,s})$, where $c_{i,t} = \sqrt{T_{i,t}/(T_{i,t}+1)}$. This transformation is suggested by Arellano and Bover (1995) to minimize data losses due to data gaps. The first difference transformation exists for $t \in \{p+2, \cdots, T\}$ and the forward orthogonal transformation exists for $t \in \{p+1, \cdots, T-1\}$. We denote the set of indexes t, for which the transformation exists by $\mathbb{T}_{\Delta}$.

Table 4: Model specifications

Type	Variable
y (endogenous)	(1) log(HH deposits)
	(2) log(NFC deposits)
	(3) log(Liab. securities)
	(4) log(Liabilities CB)
	(5) log(NFC loans)
	(6) log(Mortgage loans)
	(7) log(Consumption loans)
	(8) log(Assets securities)
	(9) log(Assets CB)
x (predetermined)	(1) DR NFC
	(2) DR HH
	(3) LR Mortgage HH
	(4) LR Consumption HH
	(5) LR NFC
s (exogenous)	(1) CB refinancing rate
	(2) 3M-Euribor
	(3) 2-year gov bond yield
	(4) 10-year gov. bond yield
	(5) Dummy neg. Euribor
	(6) Dummy neg. Euribor x Euribor
	(3) PSPP CSPP PEPP

This table lists the variables by type as used in the different estimated model specifications.
The summary statistics of these variables can be found in Table 3.
In accordance with Table 2, we have nine endogenous variables in the PVAR model.
We treat all bank-specific interest rates as predetermined in the PVAR model in equation (11), since they might depend on past shocks $\epsilon_{i,t}$.
We assume that all other variables are exogenous.
In Appendix C, we add equity as an additional endogenous variable.

6. Simulation and estimation of the bank's dynamic optimization model

In this section, we simulate three data sets generated by our model for Case 1, Case 2, and Case 3. This simulation exercise has several goals. First, we want to analyze how banks react to the ever-improving TLTRO conditions. Second, we want to compare the three cases. Third, we want to estimate the policy functions and understand what estimated coefficients in $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$, and $\tilde{\mathbf{C}}$ in equation (11) can be expected in the three cases. In particular, we are interested in testing Proposition 1 and Proposition 2, knowing the expected outcome.

To simulate data sets, we use the concept of approximate dynamic programming that has been introduced to solve, find, or approximate the solution of infinite-horizon dynamic optimization problems (Powell, 2007). There are many ideas on how to approach approximate dynamic programming. We proceed as follows. The first-order conditions for all controls and the resource constraints are stacked for all periods into a single-equation system, which is then solved using a standard Newton-Raphson method. The complementary problem (i.e., potentially binding non-negativity constraints) can be handled by adding the Min-Max approach as described in Miranda and Fackler (2004). Introducing unanticipated shocks every period then allows us to generate a stochastic simulation ("extended path" approach by Fair and Taylor, 1983).[17]

A simulation of the optimal behavior or the approximately optimal behavior over time might also help reveal the shortcomings of the model. One shortcoming of the model by Elyasiani et al. (1995) is that it does not include quadratic holding costs for bank liabilities and assets. After introducing the TLTRO III program with the 60% of the total asset rule under certain parameter configurations, a bank would just enhance its balance sheet with interbank assets and liabilities to be able to increase its TLTROs without breaching the 60% rule. Such a behavior can neither be observed in our data set nor is realistic. At a certain point, interbank liabilities have to be secured, and therefore cause holding costs. To address this, we add two complementary constraints in our simulations: $D_t^{CB} \leq 0.6 \cdot L_t$ and $F_t \in \left[-\underline{F}, \overline{F}\right]$.

We simulate exogenous interest rate shocks that replicate the observed development between the end of 2008 and 2021 in a stylized way. This period is normalized to 100 periods. Inspired by the data, the shock is modeled as a piece wise-linear decline in interest rates: 50% in the first 10 periods and 50% in the following 80 months, before leveling out. Starting from the annualized rates $l_0 = 5.3$, $d_0 = 2.3$, $r_0 = 3.4$, $d_0^{CB} = 4.1$ and $l_0^{CB} = 2.1$, the assumed shock looks as follows. For loan, deposit and interbank rates (l_t, d_t, and r_t), the total decrease is set at 2.5 percentage points, while the fall in the rate for deposits from the central bank (d_t^{CB}) was much stronger and is therefore set at 5.5 percentage points. In contrast, the loan rate vis-Ã -vis the central bank (l_t^{CB}) has seen a smaller decline and is set to 1 percentage point.

This is an important step, since in the last periods of our simulation we have $d_t^{CB} < l_t^{CB}$. To use the simulation results for vector autoregression estimations, we add noise to the market rates l_t, d_t and r_t in the form of an additive AR(1)-process term $u_t = 0.7u_{t-1} + \epsilon_t^u$ with $\epsilon^u \sim N(0, 0.1)$. The simulated path of interest rates is shown in the upper panel of Figure 7. Our simulation setup allows for expectations of future interest rate developments. If banks can anticipate changes in advance, then they will start their adjustment earlier.

Early anticipation of TLTRO programs could be possible, but the effects on banks' balance sheets should be limited before the starting date of the program. For example, banks do not need to take up CB liabilities under TLTRO I conditions to be able to take up CB liabilities under TLTRO II conditions at a later stage. Only the date decides which conditions apply. Indeed, when there were changes in the TLTRO conditions, banks paid back all CB liabilities under the old conditions and took up the requested CB liabilities when the new conditions were in place. Hence, any anticipatory effects must be reflected in other assets or liabilities. However, an expansion of the credit supply requires refinancing through other

[17]The described method is implemented in a self-written R-package ('Rmod') that can efficiently solve a wide range of dynamic, linear and non-linear, rational expectation models.

liabilities. The reduction and build-up of these liabilities cause adjustment costs, which would slow down the original lending.

We simulate the described interest rate shocks for Case 1, Case 2, and Case 3. The following set of parameters is chosen for all cases. We set $\phi_1 = \theta_1 = \zeta_1 = \kappa_1 = 0.01486$ such that D_0 is normalized to 100. For Case 2 and Case 3, the adjustment cost parameters for regular deposits and loans are set to $\phi_2 = \theta_2 = 0.1$, while half the value is used for deposits from and loans to the central bank, i.e., $\zeta_2 = \kappa_2 = 0.05$. The parameter δ is set to $\phi_1 \cdot 10^{-1}$ (Case 3) or 0 (Case 2). In Case 1 with no adjustment costs, we simply set $\phi_2 = \theta_2 = \zeta_2 = \kappa_2 = 0$. Further, the (annualized) discount factor, the reserve rate, and the bounds on interbank volumes are set to $\beta = 0.97$, $\rho = 0.01$, $\underline{F} = 0.1 \times D_0$ and $\overline{F} = 0.5 \times D_0$.

Next, we discuss how, according to our model, banks react to the introduction of TLTROs and a lower rate for deposits at the central bank. We study the responses for Case 1, Case 2, and Case 3.

Before analyzing the simulation results, recall that the optimal portfolio decision always depends on the relative change of an interest rate relative to the interbank rate r_t (Theorem 1). Hence, while the simulated interest rate paths in Figure 7 reflect the historic development, a normalized shock that is introduced only relative to r_t, i.e., $\tilde{d}_t^{CB} = d_t^{CB} - (r_t - r_0)$, etc., would result in exactly the same portfolio and volume choices by the bank.[18] As we assume the same trends for d_t, l_t and r_t, this means that the volume changes in Figure 7 can be interpreted as the isolated consequence of central bank policy, i.e., the change in the relative difference of d_t^{CB} and l_t^{CB} to r_t.

[18] In contrast to the optimal portfolio choice the resulting profits are not invariant to such a shock normalization.

Figure 7: Simulation: No Adjustment Costs, Portfolio Separation and No Portfolio Separation

The figure shows three simulations of the optimization problem in equation (4). We only include one type of deposits, one type of loans, interbank deposits, interbank loans, and central bank liabilities. In the top graph, we illustrate the interest rate trends to reflect real-world scenarios. For the first 25 periods, the central bank deposit rate is 1.5%. For the next 25 periods, it decreases to 1%. For the next 25 periods, it decreases to 0%. For the last 25 periods, it is −1%. The three bottom graphs show the volumes of loans, deposits, and central bank deposits over 100 time periods. The left-bottom graph shows the evolution of volumes for Case 1. The middle-bottom graph presents Case 2. The graph on the right-bottom reflects the developments for Case 3.

Figure 7 reveals that the central bank intervention has the same effect in all three cases in qualitative terms. Banks heavily use cheaper deposits from the central bank through TLTROs. This slightly boosts the loan supply, though not one-for-one, as central bank deposits are also used to replace other deposits. In addition, the relative increase in attractiveness of lending to the central bank results in central bank loans voluntarily exceeding the reserve requirement ($L_t^{CB} > 0$).

The differences between the three cases are more subtle (lower panels in Figure 7). In the absence of adjustment costs, banks restructure their portfolio more aggressively in response to shocks. If holding costs for interbank (net) loans are present (Case 3), then banks will be more reluctant to use interbank loans, which results in a slightly lower volume of granted regular loans.

Next, we estimate the policy function with a vector autoregression (VAR) model for each of the three cases, since we only simulate data for one representative bank. In Table 5, we present the results for Case 1. Given the policy functions in equation (6), we would expect small and insignificant lagged coefficients of the dependent variables (Proposition 1). Since the parameters in our simulation are such that the CB deposits are becoming more attractive over time, we could find some positive dependent variables lagged in column "log(CD deposits)". The interbank market interest rate r_t should be significant in all columns, while only the asset or liability interest rate should be significant in the corresponding equation (e.g., the deposit rate d should be significant in the column "log(deposit)").

Table 5: VAR Model for Case 1

	log(loans)	log(deposits)	log(CB deposits)	log(CB assets)
log(loans)(-1)	0.0845**	−0.1731*	−0.3350**	1.9683**
	(0.0388)	(0.0980)	(0.1449)	(0.8101)
log(deposits)(-1)	0.0116	0.0852*	−0.0883	0.2066
	(0.0198)	(0.0500)	(0.0740)	(0.4136)
log(CB deposits)(-1)	−0.0279**	−0.0464*	0.6639***	−0.3726
	(0.0107)	(0.0271)	(0.0401)	(0.2242)
log(CB assets)(-1)	0.0045	0.0006	−0.0178	0.6511***
	(0.0037)	(0.0093)	(0.0138)	(0.0772)
intercept	3.5025***	4.2161***	2.6796***	−5.2011
	(0.2031)	(0.5137)	(0.7595)	(4.2457)
l	0.3877***	0.0923**	0.2398***	−0.5595
	(0.0166)	(0.0419)	(0.0619)	(0.3462)
d	−0.0489***	−0.6390***	−0.0048	0.2527
	(0.0164)	(0.0415)	(0.0613)	(0.3429)
d^{CB}	−0.1679***	0.0370	−0.2521***	0.4351
	(0.0170)	(0.0429)	(0.0634)	(0.3543)
r	−0.1451***	0.4641***	0.0292	−1.1990***
	(0.0184)	(0.0466)	(0.0689)	(0.3851)
Number of Observations	100	100	100	100

Source: Own calculations. $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$.
We use the simulated dataset with no adjustment costs (Case 1).
We estimate a vector autoregression model for this dataset to make it comparable to the other tables in this subsection.
The first dependent variable is log(loans) and refers to the logarithm of loans. In our simulation study, these loans would be the sum of consumption and NFC loans in Table 2, which count towards the TLTRO programs.
The second dependent variable is log(deposits). In our simulation study, these deposits are the sum of household and NFC deposits in Table 2.
The third dependent variable is log(CB deposits). The fourth dependent variable is log(CB assets).
For each dependent variable, we include the corresponding interest rate: l refers to the lending rate. d refers to the deposit rate. d^{CB} refers to the TLTRO interest rate. r refers to the interbank rate. We cannot include the l^{CB} (e.g., the ECB DFR), since it is perfectly collinear with the d^{CB}.

According to Proposition 1, we expect highly significant lagged dependent variables in the corresponding equations (e.g., the coefficient of "log(loans)(-1)" in column "log(loans)") for Case 2. This can be clearly seen in Table 6.

Table 6: VAR Model for Case 2

	log(loans)	log(deposits)	log(CB deposits)	log(CB assets)
log(loans)(-1)	0.7176***	0.0135	−0.2164**	0.8461
	(0.0225)	(0.0449)	(0.1040)	(0.7643)
log(deposits)(-1)	0.0249**	0.6814***	−0.0750	0.4409
	(0.0116)	(0.0232)	(0.0537)	(0.3944)
log(CB deposits)(-1)	−0.0006	−0.0151***	0.7055***	−0.0586
	(0.0028)	(0.0056)	(0.0130)	(0.0952)
log(CB Assets)(-1)	0.0045***	−0.0019	−0.0165***	0.9185***
	(0.0013)	(0.0026)	(0.0060)	(0.0437)
intercept	1.0696***	1.2205***	2.4211***	−4.1590
	(0.1289)	(0.2568)	(0.5953)	(4.3750)
l	0.0809***	0.0082	0.0579***	−0.2375*
	(0.0041)	(0.0082)	(0.0190)	(0.1395)
d	−0.0022	−0.1460***	−0.0075	0.1000
	(0.0042)	(0.0084)	(0.0195)	(0.1430)
d^{CB}	−0.0344***	0.0102	−0.1591***	0.2753
	(0.0053)	(0.0106)	(0.0246)	(0.1807)
r	−0.0405***	0.1168***	0.0548**	−0.5698***
	(0.0049)	(0.0098)	(0.0227)	(0.1666)
Number of Observations	100	100	100	100

Source: Own calculations. $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$.
We use the simulated dataset with adjustment costs and portfolio separation (Case 2).
We estimate a vector autoregression model for this dataset to make it comparable to the other tables in this subsection.
The first dependent variable is log(loans) and refers to the logarithm of loans. In our simulation study, these loans would be the sum of consumption and NFC loans in Table 2, which count towards the TLTRO programs.
The second dependent variable is log(deposits). In our simulation study, these deposits are the sum of household and NFC deposits in Table 2.
The third dependent variable is log(CB deposits). The fourth dependent variable is log(CB assets).
For each dependent variable, we include the corresponding interest rate: l refers to the lending rate. d refers to the deposit rate. d^{CB} refers to the TLTRO interest rate. r refers to the interbank rate. We cannot include the l^{CB} (e.g., the ECB DFR), since it is perfectly collinear with the d^{CB}.

According to Proposition 1 and Proposition 2 in equation (8), we expect highly significant lagged dependent variables in all equations for Case 3, e.g., the coefficient of "log(loans)(-1)" in all columns in Table 7. Moreover, we expect highly significant interest rates in all equations.

Table 7: VAR Model for Case 3

	log(loans)	log(deposits)	log(CB deposits)	log(CB assets)
log(loans)(-1)	0.6404***	0.0084	−0.1293	1.4328**
	(0.0096)	(0.0678)	(0.1071)	(0.6544)
log(deposits)(-1)	0.0222***	0.8982***	−0.1582***	−0.6816**
	(0.0039)	(0.0274)	(0.0433)	(0.2646)
log(CB deposits)(-1)	0.0050***	0.0079	0.6971***	−0.1547*
	(0.0011)	(0.0081)	(0.0128)	(0.0783)
log(CB Assets)(-1)	−0.0038***	0.0100***	−0.0076*	1.0052***
	(0.0004)	(0.0029)	(0.0045)	(0.0277)
intercept	1.4440***	0.2527	2.3406***	−2.5790
	(0.0415)	(0.2946)	(0.4654)	(2.8445)
l	0.0713***	0.0371***	0.0740***	−0.1757**
	(0.0012)	(0.0088)	(0.0140)	(0.0854)
d	−0.0232***	−0.0918***	0.0277**	−0.2250***
	(0.0012)	(0.0087)	(0.0137)	(0.0839)
d^{CB}	−0.0517***	0.0445***	−0.1524***	0.3963***
	(0.0019)	(0.0136)	(0.0214)	(0.1311)
r	−0.0065***	−0.0062	0.0281*	−0.1990**
	(0.0014)	(0.0100)	(0.0159)	(0.0970)
Number of Observations	100	100	100	100

Source: Own calculations. $^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$.
We use the simulated dataset with adjustment costs and no portfolio separation (Case 3).
We estimate a vector autoregression model for this dataset to make in comparable to the other tables in this subsection.
The first dependent variable is log(loans) and refers to the logarithm of loans. In our simulation study, these loans would be the sum of consumption and NFC loans in Table 2, which count towards the TLTRO programs.
The second dependent variable is log(deposits). In our simulation study, these deposits are the sum of household and NFC deposits in Table 2.
The third dependent variable is log(CB deposits). The fourth dependent variable is log(CB assets). For each dependent variable, we include the corresponding interest rate: l refers to the lending rate. d refers to the deposit rate. d^{CB} refers to the TLTRO interest rate. r refers to the interbank rate. We cannot include the l^{CB} (e.g., the ECB DFR), since it is perfectly collinear with the d^{CB}.

7. Estimation results with euro area bank data

In this section, we present the estimation results for the policy functions in equation (8) for the data set described in Section 4. We estimate the PVAR model with the specification described in Table 4 with nine endogenous variables and present the results in two tables. In Table 8, we present the four equations for the liabilities, while Table 9 showcases the five equations for the assets.

In Table 8, we once again observe compelling evidence of adjustment costs for liabilities, as indicated by the positive and statistically significant coefficients of the own lagged dependent variable (see Proposition 1). Moreover, in some equations, we also estimate statistically significant coefficients for the lags of other endogenous variables. We interpret this as evidence for adjustment costs and no portfolio separation (see Proposition 2). This is further supported by the fact that many interest rates from other endogenous variables are significant as well. For example, DR NFC is statistically significant in the log(HH deposit) equation.

In Table 9, there is also strong evidence for the existence of adjustment costs and no portfolio separation for the assets equations. The arguments are similar to those described for the liabilities.

With our PVAR model, we can answer which optimal strategies the average bank follows after the introduction of TLTRO. We therefore go beyond the ideas of Castillo Lozoya et al. (2022), who identify four strategies, NFC and HH lending, holding reserves at the central bank, buying government securities, and substituting for market funding. Based on our results, we identify a fifth strategy, granting interbank market loans, which is also in line with the developments shown in Figure 2, especially under TLTRO I and TLTRO II.

Since we eliminated the net interbank position in solving the theoretical model in Section 3.1, we note that banks also increase their share of interbank assets in their balance sheet to distribute some funds to other banks, as shown in Figure 4. As mentioned in the data section, we analyze large European banks at the unconsolidated level. This means that the potential effects of bidder groups for TLTROs are covered by the interbank market liabilities and assets.

The initial impact of TLTRO uptake, which expands the bank's balance sheet by an increase in CB liabilities and in CB assets, banks, then, decide how to rebalance their assets and liabilities. These rebalancing can be seen by looking at the coefficients of "log(CB deposits)(-1)" and "log(CB assets)(-1)" in Table 8 and Table 9. In Table 9, the coefficients of "log(CB assets)(-1)" show how banks optimally distribute rebalance their funds. The majority of them (56%) is kept as CB assets. Only around 7% flows into NFC loans, followed by 4% to household consumption loans, the rest flows to interbank market loans, which is the residual quantity.

One could argue that these L_t^{CB} (central bank assets above the minimum reserve requirements) created by the initial TLTRO update have to stay on any bank's balance sheet, independently of its usage by the initial up-taking bank. However, this is not the case. Suppose that bank i takes up TLTRO and therefore simultaneously increases L_t^{CB} and D_t^{CB}. Then, i grants a loan to a non-financial corporation that has a deposit account at a bank j. Then, the minimum reserve requirement for the bank j increases and therefore the excess reserves in the system should be slowly absorbed by the increasing minimum reserve requirements in the system. Next bank j grants a loan to another non-financial corporation which has a deposit account at bank k and after $n \to \infty$ steps, the excess reserves in the system converge to zero if banks use their CB assets to grant loans.

Our structural model can be used for various counterfactual analyses. The most interesting counterfactual analysis is about the ever improving TLTRO conditions and the discussion in Section 3.2. What would have happened if the ECB had not set $d_t^{CB} < l_t^{CB}$ and had not allowed a risk-free carry trade under TLTRO III? The answer can be calculated by setting the CB refinancing rate to the MRO rate, but can already be seen by analyzing the data under TLTRO I and TLTRO II.

Under TLTRO I and II, most banks were not interested in participating, as can be seen in Figure 5. The average share of CB liabilities was around 5% between 2007 and 2020. At the same time, there was no breakdown in loan supply. Under TLTRO III, there was a massive increase in demand for CB liabilities and also a massive increase in the supply of CB assets. Banks understood the potential of the carry trade by just keeping the lending growth-benchmark. All of these effects can be seen in Figure 4, Figure 5 and are captured by the estimated coefficients in Table 8 and Table 9.

Table 8: PVAR: Balance Sheet Liabilities

	log(HH deposits)	log(NFC deposits)	log(liab. securities)	log(CB deposits)
log(HH deposits) (-1)	0.5506***	0.2080***	−0.0861	0.0513
	(0.0505)	(0.0739)	(0.0526)	(0.0748)
log(NFC deposits) (-1)	0.3228***	0.3943***	−0.0244	0.1261**
	(0.0645)	(0.1081)	(0.0530)	(0.0530)
log(liab. securities) (-1)	0.1319***	−0.0140	0.9011***	−0.0843
	(0.0480)	(0.0321)	(0.0351)	(0.0597)
log(CB deposits)(-1)	0.0179**	−0.0046	−0.0128**	0.8728***
	(0.0086)	(0.0091)	(0.0063)	(0.0242)
log(HH Con. Loans) (-1)	0.0481	0.0419	0.0877	0.1090
	(0.0678)	(0.1058)	(0.0595)	(0.1050)
log(HH Mortgage loans) (-1)	0.0700	−0.0747	0.1091**	−0.0639
	(0.0575)	(0.0708)	(0.0521)	(0.0908)
log(NFC loans) (-1)	0.0904*	0.2663***	0.1821**	−0.0381
	(0.0545)	(0.0607)	(0.0839)	(0.0787)
log(assets securities) (-1)	−0.0610	0.1830**	−0.2092***	0.0252
	(0.0528)	(0.0715)	(0.0514)	(0.0545)
log(CB assets) (-1)	−0.0277	−0.0163	0.0106	−0.0185
	(0.0279)	(0.0218)	(0.0278)	(0.0312)
DR NFC	−0.3007***	−0.0285	0.1949***	0.2099***
	(0.0710)	(0.0742)	(0.0651)	(0.0714)
DR HH	−0.2119***	−0.0349	−0.0091	0.0785
	(0.0591)	(0.0688)	(0.0753)	(0.0682)
LR Consumption HH	0.0197	0.0017	0.0051	−0.0272
	(0.0221)	(0.0189)	(0.0236)	(0.0334)
LR Mortgage HH	−0.0339	−0.0889*	0.0743	−0.1288*
	(0.0366)	(0.0519)	(0.0467)	(0.0745)
LR NFC	0.0306	0.0479	−0.0542	−0.0891
	(0.0384)	(0.0561)	(0.0575)	(0.0549)
CB refinancing rate	−0.0641	−0.0361	0.0261	−0.1246
	(0.0412)	(0.0460)	(0.0465)	(0.0819)
Euribor	0.0692	0.0404	−0.0386	0.0401
	(0.0460)	(0.0604)	(0.0519)	(0.0590)
2-year gov bond yield	0.1609***	0.0114	−0.0665	0.0228
	(0.0510)	(0.0481)	(0.0427)	(0.0527)
10-year gov bond yield	0.0697**	−0.0827***	−0.0204	0.0397
	(0.0343)	(0.0307)	(0.0292)	(0.0460)
Dummy neg. Euribor	−0.1488***	−0.1036**	0.0693	0.1187
	(0.0441)	(0.0444)	(0.0713)	(0.1053)
Dummy neg. Euribor x Euribor	−0.0475**	0.0031	0.0386	−0.0932
	(0.0234)	(0.0332)	(0.0406)	(0.0634)
PSPP CSPP PEPP	−0.1296***	−0.0804	0.0445	−0.0351
	(0.0444)	(0.0643)	(0.0711)	(0.0688)
const	0.0204***	0.0152	0.0035	−0.0143
	(0.0068)	(0.0109)	(0.0051)	(0.0105)
Number of Observations	11,493	11,493	11,493	11,493
Number of Groups	140	140	140	140
Obs per group: min	4	4	4	4
avg	82.10	82.10	82.10	82.10
max	148	148	148	148
Hansen test of overid: statistics:	98.34	98.34	98.34	98.34
nof para:	729	729	729	729
p-value:	1.00	1.00	1.00	1.00
RMSE	0.10	0.05	0.09	1.24

Source. IMIR. IBSI. Own calculations.
*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$. We apply the two-step system GMM estimator by Blundell and Bond (1998) with Windmeijer corrected standard errors (Windmeijer, 2005). All models are estimated by Sigmund and Ferstl (2021).
The dependent variable is log(HH deposits) in the first column. Log(HH deposits) refers to the logarithm of household deposits. The dependent variable is log(NFC deposits) in the second column. Log(NFC deposits) refers to the logarithm of non-financial corporate deposits.
The dependent variable is log(CB deposits) in the third column. Log(CB deposits) refers to logarithm of central bank deposits. The dependent variable is log(liab. securities) in the fourth column. Log(liab. securities) refers to logarithm of securities on the liability side.
For each dependent variable on the liability side, we include the corresponding interest rate: DR HH refers to the refers to the average deposit rate of households. DR NFC refers to the average deposit rate of non-financial corporations. 2-year gov bond yield refers to the two-year zero coupon government bond yield and should approximate the bond yield of securities on the liability side. CB refinancing rate refers to the most favorable interest rate paid on TLTROs.
For each dependent variable on the asset side, we include the corresponding interest rate: LR Consumption HH refers to the refers to the average lending rate of household consumption loans. LR Mortgage HH refers to the average lending rate of mortgage loans. LR NFC refers to the average lending rate of non-financial corporations. 10-year gov bond yield refers to the ten-year zero coupon government bond yield and should approximate the bond yield of securities on the asset side. ECB DFR defines the interest banks receive for depositing money with the central bank overnight.

Table 9: PVAR: Balance Sheet Assets

	log(HH Con. Loans)	log(HH Mortgage loans)	log(NFC loans)	log(assets securities)	log(CB assets)
log(HH deposits) (-1)	−0.0386	−0.0595	−0.0179	0.1142**	0.0685
	(0.0532)	(0.0555)	(0.0418)	(0.0548)	(0.0986)
log(NFC deposits) (-1)	0.2665***	0.1567***	0.2656***	−0.0424	0.1184**
	(0.0735)	(0.0313)	(0.0512)	(0.0510)	(0.0540)
log(liab. securities) (-1)	0.0314	0.0531	−0.0133	−0.0406*	0.0470
	(0.0488)	(0.0326)	(0.0342)	(0.0223)	(0.1087)
log(CB deposits)(-1)	−0.0052	−0.0060	0.0327***	−0.0039	−0.0353
	(0.0086)	(0.0082)	(0.0091)	(0.0102)	(0.0245)
log(HH Con. Loans) (-1)	0.4059***	0.2728***	0.0947**	0.2678***	0.0093
	(0.0544)	(0.0381)	(0.0433)	(0.0403)	(0.0597)
log(HH Mortgage loans) (-1)	0.2649***	0.6278***	0.1866***	−0.0294	0.1234
	(0.0673)	(0.0584)	(0.0542)	(0.0611)	(0.0931)
log(NFC loans) (-1)	0.0009	0.1745***	0.3523***	0.0730	0.0643
	(0.0375)	(0.0362)	(0.0462)	(0.0633)	(0.1083)
log(assets securities) (-1)	0.0583	−0.1138*	0.0226	0.6325***	−0.0269
	(0.0719)	(0.0614)	(0.0504)	(0.0429)	(0.0810)
log(CB assets) (-1)	0.0347	−0.0402	0.0740***	−0.0077	0.5640***
	(0.0286)	(0.0478)	(0.0275)	(0.0288)	(0.0787)
DR NFC	0.0025	−0.1649***	−0.1479***	0.2135***	0.2100**
	(0.0568)	(0.0444)	(0.0487)	(0.0442)	(0.1028)
DR HH	0.0322	−0.0002	0.0633	0.0847	0.0507
	(0.0589)	(0.0454)	(0.0678)	(0.0695)	(0.1309)
LR Consumption HH	0.0008	0.0198	−0.0007	−0.0257	0.0533
	(0.0268)	(0.0281)	(0.0200)	(0.0244)	(0.0698)
LR Mortgage HH	−0.0709	−0.0315	0.2366***	−0.0079	−0.1582
	(0.0646)	(0.0517)	(0.0481)	(0.0468)	(0.1274)
LR NFC	−0.1493***	−0.0392	−0.0015	0.1216***	−0.0875
	(0.0253)	(0.0380)	(0.0468)	(0.0406)	(0.1196)
CB refinancing rate	0.0778*	−0.0220	0.0092	0.1081**	−0.2788***
	(0.0441)	(0.0471)	(0.0364)	(0.0463)	(0.0917)
Euribor	0.1023***	−0.0101	0.0100	−0.1294***	0.0581
	(0.0340)	(0.0372)	(0.0332)	(0.0365)	(0.0875)
2-year gov bond yield	0.0367	0.0374	0.0043	−0.2283***	0.1405
	(0.0372)	(0.0392)	(0.0320)	(0.0291)	(0.0874)
10-year gov bond yield	0.0942**	0.0697**	−0.0299	−0.0687*	−0.1983***
	(0.0375)	(0.0330)	(0.0296)	(0.0394)	(0.0592)
Dummy neg. Euribor	−0.0876**	−0.0046	−0.1279***	0.1219***	0.1012
	(0.0342)	(0.0285)	(0.0234)	(0.0323)	(0.0796)
Dummy neg. Euribor x Euribor	0.0065	−0.0043	0.0351	0.0890***	−0.1926***
	(0.0292)	(0.0246)	(0.0234)	(0.0252)	(0.0455)
PSPP CSPP PEPP	−0.0380	0.0842***	0.0785**	0.0359	0.1147*
	(0.0384)	(0.0317)	(0.0381)	(0.0433)	(0.0599)
const	−0.0057	0.0247***	0.0213***	0.0170***	0.0127*
	(0.0065)	(0.0045)	(0.0051)	(0.0056)	(0.0072)
Bank fixed effects	*yes*	*yes*	*yes*	*yes*	*yes*
Number of Observations	11,493	11,493	11,493	11,493	11493
Number of Groups	140	140	140	140	140
Obs per group: min	4	4	4	4	4
avg	82.10	82.10	82.10	82.10	82.10
max	148	148	148	148	148
Hansen test of overid: statistics:	98.34	98.34	98.34	98.34	98.34
nof para:	729	729	729	729	729
p-value:	1.00	1.00	1.00	1.00	1.00
RMSE	0.07	0.06	0.09	0.07	1.00

Source. IMIR. IBSI. Own calculations.
*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$. We apply the two-step system GMM estimator by Blundell and Bond (1998) with Windmeijer corrected standard errors (Windmeijer, 2005). All models are estimated by Sigmund and Ferstl (2021).
The dependent variable is log(HH Con. Loans) in the first column. Log(HH Con. Loans) refers to the logarithm of consumption loans.
The dependent variable is log(HH Mortgage loans) in the second column. Log(HH Mortgage loans) refers to the logarithm of household mortgage loans.
The dependent variable is log(NFC loans) in the third column. Log(NFC loans) refers to logarithm of non-financial corporate loans.
The dependent variable is log(CB assets) in the fourth column. Log(CB assets) refers to logarithm of central bank assets.
The dependent variable is log(assets securities) in the fourth column. Log(assets securities) refers to logarithm of securities on the asset side.
For each dependent variable on the liability side, we include the corresponding interest rate: DR HH refers to the refers to the average deposit rate of households. DR NFC refers to the average deposit rate of non-financial corporations. 2-year gov bond yield refers to the two-year zero coupon government bond yield and should approximate the bond yield of securities on the liability side. CB refinancing rate refers to the most favorable interest rate paid on TLTROs.
For each dependent variable on the asset side, we include the corresponding interest rate: LR Consumption HH refers to the refers to the average lending rate of household consumption loans. LR Mortgage HH refers to the average lending rate of mortgage loans. LR NFC refers to the average lending rate of non-financial corporations. 10-year gov bond yield refers to the ten-year zero coupon government bond yield and should approximate the bond yield of securities on the asset side. ECB DFR defines the interest banks receive for depositing money with the central bank overnight.

8. Summary and conclusions

We constructed a dynamic programming optimization model with the possibility of incorporating adjustment costs and portfolio separation. The model was used to study how banks responded to the TLTROs and their improving conditions. We theoretically derived three policy functions for no adjustment costs (Case 1), adjustment costs and portfolio separation (Case 2), and adjustment costs and no portfolio separation (Case 3). We simulated data for each of the three cases and also estimated the resulting policy functions. We then estimated the policy functions on real-world data of 200 large banks in the euro area for the period 2007-2021.

Our estimation results allow us to evaluate the effect of the TLTROs on banks' balance sheet structures. We find strong evidence that banks face adjustment costs (Case 2 or Case 3) and also evidence that there is no portfolio separation (Case 3). This implies, as seen in the data, that banks did not expand their balance sheets in response to the TLTROs as intended by mechanically increasing the TLTROs on the liabilities side and lending to consumers and NFCs on the assets side. Instead, there was a gradual rebalancing of balance sheet positions over time, driven by adjustment costs and interest rate differentials. Banks primarily expanded their central bank assets in response to steadily improving TLTRO conditions and other unconventional monetary policy measures. The complex flows of funds approximated in our PVAR model also imply that any reduced-form model, which analyzes, for instance, only the growth of loans to NFCs, is likely to be incomplete.

The model also shows that it was rational for banks to take up central bank liabilities under TLTRO III and keep them as central bank assets until the TLTRO III had to be repaid. Banks fulfilled only the minimum requirements for lending to the private sector to obtain the most favorable TLTRO rate. This results in a risk-free carry trade with an interest rate margin between 0.5% to 1% depending on how many assets a bank holds at the CB in excess of the minimum reserve requirements during the exemption scheme.

Our model can be used to explain many empirical facts discussed in the literature. The results in Fricke et al. (2024) show that the net worth of reserve-rich banks increases when the interest rate on reserves increases sharply, directly resulting from the fact that the reversal of central bank policy rates remains, offering a risk-free carry trade until the TLTRO III program is paid back.

Our model also provides an alternative "deposit channel" of monetary policy. Drechsler et al. (2017) show that when the Fed funds rate increases, banks widen the spreads they charge on deposits, and deposits flow out of the banking system. Deposit spreads increase more, and deposits flow out at a higher rate in concentrated markets. In the euro area, banks have expanded their deposit spread (ECB DFR - deposit rate) in the recent policy rate hike because their market power in the deposit market has increased, at least in the short run, due to TLTRO III as an alternative cheap funding source.

We argue that there is a bank balance sheet channel of monetary policy. This channel depends not only on the spreads between lending or deposit rates and the interbank rate, but also on the interest rates of alternative or even new balance sheet items, as well as adjustment and holding costs. This demonstrates that monetary policy measures cannot be analyzed in isolation, focusing solely on the bank lending channel or the bank deposit channel, as these channels are interconnected.

In summary, we find that the TLTROs did not have a particularly strong effect on lending to the real economy. On the other hand, we show theoretically and empirically that the TLTRO program was effective with regard to banks' profitability: banks take it and leave it.

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Appendix A. Proof of Theorem 1

We first show that the Euler equations (5) are sufficient conditions for an inner plan to solve the optimization problem (4).

Let $\tilde{\xi} \in \Xi(X_0)$ solve (5), and let $\hat{\xi} \in \Xi(X_0)$ be another plan. Write $\tilde{X}_t := \tilde{\xi}_t[Z_1, ..., Z_t]$ and $\hat{X}_t := \hat{\xi}_t[Z_1, ..., Z_t]$ for $(Z_1, \dots, Z_t) \in B^t$, and consider

$$V := \lim_{n\to\infty} \mathbb{E}_0 \left(\sum_{t=0}^{n} \beta^t \left[N\left(\tilde{X}_t, \tilde{X}_{t+1}, Z_{t+1}\right) - N\left(\hat{X}_t, \hat{X}_{t+1}, Z_{t+1}\right)\right]\right),$$

the expected difference in the discounted total return between the plans $\tilde{\xi}$ and $\hat{\xi}$ along a sequence of shocks $\{Z_t \in B\}_{t\geq 1}$. As N is concave, $\tilde{X}_0 = \hat{X}_0$, and $\{\tilde{X}_t\}$ solves (5), we find

$$\begin{aligned}
V \geq{}& \lim_{T\to\infty} \mathbb{E}_0 \left(\sum_{t=0}^{T} \beta^t \left[\nabla_1 N\left(\tilde{X}_t, \tilde{X}_{t+1}, Z_{t+1}\right)\left(\tilde{X}_t - \hat{X}_t\right) + \nabla_2 N\left(\tilde{X}_t, \tilde{X}_{t+1}, Z_{t+1}\right) \cdot \left(\tilde{X}_{t+1} - \hat{X}_{t+1}\right)\right]\right) \\
={}& \lim_{T\to\infty} \mathbb{E}_0 \left(\sum_{t=0}^{T} \beta^t \left[\beta \nabla_1 N\left(\tilde{X}_{t+1}, \tilde{X}_{t+2}, Z_{t+2}\right) + \nabla_2 N\left(\tilde{X}_t, \tilde{X}_{t+1}, Z_{t+1}\right)\right] \cdot \left(\tilde{X}_{t+1} - \hat{X}_{t+1}\right)\right) \\
&- \lim_{T\to\infty} \mathbb{E}_0 \left(\beta^{T+1} \nabla_1 N\left(\tilde{X}_{T+1}, \tilde{X}_{T+2}, Z_{T+2}\right) \cdot \left(\tilde{X}_{T+1} - \hat{X}_{T+1}\right)\right) \\
={}& \lim_{T\to\infty} \mathbb{E}_0 \left(\sum_{t=0}^{T} \beta^t \left[\beta \mathbb{E}_{t+1} \nabla_1 N\left(\tilde{X}_{t+1}, \tilde{X}_{t+2}, Z_{t+2}\right) + \nabla_2 N\left(\tilde{X}_t, \tilde{X}_{t+1}, Z_{t+1}\right)\right] \cdot \left(\tilde{X}_{t+1} - \hat{X}_{t+1}\right)\right) \\
&- \lim_{T\to\infty} \mathbb{E}_0 \left(\beta^{T+1} \nabla_1 N\left(\tilde{X}_{T+1}, \tilde{X}_{T+2}, Z_{T+2}\right) \cdot \left(\tilde{X}_{T+1} - \hat{X}_{T+1}\right)\right) \\
={}& - \lim_{T\to\infty} \mathbb{E}_0 \left(\beta^{T+1} \nabla_1 N\left(\tilde{X}_{T+1}, \tilde{X}_{T+2}, Z_{T+2}\right) \cdot \left(\tilde{X}_{T+1} - \hat{X}_{T+1}\right)\right).
\end{aligned} \tag{A.1}$$

In the last step, we apply the law of iterated expectations. Both plans $\tilde{\xi}$ and $\hat{\xi}$ are in $\Xi(X_0)$, hence $\nabla_1 N\left(\tilde{X}_{t+1}, \tilde{X}_{t+2}, Z_{t+2}\right)$ and $\tilde{X}_t - \hat{X}_t$ are uniformly bounded. With $\beta \in (0, 1)$, this implies that the limit in the last line of (A.1) is zero, thus $V \geq 0$. Note that if $\theta_2 = \phi_2 = \kappa_2 = \zeta_2 = 0$ and no adjustment costs apply, we have $\nabla_1 N\left(\tilde{X}_{t+1}, \tilde{X}_{t+2}, Z_{t+2}\right) = 0$.

As $\hat{\xi}$ was arbitrarily chosen, $V \geq 0$ implies that a solution $\tilde{\xi}$ of the Euler equations (5) gives an optimal plan, i.e., a solution of (4). This proves claim (i) of Theorem 1.

In the next step, we construct the solution of (5) for the different cases.

The Euler equations (5) read

$$
\begin{aligned}
0 &= -d_t + (1-\rho)r_t - \left(\beta\theta_2 + \delta(1-\rho)^2 + \theta_1 + \theta_2\right) D_t + \delta(1-\rho)L_t - \\
&\qquad - \delta(1-\rho)D_t^{CB} + \delta(1-\rho)L_t^{CB} + \theta_2 D_{t-1} + \beta\theta_2 \mathbb{E}_t D_{t+1}, \\
0 &= l_t - r_t + \delta(1-\rho)D_t - (\beta\phi_2 + \delta + \phi_1 + \phi_2) L_t + \delta D_t^{CB} - \delta L_t^{CB} + \phi_2 L_{t-1} + \beta\phi_2 \mathbb{E}_t L_{t+1}, \\
0 &= -d_t^{CB} + r_t - \delta(1-\rho)D_t + \delta L_t - (\beta\kappa_2 + \delta + \kappa_1 + \kappa_2) D_t^{CB} + \delta L_t^{CB} + \kappa_2 D_{t-1}^{CB} + \beta\kappa_2 \mathbb{E}_t D_{t+1}^{CB}, \\
0 &= l_t^{CB} - r_t - \delta(1-\rho)D_t + \delta L_t - \delta D_t^{CB} - (\beta\zeta_2 + \delta + \zeta_1 + \zeta_2) L_t^{CB} + \zeta_2 L_{t-1}^{CB} + \beta\zeta_2 \mathbb{E}_t L_{t+1}^{CB}.
\end{aligned}
\tag{A.2}
$$

In Case 1, one directly confirms that (6) gives the unique solution of (A.2), proving claim (ii) of Theorem 1.

Assume now that $\theta_2, \phi_2, \kappa_2, \zeta_2 > 0$, and $\delta \geq 0$, which covers Case 2 and Case 3. The Euler equations (A.2) can now be written as first-order system,

$$
\mathbb{E}_t \begin{pmatrix} X_{t+1} \\ Y_{t+1} \end{pmatrix} = A \begin{pmatrix} X_t \\ Y_t \end{pmatrix} + B \begin{pmatrix} Z_t \\ 0 \end{pmatrix} \tag{A.3}
$$

with

$$
X_t = \begin{pmatrix} D_t \\ L_t \\ D_t^{CB} \\ L_t^{CB} \end{pmatrix}, \quad Y_t = \begin{pmatrix} D_{t-1} \\ L_{t-1} \\ D_{t-1}^{CB} \\ L_{t-1}^{CB} \end{pmatrix}, \quad Z_t = \begin{pmatrix} d_t \\ r_t \\ l_t \\ d_t^{CB} \\ l_t^{CB} \end{pmatrix}, \quad A = \begin{pmatrix} A_1 & A_2 \\ I & 0 \end{pmatrix},
$$

where

$$
A_1 = \begin{pmatrix}
1 + \frac{\delta(1-\rho)^2}{\beta\theta_2} + \frac{1}{\beta} + \frac{\theta_1}{\beta\theta_2} & -\frac{\delta(1-\rho)}{\beta\theta_2} & \frac{\delta(1-\rho)}{\beta\theta_2} & -\frac{\delta(1-\rho)}{\beta\theta_2} \\
-\frac{\delta(1-\rho)}{\beta\phi_2} & 1 + \frac{\delta}{\beta\phi_2} + \frac{1}{\beta} + \frac{\phi_1}{\beta\phi_2} & -\frac{\delta}{\beta\phi_2} & \frac{\delta}{\beta\phi_2} \\
\frac{\delta(1-\rho)}{\beta\kappa_2} & -\frac{\delta}{\beta\kappa_2} & 1 + \frac{\delta}{\beta\kappa_2} + \frac{1}{\beta} + \frac{\kappa_1}{\beta\kappa_2} & -\frac{\delta}{\beta\kappa_2} \\
-\frac{\delta(1-\rho)}{\beta\zeta_2} & \frac{\delta}{\beta\zeta_2} & -\frac{\delta}{\beta\zeta_2} & 1 + \frac{\delta}{\beta\zeta_2} + \frac{1}{\beta} + \frac{\zeta_1}{\beta\zeta_2}
\end{pmatrix}, \quad A_2 = -\frac{1}{\beta} I,
$$

and

$$
B = \begin{pmatrix}
\frac{1}{\beta\theta_2} & -\frac{1-\rho}{\beta\theta_2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\beta\phi_2} & -\frac{1}{\beta\phi_2} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{\beta\kappa_2} & 0 & \frac{1}{\beta\kappa_2} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\beta\zeta_2} & 0 & 0 & -\frac{1}{\beta\zeta_2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}.
$$

The system (A.3) is of Blanchard-Kahn type with X_t non-predetermined in t, and $Y_t = X_{t-1}$ predetermined in t. By Proposition 1 in Blanchard and Kahn (1980), the Euler equations will have a solution if and only if the coefficient matrix A has exactly four explosive eigenvalues, i.e., eigenvalues with absolute values strictly greater than 1. To show that this holds, we use the following lemma.

Lemma 1. *Let $A \in \mathbb{R}^{2n\times 2n}$, $n \geq 1$, be of the form*

$$A := \begin{pmatrix} \tilde{A} & cI \\ I & 0 \end{pmatrix}$$

with $\tilde{A} \in \mathbb{R}^{n\times n}$ and $c \neq 0$. Then A is non-singular, and the eigenvalues of A are the solutions of

$$\lambda^2 - \lambda\mu_j - c = 0,$$

where μ_j is an eigenvalue of $\tilde{A}$, $j = 1, \ldots, n$.

Proof. Let $c \neq 0$. The determinant of A is c^n, thus A is non-singular. The eigenvalues of A are the roots of the characteristic polynomial. As the eigenvalues are non-zero, it is given as

$$\begin{aligned}\chi(\lambda) &= \det(A - \lambda I)\\ &= \det(-\lambda I)\det\left((\tilde{A} - \lambda I) - cI(-\lambda I)^{-1}\right)\\ &= (-\lambda)^n \det\left(\tilde{A} - \left(\lambda - \frac{c}{\lambda}\right)I\right),\end{aligned}$$

where the second factor in the last line is the characteristic polynomial of $\tilde{A}$, evaluated at $\mu = \lambda - c/\lambda$. □

Using the Gershgorin circle theorem and its generalization by Feingold and Varga (1962), we find that the eigenvalues $\mu_1, \ldots, \mu_4$ of A_1 are real, and that

$$\mu_j > \frac{2}{\sqrt{\beta}}, \quad j = 1, \ldots, 4.$$

By Lemma 1, it follows that all eigenvalues of A are real and that there are four eigenvalues greater than 1 and four less than or equal to 1. Let $\lambda_1, \ldots, \lambda_8$ denote the eigenvalues of A such that

$$\lambda_1, \ldots, \lambda_4 > 1, \quad \lambda_5, \ldots, \lambda_8 \leq 1,$$

and

$$\lambda_j + \lambda_{j+4} = \mu_j, \quad j = 1, \ldots, 4, \tag{A.4}$$

which follows from Vieta's formulas. In particular, the number of explosive eigenvalues equals the number of non-predetermined variables, and there is a unique-bounded solution of (A.3). Our solution strategy is to diagonalize matrix A to separate the stable from the unstable part of equation (A.3).

We thus need to find a basis transformation H such that $H^{-1}AH$ is diagonal. The columns of H are eigenvectors of A, and we use the specific block structure of A and (A.4) to find an appropriate H. Direct calculations yield that matrix H, given by

$$H = \begin{pmatrix} \lambda_1 & \lambda_2 h_{21} & \lambda_3 h_{31} & \lambda_4 h_{41} & \lambda_5 & \lambda_6 h_{21} & \lambda_7 h_{31} & \lambda_8 h_{41} \\ \lambda_1 h_{11} & \lambda_2 & \lambda_3 h_{32} & \lambda_4 h_{42} & \lambda_5 h_{11} & \lambda_6 & \lambda_7 h_{32} & \lambda_8 h_{42} \\ \lambda_1 h_{12} & \lambda_2 h_{22} & \lambda_3 & \lambda_4 h_{43} & \lambda_5 h_{12} & \lambda_6 h_{22} & \lambda_7 & \lambda_8 h_{43} \\ \lambda_1 h_{13} & \lambda_2 h_{23} & \lambda_3 h_{33} & \lambda_4 & \lambda_5 h_{13} & \lambda_6 h_{23} & \lambda_7 h_{33} & \lambda_8 \\ 1 & h_{21} & h_{31} & h_{41} & 1 & h_{21} & h_{31} & h_{41} \\ h_{11} & 1 & h_{32} & h_{42} & h_{11} & 1 & h_{32} & h_{42} \\ h_{12} & h_{22} & 1 & h_{43} & h_{12} & h_{22} & 1 & h_{43} \\ h_{13} & h_{23} & h_{33} & 1 & h_{13} & h_{23} & h_{33} & 1 \end{pmatrix},$$

will diagonalize A, and that its j^{th} column is an eigenvector for the eigenvalue λ_j, $j = 1, \ldots, 8$, of A. Note that H is of block form,

$$H = \begin{pmatrix} H_1 & H_2 \\ H_3 & H_3 \end{pmatrix}, \tag{A.5}$$

that aligns to the block form of A. The matrix H_3 is invertible as H is invertible. The columns of H_1 and H_2 are the columns of H_3 multiplied by an eigenvalue λ_j.

We use H to introduce new coordinates,

$$\begin{pmatrix} X_t \\ Y_t \end{pmatrix} = H \begin{pmatrix} U_t \\ S_t \end{pmatrix}, \tag{A.6}$$

in which (A.3) reads

$$\mathbb{E}_t \begin{pmatrix} U_{t+1} \\ S_{t+1} \end{pmatrix} = \Lambda \begin{pmatrix} U_t \\ S_t \end{pmatrix} + H^{-1} B \begin{pmatrix} Z_t \\ 0 \end{pmatrix} \tag{A.7}$$

with $A = H\Lambda H^{-1}$, $\Lambda = \operatorname{diag}(\lambda_1, \ldots, \lambda_8)$. Let (U_t, S_t) be the unique solution of (A.7). With $H^{-1} = (\hat{h}_{ij})_{i,j=1,\ldots,8}$, we find

$$\begin{aligned} U_{i,t} = -\sum_{k=0}^{\infty} \lambda_i^{-(k+1)} \Bigg\{ & \frac{\hat{h}_{i1}}{\beta\theta_2} \mathbb{E}_t \left[d_{t+k} - (1-\rho) r_{t+k} \right] + \frac{\hat{h}_{i2}}{\beta\phi_2} \mathbb{E}_t \left[r_{t+k} - l_{t+k} \right] + \\ & + \frac{\hat{h}_{i3}}{\beta\kappa_2} \mathbb{E}_t \left[d^{CB}_{t+k} - r_{t+k} \right] + \frac{\hat{h}_{i4}}{\beta\zeta_2} \mathbb{E}_t \left[r_{t+k} - l^{CB}_{t-k} \right] \Bigg\}, \end{aligned} \tag{A.8}$$

see Blanchard and Kahn (1980). Using (A.5) and (A.6), we have

$$X_t = H_1 U_t + H_2 S_t, \quad Y_t = H_3 U_t + H_3 S_t,$$

which gives

$$X_t = (H_1 - H_2)\, U_t + H_2 H_3^{-1} Y_t = (H_1 - H_2)\, U_t + H_2 H_3^{-1} X_{t-1}. \tag{A.9}$$

As the diagonal entries of $H_1 - H_2$ are the differences between the coupled eigenvalues, $\lambda_1 - \lambda_5, \ldots, \lambda_4 - \lambda_8$, we have established the solution (8) in claim (iv) of Theorem 1.

For $\delta = 0$, H_3 is the identity matrix, and H_1 and H_2 are diagonal matrices because A_1 is diagonal in that case. In particular, $H_2 = \mathrm{diag}(\lambda_5, \ldots, \lambda_8)$. In addition, $\hat{h}_{ij} = 0$ if $i \neq j$, and $\hat{h}_{ii} = (\lambda_i - \lambda_{i+4})^{-1}$ for $i, j = 1, \ldots, 4$. With (A.8), the solution (A.9) for $\delta = 0$ is as claimed by (7). This completes the proof of Theorem 1. □

Appendix B. Long-run equilibrium

Lemma 2. *Assume that there is a steady state for system* (4) *and let $\mathbb{E}^*$ denote the expectation with respect to the steady-state probability distribution. Then the steady-state solution of system* (4) *is*

$$\begin{aligned} D^* &= \frac{(1-\rho)\mathbb{E}^*[r_t^f - d_t]}{\theta_1}, \qquad & L^* &= \frac{\mathbb{E}^*[l_t - r_t^f]}{\phi_1}, \\ D^*_{CB} &= \frac{\mathbb{E}^*[r_t^f - d_t^{CB}]}{\kappa_1}, \qquad & L^*_{CB} &= \frac{\mathbb{E}^*[l_t^{CB} - r_t^f]}{\zeta_1} \end{aligned} \tag{B.1}$$

if and only if $\delta = 0$.

Proof. If $\phi_2, \theta_2, \kappa_2, \zeta_2, \delta = 0$, equation (B.1) is directly obtained as expected value of equation (6). For $\phi_2, \theta_2, \kappa_2, \zeta_2 > 0$, we use the notation of Appendix A and equation (A.9). The steady state X^* is the solution of

$$\left(I - H_2 H_3^{-1}\right) X^* = (H_1 - H_2) U^* \tag{B.2}$$

where U^* denotes the forward solution from equation A.8 with expected value operator $\mathbb{E}^*$. For $\delta = 0$, equation B.2 decouples as H_3 is the identity matrix. The first equation gives

$$\begin{aligned} (1-\lambda_5) D^* &= (\lambda_1 - \lambda_5) U_1^* \\ &= (\lambda_1 - \lambda_5) \sum_{k=0}^{\infty} \lambda_1^{-(k+1)} \frac{\hat{h}_{11}(1-\rho)}{\beta\theta_2} \mathbb{E}^* \left[r_{t+k} - d_{t+k}\right] \\ &= \frac{1-\rho}{\beta\theta_2} \mathbb{E}^* \left[r_t - d_t\right] \sum_{k=0}^{\infty} \lambda_1^{-(k+1)} \\ &= \frac{1-\rho}{\beta\theta_2} \mathbb{E}^* \left[r_t - d_t\right] \frac{1}{\lambda_1 - 1} \end{aligned} \tag{B.3}$$

as $\hat{h}_{11} = \lambda_1 - \lambda_5$ for $\delta = 0$ and $\mathbb{E}^*$ is the expectation with respect to the asymptotic probability measure. With Vieta's formula, we find further that, for $\delta = 0$,

$$\lambda_1 + \lambda_5 = \mu_1 = \frac{\theta_1}{\beta\theta_2}, \quad \lambda_1 \lambda_5 = \beta,$$

see also Lemma 1. Plugging this into equation (B.3) yields the expression for D^* as claimed in equation (B.1). Analogous arguments yield the other equations in (B.1).

Appendix C. Estimation results with equity

In this section, we add equity, defined as capital and reserves that include issued equity, profits and losses, and various provisions, as a fifth liability position to our model. In Table C.10, we see strong impact of log(equity)(-1) on other liabilities and assets.

Table C.10: PVAR: Balance Sheet Liabilities with Equity

	log(HH deposits)	log(NFC deposits)	log(liab. securities)	log(CB deposits)	log(equity)
log(HH deposits) (-1)	0.4080***	0.2362***	−0.1011**	0.1079	0.0047
	(0.0361)	(0.0629)	(0.0433)	(0.0753)	(0.0427)
log(NFC deposits) (-1)	0.3430***	0.3627***	0.0807*	0.0553	0.0766**
	(0.0613)	(0.0738)	(0.0418)	(0.0460)	(0.0358)
log(liab. securities) (-1)	0.0800*	0.0187	0.8023***	0.0137	−0.0389
	(0.0476)	(0.0331)	(0.0422)	(0.0598)	(0.0437)
log(CB deposits)(-1)	0.0018	−0.0033	−0.0281***	0.8719***	0.0010
	(0.0111)	(0.0096)	(0.0064)	(0.0230)	(0.0111)
log(equity)(-1)	0.0327	0.1427***	0.0996***	−0.1467**	0.3035***
	(0.0294)	(0.0450)	(0.0344)	(0.0619)	(0.0295)
log(HH Con. Loans) (-1)	0.1823**	0.1383	0.0373	0.1592	0.0812**
	(0.0754)	(0.0879)	(0.0648)	(0.1013)	(0.0342)
log(HH Mortgage loans) (-1)	−0.0014	−0.1794***	0.0429	−0.0010	0.1940***
	(0.0635)	(0.0580)	(0.0436)	(0.0654)	(0.0420)
log(NFC loans) (-1)	0.0430	0.2063***	0.0960*	−0.1054	0.2742***
	(0.0473)	(0.0539)	(0.0531)	(0.0956)	(0.0480)
log(assets securities) (-1)	0.0195	0.1039**	−0.0496	−0.0148	−0.0073
	(0.0549)	(0.0464)	(0.0655)	(0.0644)	(0.0463)
log(CB assets) (-1)	−0.0038	−0.0212	0.0443	−0.0363	0.0125
	(0.0350)	(0.0295)	(0.0283)	(0.0283)	(0.0276)
DR NFC	−0.1762***	−0.1033*	0.1693***	0.1067	0.0549
	(0.0427)	(0.0542)	(0.0539)	(0.0790)	(0.0402)
DR HH	−0.2970***	0.0593	0.0050	0.1577**	0.0323
	(0.0537)	(0.0573)	(0.0711)	(0.0628)	(0.0364)
LR Consumption HH	0.0430	−0.0150	−0.0894***	−0.0051	0.0327
	(0.0299)	(0.0243)	(0.0238)	(0.0224)	(0.0293)
LR Mortgage HH	0.0347	−0.0354	0.0188	−0.0061	0.0302
	(0.0383)	(0.0457)	(0.0370)	(0.0598)	(0.0321)
LR NFC	0.0696*	−0.0039	−0.1016***	−0.0398	0.0584
	(0.0389)	(0.0507)	(0.0278)	(0.0584)	(0.0413)
CB refinancing rate	−0.0158	−0.1090*	0.1617***	−0.3521***	0.0776**
	(0.0651)	(0.0565)	(0.0362)	(0.0702)	(0.0316)
Euribor	0.0043	−0.0127	0.0747**	−0.0765	−0.0427*
	(0.0339)	(0.0375)	(0.0346)	(0.0732)	(0.0242)
2-year gov bond yield	0.0838*	0.1211***	−0.0682**	0.1272**	−0.1458***
	(0.0478)	(0.0360)	(0.0299)	(0.0527)	(0.0359)
10-year gov bond yield	0.0319	−0.0550*	0.0204	0.0477	−0.0730**
	(0.0378)	(0.0317)	(0.0243)	(0.0473)	(0.0344)
Dummy neg. Euribor	−0.1063***	−0.1337***	0.0617*	0.0994	0.0689***
	(0.0328)	(0.0377)	(0.0363)	(0.0854)	(0.0194)
Dummy neg. Euribor x Euribor	0.0158	−0.0152	0.0663***	−0.2162***	0.0701***
	(0.0333)	(0.0292)	(0.0176)	(0.0399)	(0.0153)
PSPP CSPP PEPP	0.0038	0.0188	0.0836**	0.0395	0.0994***
	(0.0460)	(0.0564)	(0.0382)	(0.0686)	(0.0261)
const	0.0055	0.0065	−0.0105**	−0.0016	0.0171***
	(0.0046)	(0.0073)	(0.0048)	(0.0064)	(0.0033)
Number of Observations	11,201	11,201	11,201	11,201	11,201
Number of Groups	136	136	136	136	136
Obs per group: min	4	4	4	4	4
avg	82.40	82.40	82.40	82.40	82.40
max	148	148	148	148	148
Hansen test of overid: statistics:	91.82	91.82	91.82	91.82	91.82
nof para:	900	900	900	900	900
p-value:	1.00	1.00	1.00	1.00	1.00
RMSE	0.09	0.06	0.10	1.25	0.04

Source. IMIR. IBSI. Own calculations.

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$. We apply the two-step system GMM estimator by Blundell and Bond (1998) with Windmeijer corrected standard errors (Windmeijer, 2005). All models are estimated by Sigmund and Ferstl (2021).

The dependent variable is log(HH deposits) in the first column. Log(HH deposits) refers to the logarithm of household deposits. The dependent variable is log(NFC deposits) in the second column. Log(NFC deposits) refers to the logarithm of non-financial corporate deposits. The dependent variable is log(CB deposits) in the third column. Log(CB deposits) refers to logarithm of central bank deposits. The dependent variable is log(liab. securities) in the fourth column. Log(liab. securities) refers to logarithm of securities on the liability side. The dependent variable is log(equity) in the fifth column. Log(equity) refers to logarithm of capital and reserves that includes issued equity, profits and losses and various provisions.

DR HH refers to the refers to the average deposit rate of households. DR NFC refers to the average deposit rate of non-financial corporations. 2-year gov bond yield refers to the two-year zero coupon government bond yield and should approximate the bond yield of securities on the liability side. CB refinancing rate refers to the most favorable interest rate paid on TLTROs.

LR Consumption HH refers to the refers to the average lending rate of household consumption loans. LR Mortgage HH refers to the average lending rate of mortgage loans. LR NFC refers to the average lending rate of non-financial corporations. 10-year gov bond yield refers to the ten-year zero coupon government bond yield and should approximate the bond yield of securities on the asset side. ECB deposit facility defines the interest banks receive for depositing money with the CB overnight.

Table C.11: PVAR: Balance Sheet Assets with Equity

	log(HH Con. Loans)	log(HH Mortgage loans)	log(NFC loans)	log(assets securities)	log(CB assets)
log(HH deposits) (-1)	0.0157	−0.0267	0.2256***	0.0114	−0.0365
	(0.0435)	(0.0419)	(0.0429)	(0.0300)	(0.0829)
log(NFC deposits) (-1)	0.0230	0.0325	0.1929***	0.0176	0.0446
	(0.0389)	(0.0339)	(0.0345)	(0.0312)	(0.0558)
log(liab. securities) (-1)	0.0129	0.0264	−0.1470***	−0.1352***	0.0290
	(0.0498)	(0.0398)	(0.0521)	(0.0388)	(0.1001)
log(CB deposits)(-1)	−0.0141	−0.0170	0.0065	0.0114	−0.0246
	(0.0103)	(0.0172)	(0.0129)	(0.0119)	(0.0301)
log(equity)(-1)	0.2285***	0.2608***	0.2207***	0.0859***	−0.0055
	(0.0247)	(0.0313)	(0.0350)	(0.0194)	(0.0558)
log(HH Con. Loans) (-1)	0.2986***	0.2911***	0.1107***	0.2395***	0.0359
	(0.0443)	(0.0428)	(0.0429)	(0.0434)	(0.0708)
log(HH Mortgage loans) (-1)	0.3523***	0.3884***	0.0644	0.0521	0.0894
	(0.0549)	(0.0434)	(0.0499)	(0.0495)	(0.1010)
log(NFC loans) (-1)	0.0251	0.0803*	0.2704***	0.0230	0.0439
	(0.0459)	(0.0455)	(0.0461)	(0.0345)	(0.0740)
log(assets securities) (-1)	0.0589	0.0428	0.0637	0.5696***	0.0953
	(0.0475)	(0.0640)	(0.0605)	(0.0335)	(0.0624)
log(CB assets) (-1)	−0.0274	0.0377	0.0109	0.0913***	0.6227***
	(0.0324)	(0.0371)	(0.0352)	(0.0176)	(0.0667)
DR NFC	0.0241	−0.0970***	0.0087	0.1948***	0.1977**
	(0.0399)	(0.0361)	(0.0519)	(0.0321)	(0.0899)
DR HH	0.2065***	−0.1035**	0.0392	0.0718	0.0718
	(0.0394)	(0.0488)	(0.0591)	(0.0524)	(0.1386)
LR Consumption HH	0.0101	−0.0081	−0.1204***	−0.0051	0.0592
	(0.0361)	(0.0520)	(0.0377)	(0.0257)	(0.0720)
LR Mortgage HH	−0.1471***	−0.0165	0.0953**	0.1280***	−0.0449
	(0.0373)	(0.0379)	(0.0388)	(0.0297)	(0.1298)
LR NFC	−0.1365***	−0.0945***	0.0796***	0.0613**	−0.1543
	(0.0280)	(0.0287)	(0.0237)	(0.0242)	(0.1052)
CB refinancing rate	0.0390	0.1361***	0.0116	0.0973***	−0.2678***
	(0.0321)	(0.0348)	(0.0424)	(0.0331)	(0.0754)
Euribor	0.0884***	−0.0129	0.0366*	0.0258	0.0205
	(0.0303)	(0.0297)	(0.0213)	(0.0214)	(0.0759)
2-year gov bond yield	0.0774**	−0.0419	0.0345	−0.0937***	0.0258
	(0.0387)	(0.0294)	(0.0462)	(0.0297)	(0.0461)
10-year gov bond yield	−0.0106	0.1218***	−0.0483	−0.1197***	−0.1212*
	(0.0343)	(0.0328)	(0.0354)	(0.0320)	(0.0651)
Dummy neg. Euribor	−0.0436	0.0809***	−0.0830***	0.0469**	0.1245**
	(0.0307)	(0.0249)	(0.0198)	(0.0204)	(0.0525)
Dummy neg. Euribor x Euribor	−0.0135	0.0680***	0.0137	0.0282*	−0.1634***
	(0.0198)	(0.0197)	(0.0222)	(0.0168)	(0.0256)
PSPP CSPP PEPP	−0.0764***	−0.0082	0.0051	−0.0288**	0.1584**
	(0.0193)	(0.0181)	(0.0351)	(0.0144)	(0.0617)
const	0.0086***	0.0060*	0.0207***	0.0108***	0.0001
	(0.0031)	(0.0037)	(0.0037)	(0.0036)	(0.0088)
Number of Observations	11,201	11,201	11,201	11,201	11,201
Number of Groups	136	136	136	136	136
Obs per group: min	4	4	4	4	4
avg	82.40	82.40	82.40	82.40	82.40
max	148	148	148	148	148
Hansen test of overid: statistics:	91.82	91.82	91.82	91.82	91.82
nof para:	900	900	900	900	900
p-value:	1.00	1.00	1.00	1.00	1.00
RMSE	0.09	0.09	0.09	0.12	1.00

Source. IMIR. IBSI. Own calculations.
*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$. We apply the two-step system GMM estimator by Blundell and Bond (1998) with Windmeijer corrected standard errors (Windmeijer, 2005). All models are estimated by Sigmund and Ferstl (2021).
The dependent variable is log(HH Con. Loans) in the first column. Log(HH Con. Loans) refers to the logarithm of consumption loans.
The dependent variable is log(HH Mortgage loans) in the second column. Log(HH Mortgage loans) refers to the logarithm of household mortgage loans.
The dependent variable is log(NFC loans) in the third column. Log(NFC loans) refers to logarithm of non-financial corporate loans.
The dependent variable is log(CB assets) in the fourth column. Log(CB assets) refers to logarithm of central bank assets.
The dependent variable is log(assets securities) in the fourth column. Log(assets securities) refers to logarithm of securities on the asset side.
For each dependent variable on the liability side, we include the corresponding interest rate: DR HH refers to the refers to the average deposit rate of households. DR NFC refers to the average deposit rate of non-financial corporations. 2-year gov bond yield refers to the two-year zero coupon government bond yield and should approximate the bond yield of securities on the liability side. CB refinancing rate refers to the most favorable interest rate paid on TLTROs.
For each dependent variable on the asset side, we include the corresponding interest rate: LR Consumption HH refers to the refers to the average lending rate of household consumption loans. LR Mortgage HH refers to the average lending rate of mortgage loans. LR NFC refers to the average lending rate of non-financial corporations. 10-year gov bond yield refers to the ten-year zero coupon government bond yield and should approximate the bond yield of securities on the asset side. ECB deposit facility defines the interest banks receive for depositing money with the central bank overnight.

Index of Working Papers:

July 1, 2022	Marcel Barmeier	242	The new normal: bank lending and negative interest rates in Austria
July 14, 2022	Pavel Ciaian, Andrej Cupak, Pirmin Fessler, d'Artis Kancs	243	Environmental-Social-Governance Preferences and Investments in Crypto-Assets
October 18, 2022	Burkhard Raunig, Michael Sigmund	244	The ECB Single Supervisory Mechanism: Effects on Bank Performance and Capital Requirements
April 5, 2023	Norbert Ernst, Michael Sigmund	245	Are zombie firms really contagious?
May 8, 2023	Richard Sellner, Nico Pintar, Norbert Ernst	246	Resource Misallocation and TFP Gap Development in Austria
September 5, 2023	Katharina Allinger, Fabio Rumler	247	Inflation Expectations in CESEE: The Role of Sentiment and Experiences
October 16, 2023	Pietro Saggese, Esther Segalla, Michael Sigmund, Burkhard Raunig, Felix Zangerl, Bernhard Haslhofer	248	Assessing the Solvency of Virtual Asset Service Providers: Are Current Standards Sufficient?
October 20, 2023	Pirmin Fessler, Severin Rapp	249	The subjective wealth distribution: How it arises and why it matters to inform policy?
October 27, 2023	Burkhard Raunig, Michael Sigmund	250	Watching over 21,000 Billion Euros: Does the ECB Single Supervisory Mechanism Affect Bank Competition in the Euro Area?
December 5, 2023	Markus Knell	251	Housing and the secular decline in real interest rates
December 14, 2023	Niko Hauzenberger, Florian Huber, Thomas O. Zörner	252	Hawks vs. Doves: ECB's Monetary Policy in Light of the Fed's Policy Stance
February 28, 2024	Lukas Olbrich, Elisabeth Beckmann, Joseph W. Sakshaug	253	Multivariate assessment of interviewer-related errors in a cross-national economic survey