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FORECASTING AUSTRIAN GDP USING THE
GENERALIZED DYNAMIC FACTOR MODEL

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Editorial

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Forecasting Austrian GDP using the generalized dynamic factor model

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September 17, 2004

Abstract

In this paper, a generalized dynamic factor model is utilized to produce short-term forecasts of real Austrian GDP. The model follows the frequency domain approach proposed by Forni, Hallin, Lippi and Reichlin (2000, 2003). The forecasting performance of the model with a large data set of 143 variables has been assessed relative to simple univariate time-series forecasts. The results show that the factor model can barely outperform the much simpler benchmark model, given the usuall levels of significance. Thus we followed a line of research proposed by Boivin and Ng (2003) and Watson (2000), who suggested that the use of a small data set may increase the forecasting performance. The main finding from our extensive out-of-sample forecasting experiment that we have conducted is that the best forecasting performance can be achieved with small data sets with a handful of variables only. These models perform significantly better than the large model. This result seems to contradict the basic idea of dynamic factor models, which have been constructed to exploit the potentially useful information of a large data set.

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1 Introduction

The need for producing accurate forecasts of key macroeconomic variables has been a strong driving force for empirical research. Much effort has been put in the development of various kinds of forecasting models utilizing the availability of monthly or quarterly data. Besides informal methods, which are widely used in most forecasting institutions, short term economic forecasts are often based on the results of small-scale time-series models. The family of time series models ranges from univariate versions as proposed by Kalman (1960) or Box and Jenkins (1976) to multivariate VARs and cointegrated systems.

Parallel to the theoretical progress in time series modelling researchers face a substantial increase in the amount and quality of economic data available. However, instead of making predictions easier, the researcher is immediately confronted with identification problems if he wants to exploit the potential benefits of a larger data set with a limited amount of observations. Although economic theory helps and the statistical literature suggests ways to deal with that problem, the researcher is left with a certain degree of arbitrariness.

The need for novel methods to deal with large data sets has concentrated research efforts on the field of factor models. The seminal contribution for static factor models came from Burns and Mitchell (1946). A desirable improvement of the static factor model methodology was introduced by Sargent and Sims (1977) and Geweke (1977) by the generalization to the dynamic case exploiting the dynamic interrelationship of the variables and by reducing the number of common factors even further. Recently, Forni and Reichlin (1998), Forni, Hallin, Lippi, and Reichlin (2000) and Forni and Lippi (1999) have introduced the generalized dynamic factor model allowing for a limited amount of cross correlation among the idiosyncratic components and proposed this method for exploiting the potentially useful information in large panels of time series.

Generally, factor models offer a tool to summarize the information available in a large data set by a small number of factors. These factors are weighted linear combinations of all variables in the data set. The basic idea that stands behind the factor model is that the movement of a time series can be characterized as the sum of two mutually orthogonal components: The common component which should explain the main part of the variance of the time series and is a linear combination of the common factors. The second component, the idiosyncratic component, contains the remaining variable specific information and is only weakly correlated across the panel. Obviously, neither the common nor the idiosyncratic component can be observed directly and have to be estimated. Commonly used estimation procedures are principal components methods (Stock and Watson, 1998), state space models (Harvey (1989), Stock and Watson (1998)) and cointegration frameworks (Gonzalo and Granger (1995)). One recently developed approach, utilized in this paper, is the dynamic approach using frequency domain analysis as proposed by Forni and Reichlin (1998) and Forni, Hallin, Lippi, and Reichlin (2000). During the last years there are a growing number of forecasts of macroeconomic variables which rely on dynamic factor models (see e.g., Stock and Watson (1998), Gosselin and Tkacz (2001), Artis, Banerjee, and Marcellino (2002)).

The currently available empirical literature suggests that factor-based forecasts usually outperform simpler time-series methods such as univariate models and VARs, although the benefits are not always statistically significant (Artis, Banerjee, and Marcellino (2002), Dreger and Schumacher (2002)). An important empirical question which has not been answered thoroughly until yet now concerns the size and the composition of the optimal data set. Although factor models have been developed to deal with large panels with hundreds

of variables, there are some results which indicate that increasing the number of variables over a certain size do not improve or even worsen forecasting results (Boivin and Ng (2003), Watson (2000)).

The main purpose of this paper is to develop a framework for short-term forecasting of real Austrian GDP. It utilizes the generalized dynamic factor model proposed by Forni, Hallin, Lippi and Reichlin (referred to as FHLR) (2000, 2001a, 2001b, 2001c) and embeds it into the environment needed to conduct short-term forecasting. The paper is organized as follows. Section 2 gives a brief description of the generalized dynamic factor model. Details of the model can be found in appendix A. The data set is described in section 3. This section also presents the method to deal with the problem of different frequencies and timeliness of the data needed for forecasting. In section 4 the forecasting performance of the factor model is assessed. The first subsection gives an overview of the out-of-sample forecasting exercise. The next subsection presents the performance of the model with the full data set, whereas in the last subsection we propose a method to construct sub sets of the data and assess their forecasting performance. Section 5 draws some conclusions and puts forward directions for future improvements.

2 The dynamic factor model

The model utilized in this paper is the so-called generalized dynamic factor model proposed by Forni, Hallin, Lippi and Reichlin ((2000), (2001), (2003)), henceforth FHLR. The representation theory of the dynamic factor model can be found in Forni and Lippi (1999). This approach has been developed to deal with large panels of time series, i.e. when the number of variables becomes large compared to the number of observations. Each time series is represented as the sum of two components: the common component and the idiosyncratic component. The common component of the time series is driven by a few underlying uncorrelated and unobservable common factors. The estimated factors can be derived by applying a linear (time-invariant) filter to the data set (possibly with lags). The generalized dynamic factor model exploits the dynamic covariance structure of the data, i.e. the relation between different variables at different points in time. This makes an important difference to the forecasting model proposed by Stock and Watson (2002). Their forecast is based on a projection onto the space spanned by the static principal components of the data. Thus, being based on contemporaneous covariances only, their approach fails to exploit the dynamic relations between the variables of the panel. FHLR (2003) worked out the theoretical advantage of the dynamic approach compared to the static one.

In traditional factor analysis ((Sargent and Sims 1977) and (Geweke 1977)), it is assumed that there is no cross-correlation among the idiosyncratic components at any lead and lag. This assumptions allows for identification of common and idiosyncratic components but represents a strong restriction. The following two examples illustrate that this assumption could represent a serious weakness of traditional factor models. First, consider the output of two different industries. Each sectoral output consists of a common and an idiosyncratic component. Now suppose that the industries are linked by input-output relations (possibly with a lag). An idiosyncratic shock in industry B will therefore propagate to industry A. The resulting correlation violates the assumption of uncorrelated idiosyncratic shocks. A similar situation arises for shocks which are neither strictly common nor strictly idiosyncratic. This might be the case for productivity shocks affecting only a subset of industries. A second example is given by a regional data set including data for different regional aggregates (e.g.

employment and income). In the aggregate each variable is driven by a national and a regional component which is orthogonal to the first. However the two variables are likely to be correlated for the same region, again violating the above assumption ((Forni and Lippi 1999))¹.

The FHLR approach allows for both contemporaneous and lagged correlation between the idiosyncratic terms and has been increasingly used for business cycle analysis and forecasting (e.g. 'EuroCoin', Altissimo et al., 2001 or Cristadoro et al., 2001).

2.1 The model

Our panel consists of i time series, which are assumed to be a realization of a zero mean, wide-sense stationary process $\{x_{it}; t \in \mathbb{Z}\}$. Stationarity can be achieved by suitable transformations of the raw data. Each process of the panel is thought of as an element from an infinite sequence, indexed by $i \in \mathbb{N}$. All processes are co-stationary, i.e. stationarity holds for any of the n-dimensional vector processes $\{x_{nt} = (x_{1t} \dots x_{nt})'; t \in \mathbb{Z}, n \in \mathbb{N}\}$.

In the dynamic factor model, each variable x_{it} of the panel is decomposed into two components

$$x_{it} = \chi_{it} + \xi_{it} = b_i(L)u_t + \xi_{it} = \sum_{i=1}^{q} b_{ij}(L)u_{jt} + \xi_{it},$$
(1)

where χ_{it} is called the common component and ξ_{it} the idiosyncratic component. $b_i(L) = b_{i1}(L), \ldots, b_{iq}(L)$ is a vector of lag polynomials and $u_t = (u_{1t}, \ldots, u_{qt})'$ is a q-dimensional vector of common shocks. The q-dimensional process $\{(u_{i1}, \ldots, u_{it}); t \in \mathbb{Z}\}$ is assumed to be mutually orthonormal white noise with unit variance. ξ_{it} is orthogonal to u_{t-k} for any k and i. The infinite cross-section and two main assumptions are crucial for identification of the model. The first assumption allows for a limited amount of cross-correlation between idiosyncratic components and ensures that the variance explained by the idiosyncratic component vanishes as $N \to \infty$. The second assumption assures a minimum amount of correlation between the common components. A more accurate definition of the

generalized dynamic factor model (including all assumptions) can be found in appendix A.

2.2 Forecasting the common component

For further details see FHLR (2000).

The common and the idiosyncratic component of a variable are mutually orthogonal. Thus, forecasting a variable in a dynamic factor model can be split into two separate forecasting problems, forecasting the common component and forecasting the idiosyncratic component. Since the idiosyncratic components are mutually orthogonal or only weakly correlated, they can be forecast easily using standard univariate or low-dimensional multivariate methods like ARIMA of VAR models.

The h-step ahead forecast of the common component amounts to finding the best linear predictor for $\chi_{i,T+h}$ which is the projection onto the space spanned by the common components obtained by a linear filter of the data matrix

$$\phi_{i,T+h|T} \equiv \operatorname{proj}(\chi_{i,T+h}|\mathcal{G}(\chi,T)) = \sum_{j=1}^{q} \sum_{k=h}^{\infty} b_{ij,k} u_{jT+h-k}.$$
 (2)

¹See also Forni, Hallin, Lippi, and Reichlin (2003) for an illuminating stylised example on the differences between the static and the dynamic factor approach.

However the coefficients $b_{ij,k}$ rely on a two sided filter and are therefore not appropriate for forecasting purposes. Instead one has to estimate the span of $\mathcal{G}(\chi,T)$ by a one-sided filter of the data matrix, say $W_{nt}^{kT} \equiv Z_{nk}^T x_{nt}$. Forni, Hallin, Lippi, and Reichlin (2003) show that the weights Z_{nk}^T can be obtained as the solution of the following generalized eigenvalue problem:

$$Z_{nl}^{T} := \arg \max_{\mathbf{a} \in \mathbb{R}^{n}} \operatorname{var} \left(\mathbf{a} \chi_{nt}^{T} \right)$$
subject to
$$\operatorname{var} \left(\mathbf{a} \xi_{nt}^{T} \right) = 1$$

$$\mathbf{a} \xi_{nt}^{T} \perp \mathbf{Z}_{nm}^{T} \xi_{nt}^{T} \quad \text{for} \quad 1 \leq m < \ell, \quad 1 \leq \ell \leq n.$$

$$(3)$$

a denotes the eigenvalues resulting from the solution of the generalized eigenvalue problem and $\Gamma_{n0}^{\chi T}$ and $\Gamma_{n0}^{\xi T}$ denote the contemporaneous variance-covariance matrices of the common and the idiosyncratic components, respectively. The intuition behind this approach is that the solution of the generalized eigenvalue problem gives us weights Z_{nl}^T that maximize the ratio between the variance of the common and the idiosyncratic component in the resulting aggregates. In other words, the two variance-covariance matrices can help to construct averages of the data matrix which put a larger weight on variables that have a larger 'commonality' (Forni, Hallin, Lippi, and Reichlin (2003)). The proposed projection matrix is then

$$\phi_{i,T+h|T}^{n,T} = \Gamma_{nh}^{\chi T} Z_n^T (\tilde{Z}_n^T \Gamma_{n0}^{\chi T} Z_n^T)^{-1} \tilde{Z}_n^T x_{nT}$$
(4)

where Γ_{n0}^{xT} denotes the contemporaneous variance-covariance matrices of the data matrix. As the sample size increases, the estimate $\phi_{i,T+h|T}^{n,T}$ converges in probability to χ_{it} .².

2.3 Estimating the variance covariance matrices of the common and the idiosyncratic component

Since the covariance matrices $\Gamma_{n0}^{\chi T}$ and $\Gamma_{n0}^{\xi T}$ in the generalized eigenvalue problem in equation (3) are not known, they have to be estimated in advance in a separate step utilizing dynamic principal component analysis (Forni and Lippi (1999)). This approach is based on the spectral density matrices of the data $\Sigma(\theta)$, which are decomposed into common and idiosyncratic components by a dynamic principal component decomposition for each frequency θ . Applying the inverse Fourier transformation to the matrices of eigenvectors gives weights, $b_{ij,k}$, for the two-sided filter above. With these weights the common and the idiosyncratic components of each variable can be calculated and this gives the variance-covariance matrices $\Gamma_{n0}^{\chi T}$ and $\Gamma_{n0}^{\xi T}$ needed in the eigenvalue problem (3). For a detailed illustration see appendix A.

2.4 Selecting the number of common factors

In addition to the determination of the size of the lag window for the Fourier transformation (M) and the number of leads and lags, the number of common factors q has to be chosen.

²Computing a 0-step ahead forecast gives the in-sample estimator of χ_{it}^{nT}

Table 1: Percentage total variance explained by the first q common factors

\overline{q}	Cumulative % variance explained	Contribution to explained variance
1	0.30	0.30
2	0.48	0.18
3	0.58	0.10
4	0.64	0.06
5	0.69	0.05
6	0.73	0.04
7	0.77	0.04
8	0.80	0.03
9	0.83	0.03
10	0.85	0.02

Several strategies are available for determining the 'proper' q. First, one can use information criteria such as Akaike's information criterion or the modification proposed by Bai and Ng (2002). Second, a scree plot is often used in principal components analysis. The eigenvalues for each common component are plotted in descending order. Such a graphic usually is elbow-shaped. The optimal number of common components is determined by the last point before the kink. In frequency domain analysis, this approach has to be extended to the different frequencies of the spectral density function. This approach has the disadvantage that there it has no formal basis and that the number of common components usually varies over the frequencies (Mansour (2003)). Forni and Lippi (1999) proposed a method based on a heuristic inspection of the eigenvalues against the number of series n. If T observations are available for a n variables x_{it} , the spectral density matrices σ_r^T , $r \leq n$ can be estimated and the resulting empirical dynamic eigenvalues λ_{rj}^T can be computed for a grid of frequencies. Two features of the computed eigenvalues are then considered to determine the number of common factors:

- 1. The average over frequencies θ of the first q eigenvalues diverges, whereas the average of the (q+1)-th eigenvalue remains relatively stable.
- 2. At r = n there should be a substantial gap between the variance explained by the q-th principal component and the variance explained by the q + 1-th one.

We have calculated the eigenvalues and the explained variance for our data set (see section 3). As figure 1 shows, the first three eigenvalues diverge most probably, whereas the following 140 remain bounded. These first three eigenvalues explain 58 % of the total variance. The fourth common factor would add only 6 percentage points to the total variance explained (see table 1).

Another way to determine the number of common factors q is to utilize the out-of-sample forecasting performance of the model. This approach has been used in this paper.

3 The data set

The data set includes 105 variables of monthly or quarterly frequency. Some variables have been included in the model in levels as well as in differences. So the total number of series

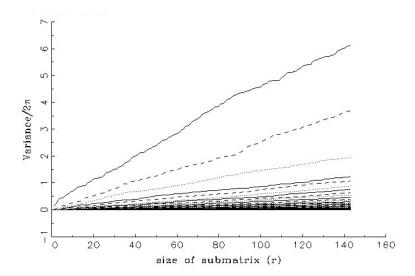


Figure 1: The FHLR identification criterion (mean eigenvalues of the spectral density matrices over frequencies)

included in the factor model is 143. The quarterly data set ranges from 1988Q1 until 2003Q2, i.e. it contains 62 observations. The variables in the list below can be attributed to different categories. Numbers and letters in parenthesis refer to the number of variables per category and their frequency (monthly or quarterly). The detailed list of variables can be found in annex B.

- National account data (15, Q): Real GDP and its components and deflators
- WIFO quarterly survey (8, Q): Quarterly survey of the Austrian Institute of Economic Research
- Monthly survey data (18, M): Economic sentiment indicator of the European Commission including sub indices, Ifo index for Germany and purchasing manager index for the US
- Prices (30, M): Consumer prices, producer prices, oil price, exchange rates
- Foreign trade (14, M): Real exports and imports
- Labour market (6, M): Unemployment, employment, vacancies
- Financial variables (13, M): Money aggregates, interest rates, stock prices, credits
- Miscellaneous (1, M): Industrial production

The following steps have been undertaken to prepare the data. First, the data have been seasonally adjusted using Tramo/Seats. Second, if not already stationary, the variables have been differenced by taking either first differences or calculating the percentage change.

Third, outliers have been removed from the data set. This has been done with a semiautomatic procedure. For every data point, a moving average has been calculated. If the difference between the point and the moving average exceeds a predetermined threshold defined in units of the standard deviation of the series, then the point is treated as an outlier and replaced by an interpolation between its adjacent points. The threshold has been defined for each variable separately by visual inspection. Finally, the variables have been standardized to obtain zero mean and unit standard deviation.

One important problem in real-time forecasting is the different timeliness of the variables. This problem is especially important in factor model applications, because these models require a complete panel without missing data. Our approach to deal with that problem is to shift incomplete series in the data matrix so that the latest available observation for each series can now be obtained at time T. For example, if the variable x is available until period T-1 only, it will be shifted such that the last observation is now in the row for period T. Finally, all periods with missing observations at the beginning of the sample are deleted. Consequently, the final data set now contains 59 observations only. After performing the forecast the shifting is then to be reversed in order to re-establish the original ordering of the data.

A second important problem is that not all variables of the data set are available at the same frequency. We aggregate monthly series to a quarterly frequency. Missing monthly observations within the last quarter are forecast by a monthly factor model. These monthly series are aggregated to quarters and are then concatenated to the quarterly data to build the final data set.

4 Forecasting Austrian GDP

4.1 Overview

This section presents the performance of the dynamic factor model in forecasting Austrian real GDP³. Since the results of Boivin and Ng (2003) and Watson (2000) suggest that a factor model with a small data set might outperform a model with a large data set, we have conducted an extensive out-of-sample forecasting exercise⁴ to shed some light on this topic. This exercise focuses on two main questions. First, which model performs best in forecasting real GDP? Second, how does this optimal model perform relative to a simple univariate time-series model (ARIMA) and to the dynamic factor model with the full data set?

The design of the simulations to answer these two questions is as follows. We have evaluated the forecasting performance of many different models. These models were obtained by varying the size of an ordered data set (see section 4.3 for details on the ordering) and the number of common factors. The forecasting performance for each model was obtained by performing out-of-sample forecasts for 30 rolling windows. The first window contains data from 1988Q1 until 2003Q2. The last three observations were omitted for estimation and were used for evaluating the out-of-sample forecasts. After computing one to three-steps-ahead forecasts, the last observation of the data set was truncated, the model was reestimated and

³The model has been implemented in GAUSS.

⁴The forecasting exercise is not an out-of-sample exercise in a strict sense. Although the forecasts for a given model are based on in-sample information only, out-of-sample information is used to select the best model.

new forecasts were computed. This procedure was repeated for all remaining windows. This gives us one vector with out-of-sample forecasts per forecasting horizon.

4.2 Assessing the forecasting performance with the full data set

In this section the forecasting performance of the factor model with the full data set is assessed. This is done by comparing the performance of the factor model with the forecasts produced by an ARIMA model. The forecast accuracy is measured by the root mean squared error (RMSE), the mean absolute error (MAE) and the Pesaran-Timmerman test. This test assesses the directional accuracy of a forecast. It has been applied to the first differences of the growth rates of GDP, i.e. it tests whether an acceleration or deceleration of growth is correctly predicted. The Diebold and Mariano (1995) test and the Wilcoxon signed rank test have been used to test for equal forecasting accuracy of the two rivalling forecasts.⁵ Finally, the test for multiple forecast encompassing proposed by Harvey, Leybourne and Newbold (1998) has been used to test whether the factor model forecast encompasses the forecast of the ARIMA model. Detailed explanations of these tests can be found in the appendix. The number of AR and MA terms and the use of a constant of the ARIMA model are determined by the minimum RMSE. Table 2 summarizes the results.

The most important finding is that the only forecasting horizon where the forecasting performance of the factor model is significantly better than of the ARIMA forecast is one quarter ahead. In this case, the RMSE of the factor model forecast is by 12.4% smaller than that of the ARIMA forecast. According to the Diebold-Mariano test, the gains are significant at the 10% level. For forecasting horizons two and three the factor model is not able to outperform the ARIMA forecast significantly. For two-steps-ahead the factor model even performs worse than the ARIMA forecast in terms of the MAE.

4.3 Selecting an optimal sub set of the data set

Factor models have been developed to deal with panels with a large number of variables. This may tempt the researcher to use as many series as possible. From a theoretical point of view, a large number of variables is convenient since in theory population results hold for an infinite cross-sectional dimension. Efficient estimates of the common and the idiosyncratic components are obtained asymptotically as the number of variables goes to infinity. However, Boivin and Ng (2003) have put forward some arguments that support the hypothesis that a carefully chosen subset of variables outperforms the full sample forecasts. Since each sample is always a subset of the potentially available variables, there is no guarantee that an arbitrary sample captures correctly the main factors that drive the variable of interest (in our case GDP).

To illustrate the problem suppose that the whole dataset is entirely driven by two common factors, u_1 and u_2 , whereas the variable of interest depends on u_1 only. If the sample includes many variables that are dominated by u_2 , then the space spanned by this sample is dominated by u_2 rather than u_1 . Estimating and forecasting the common component of the variable of interest by one factor selected by one of the conventional selection criteria will thus produce poor results. Boivin and Ng (2003) have entitled this situation where many variables are included in the dataset which have no predictive power for the variable of interest as 'oversampling'.

⁵For a collection of thoughts about the conceptual caveats of this type of test see Kunst (2003).

Table 2: Forecasting performance of the factor models and the ARIMA model

	h = 1	h = 2	h = 3	Average
Factor Model (full data set with $n = 143$	3) ^a			
number of dynamic factors	q = 2	q = 1	q = 1	-
RMSE	0.886	1.056	1.042	1.008
MAE	0.737	0.871	0.846	0.819
Pesaran-Timmerman	**0.037	*0.080	*0.096	-
ARMA				
Specification: ARMA(1,1) with constant	t			
RMSE	1.011	1.056	1.070	1.046
MAE	0.809	0.824	0.855	0.829
Pesaran-Timmerman	*0.097	*0.099	0.122	-
Factor Model (full data set) compared to A	RIMA Model			
RMSE % gain	12.4	0.0	2.6	3.6
MAE % gain	8.9	-5.7	1.1	1.2
Diebold-Mariano RMSE	*0.088	0.496	0.122	-
Diebold-Mariano MAE	0.177	-	0.376	-
Wilcoxon RMSE	0.191	0.521	0.279	-
Wilcoxon MAE	0.272	-	0.463	-
Model encompassing b	0.283	0.224	0.347	-

 $[^]a{\rm Reported}$ values are probabilities with the exception of RMSE, MAE, RMSE gain and MAE gain. $^b{\rm Harvey}$ et. al. Model encompassing test

Table 3: Cross correlation of variables at different leads with respect to GDP

Variable	Transformation a	Lead 1	Lead 2	Lead 3	Average
Number of vacancies	%d	0.62	0.46	0.25	0.44
Unemployment to vacancies ratio	%d	-0.58	-0.45	-0.24	-0.42
Construction sentiment indicator	d	0.46	0.27	0.27	0.33
Dow Jones Index	%d	0.37	0.31	0.18	0.29
Exports of machinery and vehicles	%d	0.33	0.41	0.11	0.29
Total exports	%d	0.26	0.40	0.15	0.27
DAX	%d	0.44	0.28	0.09	0.27
IFO current situation Western Germany	d	0.34	0.28	0.19	0.27
Nominal-effective exchange rate	%d	0.37	0.25	0.17	0.27
IFO business climate index Western Germany	d	0.34	0.28	0.11	0.25

 $[^]a {\rm d}$. . . difference, $\% {\rm d}$. . . percentage difference, l . . . level

In addition to oversampling, Boivin and Ng (2003) have identified two additional situations, where too many series may worsen forecasting results. The first situation refers to the dispersion of the importance of the common component. The common factors can be estimated more precisely when the common component is important relative to the idiosyncratic component. Conversely, adding data with high idiosyncratic errors reduces the precision of the estimates. The second situation is that cross correlation in the errors seem to have a negative impact on the forecasting performance. Although the tested model allows for a limited amount of cross-correlation among the idiosyncratic components, it appears that a higher amount of cross-correlation leads to less efficient estimates and forecasts of the common components. Boivin and Ng (2003) have obtained these results with a Stock-Watson type factor model within a Monte Carlo setting with simulated data. In the Stock-Watson approach, the common shocks are derived by a static principal component analysis. The forecasts are then obtained from a forecasting equation, which regresses GDP on its own history plus the common factors (='factor-augmented forecast'). They have suggested some rules to drop, weight or to group the series for the extraction of common factors to overcome this problem, whereby these rules are rather 'ad-hoc' rules than derived from theory. Their main result is that a careful pre-selection of variables based on their rules improves the forecast. But there is no formalized guide to answer the question of variable selection. Since the approach utilized in this paper differs substantially from the Stock-Watson approach, the rules cannot be applied one to one to the setting of the generalized dynamic factor model.

The basic methodology to select variables used in this paper can be outlined as follows. We have reordered the data set according to the absolute value of the average correlation coefficient of each variable with GDP at different leads (leads one to three). GDP is always the first variable in the data set. The other variables were arranged in a descending order, i.e. the variable with the highest correlation to GDP is the second variable and the variable with the lowest correlation is the last variable.

Table 3 shows the cross correlation between the ten variables with the strongest leading behaviour with respect to real GDP. The variables with the strongest correlation with GDP are the number of vacancies and the unemployment to vacancies ratio. Among the other top ten leading variables three survey series can be found. Two of them (IFO business climate index and IFO current situation for Western Germany) are related to the situation of Austria's most important trading partner, Germany. Foreign trade is also represented by exports of machinery and vehicles and total exports. Finally, two financial series (Dow Jones Index and DAX) and the nominal-effective exchange rate are ranked in the top ten variables. Extensive simulations have been performed with this reordered data set to find a subset with the lowest out-of-sample root mean squared error (RMSE).

For each subset $(n=1,\ldots,143 \text{ variables})$ of the ordered data set and for $q=1,\ldots,15$ common factors⁶, we have computed the out-of-sample forecast errors for 30 windows beginning at time t=T-30-3. For each forecasting exercise, we have reported the RMSE and MAE of GDP for $h=1,\ldots,3$ forecasting steps and the average. The results are summarized in table 4 and figure 2.

From a methodological point of view, two main results can be pointed out. First and foremost, the subsets which yield the best forecasting performance are very small. For the two-steps-ahead forecast, a subset of eleven variables with two common factor provides us with the best forecasts. Forecasting one-step-ahead is best done with nine variables and three common factors. The best results for the three-step-ahead forecast are achieved with

⁶The number of common factors (q) must not exceed the number of variables (n).

Table 4: Forecasting performance of the smaller factor models compared to the large factor model

	h = 1	h = 2	h = 3	Average
Performance of best subsets ^a				
size of subset	n = 9	n = 11	n = 5	
$number\ of\ principal\ components$	q = 3	q = 2	q = 5	
RMSE	0.871	0.815	0.928	0.871
MAE	0.726	0.629	0.754	0.703
Pesaran-Timmerman	*0.052	**0.026	*0.079	-
Factor Model (best subset) compared to ARIM	$MA\ Model$			
RMSE % gain	13.8	22.8	13.3	16.7
MAE $\%$ gain	10.3	23.7	11.8	15.2
Diebold-Mariano RMSE	*0.054	**0.020	**0.012	-
Diebold-Mariano MAE	0.130	***0.006	*0.069	-
Wilcoxon RMSE	0.145	**0.015	*0.060	-
Wilcoxon MAE	0.175	***0.009	0.107	-
Model encompassing	0.473	0.122	0.150	-
Factor Model (best subset) compared to factor	r model with full dat	$a \ set$		
RMSE % gain	1.7	22.8	10.9	13.6
MAE $\%$ gain	1.5	27.8	10.9	14.2
Diebold-Mariano RMSE	0.407	**0.022	*0.081	-
Diebold-Mariano MAE	0.423	***0.007	0.126	-
Wilcoxon RMSE	0.259	**0.019	0.111	-
Wilcoxon MAE	0.399	***0.006	0.111	-
Model encompassing b	0.468	0.410	0.473	-

 $[^]a \rm Reported$ values are probabilities with the exception of RMSE, MAE, RMSE gain and MAE gain. $^b \rm Harvey$ et. al. Model encompassing test

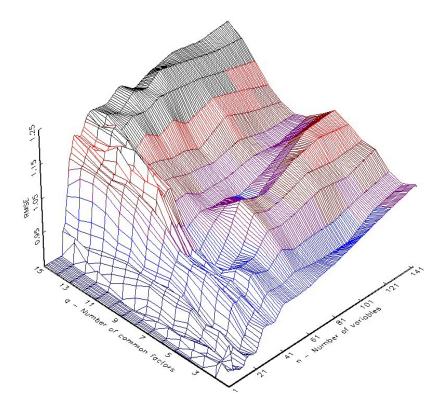


Figure 2: RMSE of the factor model for one-quarter-ahead forecasts, depending on the number of variables and dynamic common factors

only five variables and five common factors. These results are in contradiction with the intention of dynamic factor models, which have been developed to exploit the potentially fruitful information of a large panel.

The second main result is that for the bulk of subsets the best forecasting performance can be obtained with one or two dynamic common factors only. Although a higher number of dynamic common factors results in a better fit, the forecasting performance worsens if too many factors are chosen. To give an intuition why that might happen, reconsider the basic steps of the procedure. First, the covariance matrices of the common and the idiosyncratic components are estimated by means of the frequency domain approach. In a second step, these covariance matrices are then used to construct linear filters, which are used for forecasting. By construction, these filters put a higher weight on variables with a higher commonality. To be more precise, the variables used for forecasting are standardized by dividing them by the standard deviation of their idiosyncratic component in the second step (i.e. by solving the generalized eigenvalue problem from page 5) (see also FHLR (2003)). A variable which is only weakly correlated with the remainder of the data set has a larger idiosyncratic variance and hence a lower weight in the forecast. For illustrative purposes, consider a static model with one common factor and with two groups of variables which

are highly correlated within each group. If the common factor summarizes the first group, the idiosyncratic components will be larger for the second group. Increasing the number of dynamic common factors will decrease the variance of the idiosyncratic component for all variables, but especially for the variables of the second group. Thus increasing q increases the weight of the the second group of variables relative to the other variables, which will deteriorate the forecasting performance for variables belonging to the first group.

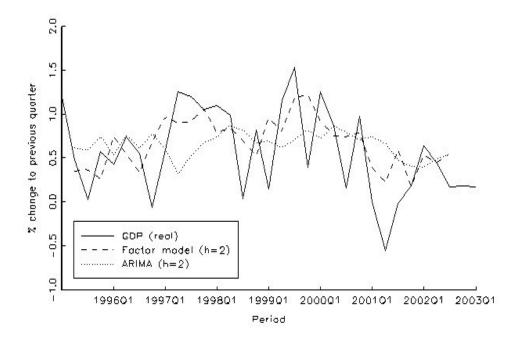


Figure 3: Factor and ARIMA model forecasts of Austrian GDP (two quarters ahead)

Figure 3 gives a visual impression of the forecasting performance of the best subsets (two quarters ahead). It shows that the broad movements of the business cycle are predicted correctly. This can be underlined by the results of the Pesaran-Timmermann test, which show that the direction of change of GDP growth rates (i.e. an acceleration or deceleration of growth) is correctly predicted. Naturally, the steep decline in activity in the beginning of 2001 was not predicted to its full extent, since it was mainly caused by a series of unpredictable shocks to the world economy. Nevertheless, the direction of change was correctly predicted. Another interesting finding is that the forecasting error of the two-steps-ahead forecast is considerably smaller than for the one-step-ahead forecast.

5 Conclusions

The aim of this paper was to develop a framework for short-term forecasting of real GDP for Austria using the generalized dynamic factor model. The forecasting performance of the model was assessed with an out-of-sample forecasting experiment over 30 windows. The simulations with the full data set with 143 variables show that the dynamic factor model was not able to perform significantly better than a simple ARIMA model for two and three quarters ahead. Hence, an extensive forecasting experiment has been conducted to find subsets of the data set that perform better. The variables have been ordered according to their lead properties with respect to GDP.

The following main empirical findings have been obtained from this experiment. First, the factor model performs significantly better with a small data set of about five to eleven variables compared with the full data set of 143 variables. Second, the number of dynamic common factors also impacts heavily on the forecasting performance. A higher number of common factors tends to worsen the forecasting performance.

Obviously, our results are just a first step in investigating the forecasting behavior of the generalized dynamic factor model. Necessary next steps in that direction would be to test the behavior of the model for other data sets. Constructing Monte Carlo simulations to shed some more light on the relation between the size of the data set and the forecasting performance would be another promising step forward. Although it is far too early to draw firm conclusions, our results indicate that the generalized dynamic factor model - although it is designed to deal with a huge number of variables - does not free the forecaster from the task of variable selection. It seems that there is plenty of room to improve the forecasting performance by carefully assembling the data set, which does not need to be huge.

A Technical annex

A.1 The model

Consider the double sequence

$$\{x_{it}, i \in \mathbb{N}, t \in \mathbb{Z}\},\$$

where

$$x_{it} = b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \dots + b_{iq}(L)u_{qt} + \xi_{it}, \tag{5}$$

L standing for the lag operator, and suppose that the following assumptions 1-4 hold.

Assumption 1.

- (I) The q-dimensional process $\{(u_{1t}u_{2t}\dots u_{qt})', t\in\mathbb{Z}\}$ is orthonormal white noise;
- (II) $\boldsymbol{\xi} = \{\xi_{it}, i \in \mathbb{N}, t \in \mathbb{Z}\}\$ is a double sequence such that, firstly, $\boldsymbol{\xi}_n = \{(\xi_{1t}\xi_{2t}\dots\xi_{1t})t \in \mathbb{Z}\}\$ is a zero-mean stationary vector process for any n, and, secondly, $\xi_{it} \perp u_{jt-k}$ for any i, j, t;
- (III) the filters $b_{ij}(L)$ are one-sided in L and their coefficients are square summable.

Assumption 2. For any $i \in \mathbb{N}$, there exists a real $c_i > 0$ such that $\sigma_{ii}(\theta) \leq c_i$ for any $\theta \in [-\pi, \pi]$.

Assumption 3. The first idiosyncratic dynamic eigenvalue λ_{n1}^{ξ} is uniformly bounded, i.e. there exists a real Λ such that $\lambda_{n1}^{\xi}(\theta) \leq \Lambda$ for any $\theta \in [-\pi, \pi]$ and any $n \in \mathbb{N}$.

Assumption 4. The first q common dynamic eigenvalues diverge almost everywhere in $[-\pi,\pi]$, i.e., $\lim_{n\to\infty} \lambda_{nj}^{\chi}(\theta) = \infty$ for $j \leq q$, a.e. in $[-\pi,\pi]$.

Model (5) under assumptions 1-4 is called the generalized dynamic factor model.

A.1.1 Forecasting the common component

In order to obtain a consistent estimation of the space of common factors it necessary to find r linear combinations $W_{nt}^{kT} = Z_{nk}^T x_{nt}$, where the weights can be obtained as the solution of a generalized eigenvalue problems:

$$Z_{nl}^{T} := \arg \max_{\mathbf{a} \in \mathbb{R}^{n}} \operatorname{var} \left(\mathbf{a} \chi_{nt}^{T} \right)$$
subject to
$$\operatorname{var} \left(\mathbf{a} \xi_{nt}^{T} \right) = 1$$

$$\mathbf{a} \xi_{nt}^{T} \perp \mathbf{Z}_{nm}^{T} \xi_{nt}^{T} \quad \text{for} \quad 1 \leq m < \ell, \quad 1 \leq \ell \leq n.$$

$$(6)$$

where **a** are the generalized eigenvectors. Intuitively, the formulation above maximizes the common to idiosyncratic variance ratio in the resulting aggregates W_{nt}^{kT} . An equivalent form of (6) is

$$Z_{nl}^{T} := \arg \max_{\mathbf{a} \in \mathbb{R}^{n}} \mathbf{a} \Gamma_{n0}^{\chi T} \mathbf{a}'$$
subject to
$$\mathbf{a} \Gamma_{n0}^{\xi T} \mathbf{a}' = 1$$

$$\mathbf{a} \Gamma_{n0}^{\xi T} \mathbf{Z}_{nm}^{T'} \quad \text{for} \quad 1 \le m < \ell, \quad 1 \le \ell \le n.$$

$$(7)$$

FHLR (2003) show that the space spanned by the aggregates approximates the space spanned by the principal components $\mathcal{G}(\mathbf{F},t)$. The proposed forecast for $\chi_{i,T+h}$ is the estimated projection of $x_{i,T+h}$ onto the space spanned by the r linear aggregates $W_{nt}^{kT}, k = 1..., r$, i.e.

$$\phi_{i,T+h|T}^{nT} := \left[\Gamma_{nh}^{\chi T} \mathbf{Z}_n^T \left(\tilde{\mathbf{Z}}_n^T \Gamma_{n0}^{xT} \mathbf{Z}_n^T \right)^{-1} \tilde{\mathbf{Z}}_n^T x_{nT} \right]_i$$
 (8)

where $\mathbf{Z}_n^T := (\mathbf{Z}_{n1}^{T'} \dots \mathbf{Z}_{nr}^{T'})'$. Setting h = 0 yields a consistent one-sided estimation of χ_{it} , which for fixed t avoids the end-of-sample inconsistency problem.

A.1.2 Estimating the variance covariance matrices of the common and the idiosyncratic component

Consider the double sequence $\{x_{it}, i=1,...,n,t=1,...,T\}$, with zero mean and variance one. For a fixed integer M compute the sample covariance matrix $\Gamma_{nk}^T = x_{nt}x'_{nt-k}$ for k=0,1,...,M and create the truncated two-sided sequence in form of a stacked matrix:

$$\begin{pmatrix}
\gamma_{11,-M} & \cdots & \gamma_{1n,-M} \\
\vdots & \ddots & \vdots \\
\gamma_{n1,-M} & \cdots & \gamma_{nn,-M}
\end{pmatrix}$$

$$\vdots$$

$$\begin{pmatrix}
\gamma_{11,0} & \cdots & \gamma_{1n,0} \\
\vdots & \ddots & \vdots \\
\gamma_{n1,0} & \cdots & \gamma_{nn,0}
\end{pmatrix}$$

$$\vdots$$

$$\vdots$$

$$\begin{pmatrix}
\gamma_{11,+M} & \cdots & \gamma_{1n,+M} \\
\vdots & \ddots & \vdots \\
\gamma_{n1,+M} & \cdots & \gamma_{nn,+M}
\end{pmatrix}$$

$$= \begin{bmatrix}
\Gamma_{n,-M}^T \\
\vdots \\
\Gamma_{n,0}^T \\
\vdots \\
\Gamma_{n,+M}^T
\end{bmatrix}_{[n(2M+1))\times n]}.$$
(9)

The spectral density matrices can be obtained by applying a discrete Fourier transformation to the sample covariance matrices Γ^T_{nk} . In the next step, compute for each $\theta_h = 2\pi h/(2M+1), h=0,1,...,2M$, and each n=1,2,...N, the (2M+1) discrete Fourier transform of this sequence of covariance matrices. More precisely compute for each n the sequence

$$\Sigma_n^T(\theta_h) = \sum_{k=-M}^M \Gamma_{n,k}^T \omega_k e^{-ik\theta_h}, \tag{10}$$

where

$$\omega_k = 1 - \frac{|k|}{(M+1)}$$

are the weights corresponding to the Bartlett lag window of size M=M(T). We use the rule $M=\sqrt{T}/4$ which ensures consistent estimation of Σ_n provided that $M(T)\to\infty$ and $M(T)/T\to 0$ as $T\to\infty$. Note that, after rearranging the elements, $\Sigma_n^T(\theta_h)$ has the form

$$\begin{pmatrix}
\sigma_{11}(\theta_{0}) & \cdots & \sigma_{1n}(\theta_{0}) \\
\vdots & \ddots & \vdots \\
\sigma_{n1}(\theta_{0}) & \cdots & \sigma_{nn}(\theta_{0})
\end{pmatrix}$$

$$\begin{pmatrix}
\sigma_{11}(\theta_{1}) & \cdots & \sigma_{1n}(\theta_{1}) \\
\vdots & \ddots & \vdots \\
\sigma_{n1}(\theta_{1}) & \cdots & \sigma_{nn}(\theta_{1})
\end{pmatrix}$$

$$\vdots$$

$$\vdots$$

$$\begin{pmatrix}
\sigma_{11}(\theta_{2M}) & \cdots & \sigma_{1n}(\theta_{2M}) \\
\vdots & \ddots & \vdots \\
\sigma_{n1}(\theta_{2M}) & \cdots & \sigma_{nn}(\theta_{2M})
\end{pmatrix}$$

$$\begin{pmatrix}
\sigma_{11}(\theta_{2M}) & \cdots & \sigma_{1n}(\theta_{2M}) \\
\vdots & \ddots & \vdots \\
\sigma_{n1}(\theta_{2M}) & \cdots & \sigma_{nn}(\theta_{2M})
\end{pmatrix}$$
(11)

The estimated spectral density matrix is then decomposed by a dynamic principal component decomposition. For each frequency of the frequency grid, we compute eigenvalues and eigenvectors. The eigenvectors are ordered in a descending manner according to their eigenvalues. The first q eigenvectors of each frequency h are then extracted over frequencies. For each h=0,1,...,2M compute the first q (row-) eigenvectors $\mathbf{p}_{nj}^T(\theta_h), j=1,...,q$ of $\Sigma_n^T(\theta_h)$ and construct for i=1,2,...,n the $(n\mathbf{x}q)$ matrix

$$\mathbf{K}_{ni}^{T}(\theta_h) = \tilde{p}_{n1,i}^{T}(\theta_h)\mathbf{p}_{n1}^{T}(\theta_h) + \dots + \tilde{p}_{nq,i}^{T}(\theta_h)\mathbf{p}_{nq}^{T}(\theta_h), \tag{12}$$

where $\tilde{p}_{nj,i}^T(\theta_h)$ is the i,jth entry in the complex conjugate of the matrix of eigenvectors. Defining the qxn matrix of eigenvectors in rows

$$\mathbf{V}(\theta_h) \equiv \begin{pmatrix} p_{11}(\theta_h) & \cdots & p_{1n}(\theta_h) \\ \vdots & \ddots & \vdots \\ p_{a1}(\theta_h) & \cdots & p_{an}(\theta_h) \end{pmatrix}$$
(13)

as the matrix of eigenvectors of $\Sigma_n^T(\theta_h)$ we get the (nxn) matrix of normalized eigenvectors for h = 0, 1, ..., 2M

$$\mathbf{K}_{n}^{T}(\theta_{h}) = \left[\mathbf{V}(\theta_{h}) \cdot \tilde{\mathbf{V}}(\theta_{h})\right]_{(n \times n)},\tag{14}$$

such that $\mathbf{K}_{ni}^T(\theta_h)$ from equation (12) is the *i*th column vector with dimension $(n \times 1)$ in the matrix $\mathbf{K}_n^T(\theta_h)$ from equation (14). Creating the stacked vector

$$(\mathbf{K}_n^T(\theta_0), \mathbf{K}_n^T(\theta_1), \cdots, \mathbf{K}_n^T(\theta_{2M}))$$

leads to the matrix $\mathbf{K}(\theta_h)$ with the following structure:

$$\begin{pmatrix} \mathbf{K}_{n}^{T}(\theta_{0})_{(n\times n)} \\ \vdots \\ \mathbf{K}_{n}^{T}(\theta_{2M})_{(n\times n)} \end{pmatrix}_{(n(2M+1)\times n)}, \tag{15}$$

which is in more detail

Extracting the elements over frequencies of equation 15 one gets a row vector for each element of $\mathbf{K}_{ni}^{T}(\theta_h)$ of matrix 14, which can lead to the proposed estimator by computing the inverse discrete Fourier transform

$$\underline{\mathbf{K}}_{ni,k}^{T} = \frac{1}{2M+1} \sum_{h=0}^{2M} \mathbf{K}_{ni}^{T}(\theta_h) e^{ik\theta_h}$$
(17)

for each i and k. Summing over the leads and lags $k = -M, \dots, +M$, the estimator of the filter is given by

$$\underline{\mathbf{K}}_{ni}^{T}(L) = \sum_{k=-M}^{M} \underline{\mathbf{K}}_{ni,k}^{T} L^{k}.$$
(18)

The common components are then simply

$$\chi_{nt} = \underline{\mathbf{K}}_{ni}^T(L)x_{nt} \tag{19}$$

A.2 Test for equal forecasting accuracy and forecast encompassing

A.2.1 Wilcoxon's signed rank test

The non-parametric Wilcoxon signed rank test tests the null hypothesis of equal forecast accuracy. It is an alternative to the t-test in situations, where the assumption of a normal distribution is violated, which is typically the case in small samples. The test assumes that both forecasting errors have identical distributions. Hence the distribution of the differences d_t between the loss functions $g(e_t^A)$ and $g(e_t^B)$ of the forecasting errors e_t^A and e_t^B is symmetric around zero. The null hypothesis is that the loss differential $\{d_t\}_1^T = e_t^A - e_t^B$ has

median value zero. The test is illustrated for a squared loss function, although it can be applied for an absolute loss function as well. The following steps are necessary to perform the test. Begin with calculating the loss differential series d_t

$$d_{t} = \begin{cases} (e_{t}^{A})^{2} - (e_{t}^{B})^{2} & , \text{ if } RMSE^{A} < RMSE^{B} \\ (e_{t}^{B})^{2} - (e_{t}^{A})^{2} & , \text{ othwerwise} \end{cases}$$
(20)

and remove all zero elements from d_t . Next, compute a 0/1 vector with ones for all elements of d_t which are greater than zero.

$$l_{+}(d_{t}) = \begin{cases} 1 & \text{, if } d_{T} > 0 \\ 0 & \text{, othwerwise.} \end{cases}$$
 (21)

Determine the rank numbers of all elements of d_t , disregarding the sign of d_t . Assign the rank number 1 to the smallest and T to the highest element. If tied values occur, than rank all elements with the mean of the rank numbers that would have been assigned if they would have been different. Compute the test statistic W as the sum of the positive ranks only.

$$W = \sum_{t=1}^{T} l_{+}(d_{t}) * \operatorname{rank}(|d_{t}|)$$
 (22)

Critical values for W are tabled. If W is smaller than the critical value, reject the null of equal forecasting accuracy. Asymptotically, W converges to a normal distribution

$$W \sim N\left(\mu, \sigma^2\right)$$
, with $\mu = \frac{T(T+1)}{4}$ and $\sigma^2 = \frac{T(T+1)(2T+1)}{24}$.

Therefore, if T > 20, one can compute the transformed test statistic

$$W = \frac{W - \mu}{\sigma} \tag{23}$$

and use the critical values from the standard normal distribution.

A.2.2 The Diebold and Mariano test

The Diebold and Mariano (1995) test for equal forecasting accuracy tests the null hypothesis of equal forecast accuracy of two competing forecasts. It uses a forecast error loss differential $d_t = g(e_t^B) - g(e_t^B)$, which is assumed to be a weakly stationary process with short memory. The main rationale underlying this test is that forecast errors are usually serially correlated. In multi-step forecasting (h > 1), forecasts errors are assumed to be at most (h - 1)-dependent. This is a plausible assumption, since two consecutive h-steps-ahead forecasts have h - 1 periods with similar information in common. The Diebold and Mariano test is a modified t-test, whereby the modification accounts for the serial correlation of the loss differential. The mean \bar{d} is assumed to be asymptotically normally distributed:

$$\sqrt{T(\bar{d}-\mu)} \quad \stackrel{d}{\longrightarrow} \quad N(0,V(\bar{d})), \tag{24}$$

whereby $V(\bar{d})$ stands for the serially correlated errors corrected variances of the sample mean (\bar{d}) , given by the sum of the variance and the auto-covariances up to lag h-1 assuming that there are no auto-correlations at a lag equal to or greater than h:

$$V(\bar{d}) = \frac{1}{T} \left(\gamma_0 + 2 \sum_{\tau=1}^{h-1} \gamma_\tau \right), \tag{25}$$

where T stands for the sample size and with autocovariance

$$\gamma_{\tau} = \frac{2}{T} \sum_{t=\tau+1}^{T} (d_t - \bar{d})(d_{t-\tau} - \bar{d}). \tag{26}$$

The asymptotically normally distributed test statistic DM can be obtained by

$$DM = \frac{\bar{d}}{\sqrt{V(\bar{d})}}. (27)$$

In small samples, the t-distributed modified test statistic DM^* should be preferred (Harvey, Leybourne, and Newbold 1997):

$$DM^* = \frac{DM}{\sqrt{\frac{T+1-2h+\frac{h(h-1)}{T}}{T}}}$$
 (28)

If the value of the test statistic is greater than the critical value, the null of equal forecasting accuracy should be rejected.

A.2.3 Harvey, Leybold, and Newborn test for forecast encompassing

Harvey, Leybourne, and Newbold (1998) proposed a test for forecast encompassing under the null that forecast A encompasses forecast B, i.e. forecast B adds no predictive power to forecast A. The test uses a linear combination of two competing forecasts y_t^A and y_t^B of variable y_t with a combined forecast error ϵ_t :

$$y_t = (1 - \lambda)y_t^A + \lambda y_t^B + \epsilon_t. \tag{29}$$

In terms of individual forecast errors $e_t^i = y_t - y_t^i$, for i = A, B, equation (29) can be written as

$$e_t^A = \lambda (e_t^A - e_t^B) + \epsilon_t. \tag{30}$$

This equation has to be estimated by OLS. If the null of forecast encompassing holds, than λ should equal zero.

A.2.4 The Pesaran-Timmerman non-parametric test of predictive performance

Let $x_t = E(y_t, \Omega_{t-1})$ be the predictor of y_t found with respect to the information set, Ω_{t-1} , with n observations $(y_1, x_1), (y_2, x_2), \dots, (y_n, x-n)$ available. The test proposed by Pesaran and Timmerman (1992) is based on the proportion of times that the direction of changes in y_t is correctly predicted by x_t . The test statistic is computed as

$$S_n = \frac{P - P^*}{\sqrt{V(P) - V(P^*)} 1/2} \sim N(0, 1)$$
(31)

where:

$$P = \bar{Z} = \frac{1}{n} \sum_{i=1}^{n} Z_{i}$$

$$P^{*} = P_{y}P_{x} + (1 - P_{y})(1 - P_{x})$$

$$V(P^{*}) = \frac{1}{n}P^{*}(1 - P^{*})$$

$$V(P) = n \left[(2P_{y} - 1)^{2}P_{x}(1 - P_{x}) + (2P_{x} - 1)^{2}P_{y}(1 - P_{y}) + \frac{4}{n}P_{y}P_{x}(1 - P_{y})(1 - P_{x}) \right]$$

 Z_i is an indicator variable which takes value of one when the sign of y_t is correctly predicted by x_t , and zero otherwise, P_y is the proportion of times y_t takes a positive value, P_x is the proportion of times x_t takes a positive value.

B List of variables

National account data

GDP, real
Private consumption, real
Gross fixed capital formation, real
Public consumption, real
Changes in inventories, real
Imports, real
Exports, real
GDP deflator
Private consumption deflator
Gross fixed capital formation deflator
Public consumption deflator
Changes in inventories deflator
Import deflator
Export deflator
Terms of Trade

WIFO Quarterly Survey

Assessment of order books
Assessment of export order books
Assessment of stocks of finished products
Selling-price expectations
Assessment of order books - construction
Selling-price expectations - construction
Assessment of current situation - construction
Business situation - construction

Monthly survey data

Economic sentiment indicator
Industrial confidence indicator
Production observed in recent months in industry
Order books in industry
Export order books in industry
Stocks of finished products in industry
Production expectations in industry
Selling-price expectations in industry
Selling-price expectations in construction
Construction confidence indicator
Retail trade confidence indicator
Consumer confidence indicator

Prices

Ifo - business expectations in Western Germany

Ifo - business climate index in Western Germany

Ifo - assessment of current situation in Western Germany

Purchasing manager index USA

HICP - Overall index

HICP - Food incl. alcohol and tobacco

HICP - Processed food incl. alcohol and tobacco

HICP - Unprocessed food

HICP - Goods

HICP - Industrial goods

HICP - Industrial goods excluding energy

HICP - Energy

HICP - Services

HICP - All items excluding alcoholic beverages, tobacco

HICP - All items excluding energy

HICP - All items excluding energy and food

HICP - All items excluding energy and unprocessed food

Consumer price index - overall index

Consumer price index 86 - housing

Index of agreed minimum wages, overall index

Index of agreed minimum wages, workers

Index of agreed minimum wages, salary earners

Wholesale prices 86 - Overall index

Wholesale prices 86 - excl. seasonal goods

Wholesale prices 86 - consumer goods

Wholesale prices 86 - durable commodities

Wholesale prices 86 - non-durable commodities

Wholesale prices 86 - non-durables

Wholesale prices 86 - consumer goods

Wholesale prices 86 - capital goods

Wholesale prices 86 - intermediate goods

Oil price

Nominal-effective exchange rate

Euro/Dollar exchange rate

Foreign trade

Total exports

Exports SITC 6 (basic manufactures)

Exports SITC 7 (machines, transport equipment)

Exports SITC 8 (misc. manufactured goods)

Total imports

Imports SITC 6 (basic manufactures)

Imports SITC 7 (machines, transport equipment)

Imports SITC 8 (misc. manufactured goods)

Labour market

Exports of commodities to USA
Exports of commodities to EU
Exports of commodities to Germany
Imports of commodities from USA
Imports of commodities from EU
Imports of commodities from Germany
Unemployment rate, national definition
Unemployment, male
Unemployment, femal
Vacancies
Employees

Financial variables

ATX (Austrian trading index)
Money aggregate M1
Money aggregate M2
Money aggregate M3
DAX
Dow Jones index
3-month money market rate
Secondary market yield on government bonds (9 to 10 years)
Yield spread
Direct credits to private households
Direct credits to private firms
Direct credits to government
Outstanding debt
Direct credits, total

Miscellaneous

Industrial production, overall index (excl. construction and energy)

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June 15, 1998	Canzeroni Matthew, Cumby Robert, Diba Behzad and Eudey Gwen	27	Trends in European Productivity: Implications for Real Exchange Rates, Real Interest Rates and Inflation Differentials
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June 30, 1998	Campa José and Wolf Holger	29	Goods Arbitrage and Real Exchange Rate Stationarity
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December 11, 1998	Helene Schuberth and Gert Wehinger	35	Room for Manoeuvre of Economic Policy in the EU Countries – Are there Costs of Joining EMU?
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December 20, 1999	John R. Freeman, Jude C. Hays and Helmut Stix	39	Democracy and Markets: The Case of Exchange Rates

March 01, 2000	Eduard Hochreiter and Tadeusz Kowalski	40	Central Banks in European Emerging Market Economies in the 1990s
March 20, 2000	Katrin Wesche	41	Is there a Credit Channel in Austria?
Warch 20, 2000	Raum Wesche	41	The Impact of Monetary Policy on Firms' Investment Decisions
June 20, 2000	Jarko Fidrmuc and Jan Fidrmuc	42	Integration, Disintegration and Trade in Europe: Evolution of Trade Relations During the 1990s
March 06, 2001	Marc Flandreau	43	The Bank, the States, and the Market, A Austro-Hungarian Tale for Euroland, 1867-1914
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February 28, 2002	Peter Backé, Jarko Fidrmuc, Thomas Reininger and Franz Schardax	61	Price Dynamics in Central and Eastern European EU Accession Countries
April 8, 2002	Jesús Crespo- Cuaresma, Maria Antoinette Dimitz and Doris Ritzberger- Grünwald	62	Growth, Convergence and EU Membership
May 29, 2002	Markus Knell	63	Wage Formation in Open Economies and the Role of Monetary and Wage-Setting Institutions
June 19, 2002	Sylvester C.W. Eijffinger (comments by: José Luis Malo de Molina and by Franz Seitz)	64	The Federal Design of a Central Bank in a Monetary Union: The Case of the European System of Central Banks

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July 10, 2002	David Begg (comment by Peter Bofinger)	66	Growth, Integration, and Macroeconomic Policy Design: Some Lessons for Latin America
July 15, 2002	Andrew Berg, Eduardo Borensztein, and Paolo Mauro (comment by Sven Arndt)	67	An Evaluation of Monetary Regime Options for Latin America
July 22, 2002	Eduard Hochreiter, Klaus Schmidt-Hebbel and Georg Winckler (comments by Lars Jonung and George Tavlas)	68	Monetary Union: European Lessons, Latin American Prospects
July 29, 2002	Michael J. Artis (comment by David Archer)	69	Reflections on the Optimal Currency Area (OCA) criteria in the light of EMU
August 5, 2002	Jürgen von Hagen, Susanne Mundschenk (comments by Thorsten Polleit, Gernot Doppelhofer and Roland Vaubel)	70	Fiscal and Monetary Policy Coordination in EMU
August 12, 2002	Dimitri Boreiko (comment by Ryszard Kokoszczyński)	71	EMU and Accession Countries: Fuzzy Cluster Analysis of Membership
August 19, 2002	Ansgar Belke and Daniel Gros (comments by Luís de Campos e Cunha, Nuno Alves and Eduardo Levy-Yeyati)	72	Monetary Integration in the Southern Cone: Mercosur Is Not Like the EU?
August 26, 2002	Friedrich Fritzer, Gabriel Moser and Johann Scharler	73	Forecasting Austrian HICP and its Components using VAR and ARIMA Models

September 30, 2002	Sebastian Edwards	74	The Great Exchange Rate Debate after Argentina
October 3, 2002	George Kopits (comments by Zsolt Darvas and Gerhard Illing)	75	Central European EU Accession and Latin American Integration: Mutual Lessons in Macroeconomic Policy Design
October 10, 2002	Eduard Hochreiter, Anton Korinek and Pierre L. Siklos (comments by Jeannine Bailliu and Thorvaldur Gylfason)	76	The Potential Consequences of Alternative Exchange Rate Regimes: A Study of Three Candidate Regions
October 14, 2002	Peter Brandner, Harald Grech	77	Why Did Central Banks Intervene in the EMS? The Post 1993 Experience
October 21, 2002	Alfred Stiglbauer, Florian Stahl, Rudolf Winter-Ebmer, Josef Zweimüller	78	Job Creation and Job Destruction in a Regulated Labor Market: The Case of Austria
October 28, 2002	Elsinger, Alfred Lehar and Martin Summer	79	Risk Assessment for Banking Systems
November 4, 2002	Helmut Stix	80	Does Central Bank Intervention Influence the Probability of a Speculative Attack? Evidence from the EMS
June 30, 2003	Markus Knell, Helmut Stix	81	How Robust are Money Demand Estimations? A Meta-Analytic Approach
July 7, 2003	Helmut Stix	82	How Do Debit Cards Affect Cash Demand? Survey Data Evidence
July 14, 2003	Sylvia Kaufmann	83	The business cycle of European countries. Bayesian clustering of country-individual IP growth series.
July 21, 2003	Jesus Crespo Cuaresma, Ernest Gnan, Doris Ritzberger- Gruenwald	84	Searching for the Natural Rate of Interest: a Euro-Area Perspective
July 28, 2003	Sylvia Frühwirth- Schnatter, Sylvia Kaufmann	85	Investigating asymmetries in the bank lending channel. An analysis using Austrian banks' balance sheet data

September 22, 2003	Burkhard Raunig	86	Testing for Longer Horizon Predictability of Return Volatility with an Application to the German DAX
May 3, 2004	Juergen Eichberger, Martin Summer	87	Bank Capital, Liquidity and Systemic Risk
June 7, 2004	Markus Knell, Helmut Stix	88	Three Decades of Money Demand Studies. Some Differences and Remarkable Similarities
August 27, 2004	Martin Schneider, Martin Spitzer	89	Forecasting Austrian GDP using the generalized dynamic factor model