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# Systematic Systemic Stress Tests

Thomas Breuer, Martin Summer

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# Systematic Systemic Stress Tests*

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**Abstract**

For a given set of banks, which economic and financial scenarios will lead to big losses? How big can losses in such scenarios possibly get? These are the two central questions of macro stress tests. We believe that most current macro stress testing models have deficits in answering these questions. They select stress scenarios in a way which might leave aside many dangerous scenarios and thus create an illusion of safety; and which might consider highly implausible scenarios and thus trigger a false alarm. With respect to loss evaluation most stress tests do not include tools to analyse systemic risk arising from the interactions of banks with each other and with the markets. We make a conceptual proposal how these shortcomings may be addressed and how stress tests could be made both systematic and systemic. We demonstrate the application of our concepts using publicly available data on European banks and capital markets, in particular the EBA 2016 stress test results.

**Keywords:** Stress Testing, Risk Measures, Scenario Analysis, Systemic Risk

**JEL-Classification Numbers:** C18, C44, C60, G01, G32, M48

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**Non technical summary**

This paper makes a concrete proposal to improve scenario selection as well as loss evaluation in solvency stress testing of banks and banking systems. The motivation for making such a proposal is that we believe that these improvements are needed because current stress testing methodologies fail to systematically identifying dangerous but plaubible scenarios. They also largly neglect important sources of losses that are due to behavioral responses of banks to changes in their economic environment.

We believe that systematic scenario identification is one methodological aspect of stress testing that should have a high priority. Current stress testing methods concentrate mainly on loss evaluation. Stress scenarios are discussed in an involved and opaque bureaucratic process, which is often highly politicised. The outcome consists of one or two stress scenarios. Then a complex loss evaluation procedure follows, involving supervisors as well as banks to find out the amount of potential impairments in these two scenarios.

Proceeding in this way the stress test may leave aside other dangerous scenarios which have never been considered and create an illusion of safety. It also bears the danger of considering scenarios which are very implausible. This may create a false sense of alarm.

When it comes to loss evaluation in potential scenarios, current stress testing methodology is mainly focussed on losses arising form exogenous risk sources. Experience from past crisis has shown that a large bulk of losses in financial distress arise from endogenous sources, amplifiers of shocks that are driven by the interaction and interconnectedness of banks. This part of loss mechanisms is often referred to as systemic risk.

We make two concrete proposals how to select stress scenarios systematically and how to take into account losses stemming from bank behavior that may amplify losses in distress situations. Using public data from the EBA 2016 stress testing exercise and including other publicly available data sources we develop a simple example, which serves the role of a proof of concept of our ideas. Compared with the standard approach currently used in the EBA stress test, we show that systematic scenario selection and loss evaluation augmented by systemic risk considerations significantly changes the relevant risk asessement.

## Contents

# 1 Introduction

Stress testing of banks is a form of economic and financial scenario analysis with one key question: Which plausible scenarios lead to losses that are able to substantially impair a bank or the financial system? An anwser requires identifying dangerous scenarios by evaluating potential losses in a systematic way.

We claim that current stress testing methodologies need to be improved because they fail in systematically identifying dangerous and plausible scenarios. We propose an approach which delivers a systematic identification of plausible stress scenarios.

We believe that systematic scenario identification is one methodological aspect of stress testing that should have a high priority. Current stress testing methods concentrate mainly on loss evaluation. Stress scenarios are discussed in an involved and opaque bureaucratic process, which is often highly politicised. The outcome consists of one or two stress scenarios. Then a complex loss evaluation procedure follows, involving supervisors as well as banks to find out the amount of potential impairments in these two scenarios.

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Many stress testing models currently in use are usually not considering systemic risk in evaluating losses. One reason for this might stem from the fact that the the literature has discussed many different concepts, without coming to a clear definition of systemic risk or a prioritization of its main mechanisms, see Fouque and Langsam [2013]. We argue that the literature has now moved to a new frontier and that it offers some tractable ways of including systemic risk in an otherwise traditional stress test. We point to one particular framework in the recent literature which we find very promising and show in an example how it can be integrated into the overall model.

Our paper is mainly related to the literature on quantitative risk management (McNeil et al. [2015]), coherent risk measures (Artzner et al. [1999]), model risk (Studer [1997], Studer [1999], Breuer and Krenn [1999], Breuer and Csiszár [2013], Breuer and Csiszár [2016]), fire sale modelling (Cont and Schaanning [2016], Cont and Wagalath [2013], Cont and Wagalath [2016], Braouezec and Wagalath [forthcoming]) and reverse stress testing (Glassermann et al. [2014], Flood and Korenko [2015]).

The literature on quantitative risk management is mainly focussed on statistical risk measurement. There the question is: What is the probability of big losses and how much capital is needed to make this probability sufficiently small? By contrast, in a stress test we ask: Which are the scenarios that lead to big losses? The answer to the stress testing question suggests risks reducing actions. Our paper is related to the coherent risk measure literature because our procedure of systematic scenario selection builds on a coherent risk measure. For our approach to scenario selection we need the concept of a generalized scenario, which builds on various contributions to the literature on model risk. For the systemic risk part we build on recent contributions to the literature on quantitative modeling of deleveraging processes and price impact. In contrast to the reverse stress testing literature, which usually starts from a given hypothetical loss and asks which are the most plausible scenarios generating this loss, we look for the stress scenarios in a different way.

Our paper is also related to the various efforts by stress testing practicioners of developing modern stress testing frameworks, such as the Comprehensive Capital Analysis and Review Process (CCAR), the Dodd-Frank-Act-Stress Tests (DFAST), the Stress tests of the European Banking authority (EBA) or the stress testing framework of the Bank of England and other major central banks. Our ideas on systematic and systemic stress tests are compatible with all of these approaches.

The paper is organized as follows. Section 2 introduces the basic concepts of stress testing and discusses a useful generalisation of the concept of a scenario. This discussion plays an important role in Section 3 where we describe our proposal for finding stress scenarios in a systematic way. Section 4 discusses how we can combine our stress testing framework with a loss evaluation which takes a key aspect of systemic risk - system wide deleveraging - explicitly into account. Section 5 develops an example using public bank exposure data from the EBA 2016 stress test as well as other public data. The example is simplified in various dimensions with the main aim of demonstrating the application of our ideas. Section 6 concludes.

# 2 Stress Testing: Basic Concepts

## 2.1 Stress Tests: Key concepts

A stress test is based on a model of future value changes of a given financial portfolio.[1] The key concepts of a stress testing model are: Risk factors,

[1]What the exact nature of the portfolio is in a given stress test depends on the context. In a solvency stress test such as the EBA stress tests, this portfolio are the different assets on a bank's balance sheet. This is the context we will also use in our discussion. If the context is a liquidity stress test, cash flows and liabilities as well as off-balance sheet items will become also relevant.

scenarios, portfolio valuation. We look at stress tests that have a simple inter-temporal structure, with an observation period and a single future time horizon up to which value changes in the portfolio are considered.

**Risk factors** are the determinants of the value of various positions in the portfolio. For particular positions, such as exposures in securities the choice of the risk factors is fairly obvious. Since securities are marked to market their price changes are the obvious choice. Clearly in practice such a mapping will be somewhat ambiguous. Sometimes an exact mapping between a position and a market price change is not available, sometimes positions are valued by models.

Since lending is a key business of banks, large parts of the bank balance sheet are positions for which no market prices are available. The loan portfolio is usually valued based on actuarial models, such as typical credit risk models (see McNeil et al. [2015]). In these cases risk factors can be either parameters of the actuarial model or underlying variables which are assumed to determine the model parameters.

**Scenarios** postulate possible future developments of the banks, their portfolios, and their environments. If these entities are modelled to depend on more than one factor, scenarios specify more than one factor. If one wishes to judge the plausibility of such scenarios the joint distribution of these factors is relevant. Most traditional stress tests, however abstain from quantifying the plausibility of scenarios.

**Portfolio valuation** is the third conceptual ingredient of a stress test. The valuation functions and models are usually composed of various asset pricing models of different complexity, for each of the portfolio positions.

In the remainder of this section we will discuss systematic selection of stress scenarios. In order to do so, we will need a quantification of scenario plausibilities and a concept of scenario versatile enough to handle market and credit risk in their interaction.

## 2.2 Generalized Scenarios

The distinction between marketable and non-marketable positions is not clear cut in practice. The distinction also does not work in terms of risk. In general, credit risk and market risk interact and can not be separated. The value of an instrument always carries some residual uncertainty. Even in the seemingly certain situation of an instrument maturing exactly at the future time horizon of the stress test, it might happen that the counterparty does not fully fulfil its obligations or that the sell price achieved varies on different markets or that the sell price is subject to a bid-ask spread expressing liquidity risk. For these reasons it is a blunt idealisation to assume that a scenario specifies a unique portfolio value.

In the traditional framework a scenario is represented by a simultaneous realisation of all risk factors and so determines the portfolio value uniquely. In our more general concept we take scenarios to be possible future developments of the banks, their portfolios, and their environments. They do not specify a unique portfolio value but a new distribution of future portfolio values.

We model the risky value of a position or portfolio at some fixed time horizon as

$$V(\mathbb{P}_0) = E_{\mathbb{P}_0}(X),$$

where the payoff $X$ is a random variable on some sample space and $\mathbb{P}_0$ is the presently estimated risk factor distribution at the time horizon. $E_{\mathbb{P}_0}(X)$ is the presently expected value of $X$ under the distribution $\mathbb{P}_0$. The risk factor distribution $\mathbb{P}_0$ is estimated in the same way as in any risk model, starting from assumptions about the model family, the parameters of the distribution are estimated from historical data.

A scenario is an alternative future development represented by a different distribution $\mathbb{Q}$ changing the model-based value to

$$V(\mathbb{Q}) = E_{\mathbb{Q}}(X)$$

This framework naturally accommodates not only market risk but also credit risk, where alternative default probabilities, rating transition probabilities or default correlations are considered. These are properties of distributions.

**Example: Credit risk of a loan** Suppose the exposure to a particular borrower has a face value $f$. Suppose there is one risk factor $r$ representing the repayment ability of the borrower at the maturity of the loan. Assume the presently estimated distribution $\mathbb{P}_0$ of $r$ is a standard normal distribution. Let $K$ denote the default threshold for the borrower. When the currently estimated probability of default is $p$, $K$ is chosen as $K = \Phi^{-1}(p)$. Whenever $r$ falls below this threshold the borrower is insolvent and defaults, repaying only $f - lf$. (Here $l$ is the loss given default as a percentage of the face value.) Then with respect to $\mathbb{P}_0$ we would value the loan according to the expected value of its payoff $X(r) = f - lf\mathbf{1}_{(-\infty,K)}(r)$. A generalized scenario is then an alternative distribution $\mathbb{Q}$ with respect to which we would take the expectation of the payoff function.

# 3 Systematic generalized stress scenarios: Improving Stress Scenario Selection

We call a stress test *systematic* if it provides a procedure for quantifying the plausibility of scenarios and if it considers a complete set of scenarios, i.e. *all* scenarios at or above a given plausibility threshold. In this sense current stress testing practice is not systematic. It refrains from quantifying the plausibility of scenarios. Since it only considers a baseline and one or two adverse scenarios it also looks at a highly incomplete scenario set. The set is incomplete because among a vast number of scenarios with equal plausibility only two are chosen. We propose to do systematic selection of generalized scenarios.

Breuer and Csiszár [2013] measure the plausibility of a generalized scenario $\mathbb{Q}$ by its relative entropy with respect to some presently estimated distribution $\mathbb{P}_0$, which could be interpreted as a prior distribution. (More generally, one can use various $f$-divergences as measures of plausibility, see Breuer and Csiszár [2016], Csiszár and Breuer [2018]). The presently estimated distribution $\mathbb{P}_0$ often results from assumptions about a model class and a parameter estimation procedure based on historical data. Distributions close enough to $\mathbb{P}_0$ are plausible alternative scenarios. We admit as plausible enough all distributions for which the relative entropy does not exceed some threshold $k$. The relative entropy of a probability distributions $\mathbb{Q}$ with respect to a presently estimated distribution $\mathbb{P}_0$ is defined as

$$D(\mathbb{Q}||\mathbb{P}_0) := \begin{cases} \int \log \frac{d\mathbb{Q}}{d\mathbb{P}_0}(r)\mathbb{Q}(dr) & \text{if } \mathbb{Q} \ll \mathbb{P}_0 \\ +\infty & \text{if } \mathbb{Q} \not\ll \mathbb{P}_0 \end{cases}$$

where $\mathbb{Q} \ll \mathbb{P}_0$ denotes absolute continuity of the distribution $\mathbb{Q}$ with respect to the distribution $\mathbb{P}_0$. Relative entropy can be interpreted to quantify similarity of the distributions $\mathbb{Q}, \mathbb{P}_0$ since $D(\mathbb{Q}||\mathbb{P}_0) \geq 0$ and $D(\mathbb{Q}||\mathbb{P}_0) = 0$ only if $\mathbb{P}_0 = \mathbb{Q}$.

The systematic stress test procedure searches for the worst expected value of the portfolio valuation function among the sufficiently plausible generalized scenearios:

$$\inf_{\mathbb{Q}:D(\mathbb{Q}||\mathbb{P}_0)\leq k} E_{\mathbb{Q}}(X). \tag{1}$$

The solution to this problem is a new distribution $\overline{\mathbb{Q}}$, which we call the *worst case distribution*.

The procedure of determining the worst case distribution $\overline{\mathbb{Q}}$ improves *robustness* with respect to the choice of $\mathbb{P}_0$. The set $\{\mathbb{Q} : D(\mathbb{Q}||\mathbb{P}_0) \leq k\}$ takes into account not just the risk factor distribution $\mathbb{P}_0$, which depends on modelling assumptions and historical data, and which might be prone to estimation errors. It also contains distributions with slightly different parameter values (e.g. different covariance structure) and distributions from different model classes (e.g. $t$-distributions instead of normals). The solution to problem (1) determines the worst case over all these alternative distributions.

The parameter $k$ is the "radius" of the set $\{\mathbb{Q} : D(\mathbb{Q}||\mathbb{P}_0) \leq k\}$. The larger we choose $k$, the larger the set of alternative distributions which we are willing to consider in the worst case problem (1), and the lower will be the worst portfolio value resulting in problem (1). The choice of $k > 0$ is free like the choice of a confidence level (between 0 and 1) for Value at Risk calculations. Instead of recommending general rules for the choice of $k$, we propose to determine $k$ from a benchmark, namely as the relative entropy of the EBA stress scenario from the EBA baseline scenario.

A key tool to solve problem (1) is the $G$-function, defined as

$$G(X, \theta_2) := \log \left( \int e^{\theta_2 X(r)} \mathbb{P}_0(dr) \right), \tag{2}$$

where $\theta_2$ is a negative real number. If the payoff function $X$ is clear from the context, we will simply write $G(\theta_2)$.

The solution to Problem (1) has the typical form given below if the following assumptions are satisfied:

(i) If $\operatorname{ess\,inf}(X)$ is finite, assume $k$ is smaller than $k_{\max} := -\log(\mathbb{P}_0(\{r : X(r) = \operatorname{ess\,inf}(X)\}))$.

(ii) Assume $\theta_{\min} := \inf\{\theta_2 : G(\theta_2) < +\infty\} < 0$,

(iii) If $\theta_{\min}, G(\theta_{\min})$, and $G'(\theta_{\min})$ are all finite, assume $k$ does not exceed $k_{\max} := \theta_{\min} G'(\theta_{\min}) - G(\theta_{\min})$.

Problem (1) can also be solved explicitly in the pathological cases where one or more of the three assumptions are violated. But this is not needed for the present purpose.

**Theorem 1** (Breuer and Csiszár [2013]). *Under assumptions (i)-(iii) the equation*

$$\theta_2 G'(\theta_2) - G(\theta_2) = k \tag{3}$$

*has a unique negative solution $\overline{\theta_2}$. The worst case distribution $\overline{\mathbb{Q}}$ solving* (1) *is the distribution with $\mathbb{P}_0$-density*

$$\frac{d\overline{\mathbb{Q}}}{d\mathbb{P}_0}(r) := e^{\overline{\theta_2} X(r) - G(\overline{\theta_2})}. \tag{4}$$

*The minimal expected payoff achieved by the worst case distribution $\overline{\mathbb{Q}}$ is*

$$G'(\overline{\theta_2}). \tag{5}$$

The result provides a practical procedure for calculating MaxLoss in the generic case, which is illustrated in Fig. 1:

1. Calculate $G(\theta_2)$ from (2). This involves the evaluation of an $n$-dimensional integral.

2. Starting from the point $(0, -k)$, lay a tangent to the curve $G(\theta_2)$.

3. The worst expected payoff is given by the slope of the tangent.

4. The worst case distribution is the distribution with density (4), where $\overline{\theta_2}$ is the $\theta_2$-coordinate of the tangent point.

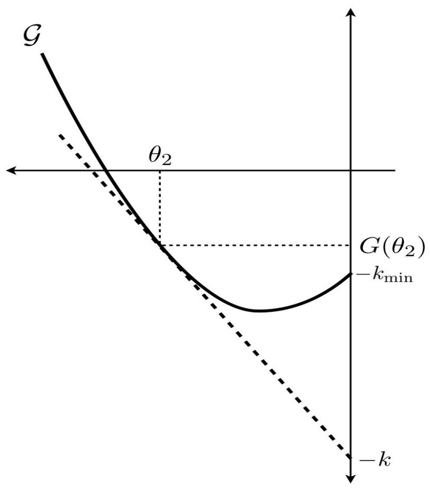


**Figure 1:** Illustration of Theorem 1: The supporting line has maximum slope $b = (k + G(\theta_2))/\theta_2$ among all lines through $(0, -k)$ that meet $\mathcal{G}$. This slope is equal to the solution of problem (1).

One possible way to choose $k$ could be for instance to calculate the relative entropy between an estimated current risk factor distribution and a risk factor distribution estimated from times of crisis.

This framework and the procedure for identifying worst case distributions (i.e. searching for worst case scenarios) naturally applies to market portfolios, credit portfolios, and portfolios exposed to both market and credit risk. It allows for a unified treatment of market and credit risk.

**Example continued: Credit risk of a loan** In this case, where $X(r) = f - lf\mathbf{1}_{(-\infty,K)}(r)$ and $p$ is the currently estimated probability of default, the function $G(\theta_2)$ in (2) equals

$$G(\theta_2) = \log\left[p \exp(\theta_2 f(1-l)) + (1-p)\exp(\theta_2 f)\right]. \tag{6}$$

For a given $k > 0$, $\overline{\theta_2}$ results from numerically solving eq. (3), which takes the form

$$\theta_2 \frac{pf(1-l)\exp(\theta_2 f(1-l)) + (1-p)f\exp(\theta_2 f)}{p\exp(\theta_2 f(1-l)) + (1-p)\exp(\theta_2 f)} \tag{7}$$

$$- \log\left[p\exp(\theta_2 f(1-l)) + (1-p)\exp(\theta_2 f)\right] \quad = k \tag{8}$$

for $\theta_2$. The worst expected payoff is

$$G'(\overline{\theta_2}) = \frac{pf(1-l)\exp(\overline{\theta_2} f(1-l)) + (1-p)f\exp(\overline{\theta_2} f)}{p\exp(\overline{\theta_2} f(1-l)) + (1-p)\exp(\overline{\theta_2} f)}.$$

As a numerical example consider a loan with loss given default $l = 0.5$, and an estimated probability of default $p = 0.1$. If the face value of the exposure is $f = 1$, the expected payoff of this loan under the estimated probability of default equals 0.95. For $k = 0.1$ the numerical solution to (3) is $\overline{\theta_2} = -2.27$. The worst expected payoff at the plausibility level $k = 0.1$ equals 0.87 (compared to 0.95 under the reference default probability). Whereas the reference payment ability distribution $\mathbb{P}_0$ results in the estimated default probability of $p = 0.1$, the worst case payment ability distribution implies a default probability of 0.257. (This is the probability mass of the worst case distribution up to the threshold $K = \Phi^{-1}(0.1) = -1.28$.) The density of the worst case distribution, given by (4), is plotted against the density of the presently estimated distribution $\mathbb{P}_0$ in Fig. 2.

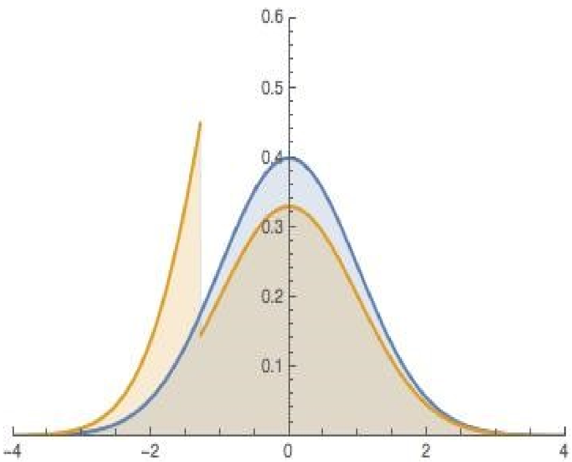


**Figure 2:** Density of presently estimated distribution $\mathbb{P}_0$ (blue) and worst case distribution $\overline{\mathbb{Q}}$ (yellow) of the repayment ability of the borrower.

In Section 5 we will use a credit risk model based on the same idea. But there, several loans with different estimated PDs depend on the same risk factor, so there will be several thresholds. Furthermore the loan portfolios there depend on more than one risk factor, namely one for each sector. Instead of Fig. 2 the worst case density looks as in Fig. 3.

Why is systematic selection of generalized scenarios an improvement over the current approach? The main advantage is that we have actually described the most detrimental outcome, once we have agreed on the risk factors, the plausibility threshold $k$ and the payoff function according to which we rank payoffs without discarding outcomes that are at least equally plausible. The discussion about how to set $k$ will raise the awareness about the plausibility of stress scenarios we want to concentrate on. An advantage that comes as a consequence is that the process of scenario selection will not only be more systematic, it will also become less prone to political influence. We still need a process that discusses where to set the plausibility threshold $k$. But once this decision is taken there will be no further meddling with the scenarios. Finally, thinking in generalized scenarios will allow for a consistent and easy joint treatment of market and credit risk as well as their interaction at a common time horizon.

# 4 Taking into account systemic risk: Improving loss assessment

Systemic risk stems from the interaction of banks. When interactions are taken into account, losses are often substantially amplified as compared to an analysis based exclusively on exogenous sources of risk. In his analysis of the great financial crisis Hellwig [2009] describes the repeated (huge) errors made by the authorities in estimating the losses that might be faced as the crisis unfolded. These errors had their root in underestimating the impacts of system wide deleveraging by banks.

For incorporating such amplifying effects on losses in a tractable, empirical way we would like to draw on recent work by Cont and Schaanning [2016][2] who analyse the deleveraging dynamics of a banking system in a practical yet credible way.

We believe that this kind of analysis could be integrated into an otherwise standard stress testing framework (with or without generalized scenarios) to think about how loss assumptions might be amplified by the banking system.

The main additional ingredient in a deleveraging model is to add some assumptions on bank behavior and a model of price impact. Assuming that banks have an explicitly or implicity given threshold of a maximal tolerable leverage which they need to maintain for unimpaired operations, we can ask for each exogenous loss whether this could force the bank to sell some part of its marketable assets to restore its acceptable leverage. Assuming that asset sales will occur proportionally accross different marketable asset classes, we can draw on models of price impact from the market micro

[2]See also Duarte and Eisenbach [2013] and Greenwood et al. [2015]. We would also like to point out the paper by Braouezec and Wagalath [forthcoming], who analyse the single bank case.

structure literature (see Kyle [1985], Obizhaeva [2012], Beritsmas and Lo [1998], Almgren and Chriss [2000], Cont et al. [2014]) to gauge potential drops in the value of marketable assets which will perhaps trigger further rounds of sales.

Price impact modelling assumes for each asset class in the portfolio an impact function $\Psi$, which maps liquidation amounts $q$ into a relative price change of the asset class. The impact function is assumed to be increasing, concave and satisfying $\Psi(0) = 0$. A common specification is a linear impact function of the form

$$\Psi(q) = \frac{q}{D} \quad \text{with} \quad D = c\frac{ADV}{\sigma}$$

where $ADV$ is average daily volume of the asset traded and $\sigma$ is the volatility of the asset price. For a given liquidation horizon $\tau$, $D$ has to be multiplied by $\sqrt{\tau}$.[3] Thus the price impact in an asset class is large when average daily turnover is low, volatility is high and the liquidation horizon is short.

This concept can be used in a stress test to assess potential amplification effects of losses: When an institution has to liquidate an amount $q$ of a particular asset with a current price $S$, this will depress the asset price to $S' = S(1 - \Psi(q))$. This in turn will lower the value of all marketable assets of this class and perhaps induce further sales. Note that this does not require a fully fledged equilibrium analysis. The modeller can choose how many potential rounds of deleveraging he would like to consider. We refer for all details of the fire sale models to the paper of Cont and Schaanning [2016].

# 5 An EBA-based example

We now apply our ideas to a real world data set, using data from the EBA stress tests and other sources.

## 5.1 Systematic scenario search

We perform a systematic stress test for a subset of the EBA risk factors, namely the exposure of the 51 EBA banks to the residential Spanish and Italian real estate sectors. (This application is just an example, our methods work for other, larger subsets of risk factors.)

In our model of credit risk for residential mortgages, which is a firm value or threshold model (see McNeil et al. [2015]), we choose house price indices from the the property price statistics of the BIS[4] as risk factors. Clearly in reality house prices are not the only (often even not the decisive) risk factors for credit risk of residential exposures. A realistic model approximation to

[3] A thorough discussion of how these formulas are derived and when they can be meaningfully applied, see Cont et al. [2014].

[4] See https://www.bis.org/statistics/pp.htm

the credit risk of European and international residential exposures taking into account the institutional heterogeneity of mortgage lending, would need a separate study.

The nominal exposures of each of the 51 EBA banks to the two risk factors are from the EBA stress test results and followed by the same processing as in Cont and Schaanning [2016] in order to handle overlaps and ensure the balance sheet identities. The structure of the total balance sheet of the 51 EBA banks is given in Table 1.

The data allow for the construction of a stylised balance sheet for each of the 51 banks included in the EBA sample. Assets are split in assets that can be sold on markets, like sovereign and corporate bonds and assets that are not (or not easily) marketable, like loans. The distinction between non-marketable and marketable assets plays an important role in fire sales situations. Fig. 7 shows the share of marketable assets in total assets across the 51 EBA banks. For each bank the rest of the total assets are non-marketable.

Within these categories assets with exposure to a particular country are considered as one asset class. So we have, for instance, for each of the 51 banks residential exposures in Austria, Belgium etc. and in the same way for all other assets. In this way we construct in total 122 asset classes in marketable assets, covering country exposures around the world. These are in our case corporate and government bonds. We have 123 non-marketable asset classes, residential lending to households and loans to enterprises. In addition we add a residual position of exposures to the non-marketable assets. The numbers shown in Table 1 are the numbers from the EBA data, where the residual category is added to the non-marketable assets. The numbers show the aggregates across all asset classes and across all banks.[5]

**Risk factors and portfolio valuation function** In our numerical example we focus on a systematic stress of $N = 2$ sectors, namely the Spanish and Italian residential property sectors. So for the two risk factors we choose $r_{ES}$, the Spanish residential property price index of the BIS, and $r_{IT}$ the corresponding index for Italy.

For the EBA stress scenario, and for the worst case scenario at the same plausibility level as the EBA stress scenario, the estimated probabilities of default of bank exposures in the two sectors (denoted further down by $p_{in}$) are backed out from the EBA data on loan portfolio impairments: Given the nominal exposure of each bank in the two sectors, an estimated probability of default can be backed out from the impairment assuming a loss given default ratio $l = 45\%$. The results are displayed in Fig. 4: For all banks and

[5] The EBA data contain some overlaps which prevent that individual positions add up exactly to the reported totals. This requires some adjustments and therefore lead to a residual position to guarantee balance sheet identity. Our adjustments follow the example of Cont and Schaanning [2016].

| Aggregate balance sheet of the 51 EBA banks | | | |
|---|---|---|---|
| Assets | | Liabilities | |
| **Maketable assets** | €7270 Billion | **Debt** | €24951 Billion |
| - Corporate bonds AA | | | |
| ⋮ | | | |
| - Corporate bonds ZZ | | | |
| - Sovereign bonds AA | | | |
| ⋮ | | | |
| - Sovereign bonds ZZ | | | |
| **Non-marketable assets** | €19049 Billion | | |
| - Residential exposures AA | | | |
| ⋮ | | | |
| - Residential exposures ZZ | | | |
| - Commercial exposures AA | | | |
| ⋮ | | | |
| - Commercial exposures ZZ | | **Equity** | €1368 Billion |
| Total | €26319 Billion | Total | €26319 Billion |

**Table 1:** A stylized bank balance sheet. Marketable assets can be valued using market prices, non-marketable assets can be valued only indirectly using an actuarial model. The numbers are aggregates over all 51 banks from the EBA data.

for both sectors, PDs in the worst case scenario are higher than PDs in the EBA stress scenario.

The loan of bank $i$ to sector $n$, whose face value is denoted by $f_{in}$, defaults if his risk factor $r_n$ drops below a default threshold $K_{in} = \Phi_n^{-1}(p_{in})$, where $\Phi_n^{-1}$ is the inverse of the marginal distribution of $r_n$ derived by integrating the density of $N(\mu, \Sigma)$ over the variables $r_1, \ldots, r_{n-1}, r_{n+1}, \ldots, r_N$. For the two sectors the estimates of $\Sigma$ and $\mu$ from the appropriate BIS time series are $\Sigma = ((16.71, 3.20), (3.20, 2.23))$ and $\mu = (241.3, 155.1)$. The time window we used was 1971Q1–2017Q2.

The final payoff to bank $i$ is

$$X_i(r_1, \ldots, r_N) = \sum_{n=1}^{N} f_{in} \left(1 - l1_{(-\infty, K_{in})}(r_n)\right). \tag{9}$$

The final value of loans given by the total banking system is

$$X(r_1, \ldots, r_N) = \sum_{i=1}^{I} X_i(r_1, \ldots, r_N). \tag{10}$$

The final value of all loans given by the total banking system is the objective function which determines the worst case distribution for the system. Formally it would be possible to take other objective functions, as for example the number of banks defaulting. (Although it makes a difference whether a small bank defaults or a large one.) Alternatively, one could take as objective function the asset value destroyed by fire sales. (But this reflects only losses in marketable securities.) Or, one could take as objective function the total value of the banking system after fire sales. This would be conceptually most gratifying, but the solution is a mathematical challenge.

To get the worst expected value of the loan portfolio at a plausibility level $k$, enter $X(r_1, \ldots, r_N)$ of (10) into (3) to get the $G$-function, solve equation (3) to get $\overline{\theta_2}$. The worst expected value of the banking system is $G'(\overline{\theta_2})$, and the density of the worst case distribution of the risk factors $r_1, \ldots, r_N$ is given by (4).

**EBA stress scenario versus Worst case stress scenario** Fig. 4 shows the probabilities of default for residential exposures in Spain (left) and residential exposures in Italy (right) for all 51 EBA banks. The PDs in the EBA stress scenario are shown as red dots, the PDs in the Worst Case scenario of equal plausibility are shown in blue dots. For all banks and for both sectors, PDs in the worst case scenario are higher than PDs in the EBA stress scenario, although the two scenarios have the same plausibility with respect to the EBA baseline scenario. (The plausibility of the EBA stress scenario with respect to the EBA baseline scenario equals $k = 0.04$. The procedure of this calculation is described in Appendix A.

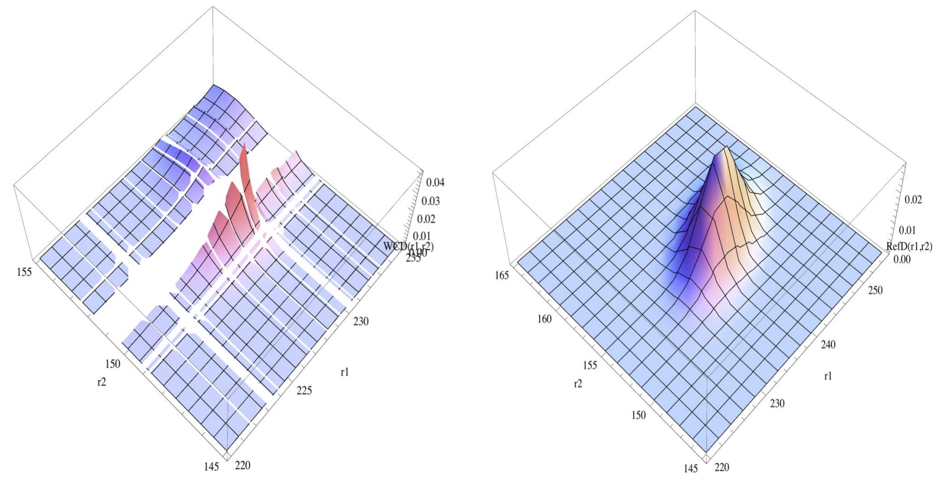


**Figure 3:** Worst case distribution (left) and presently estimated distribution (right) of the system loan portfolio. Sector payment abilities $r_1, r_2$ below the default thresholds $K_{in}$ get higher weight than in the normal presently estimated distribution.

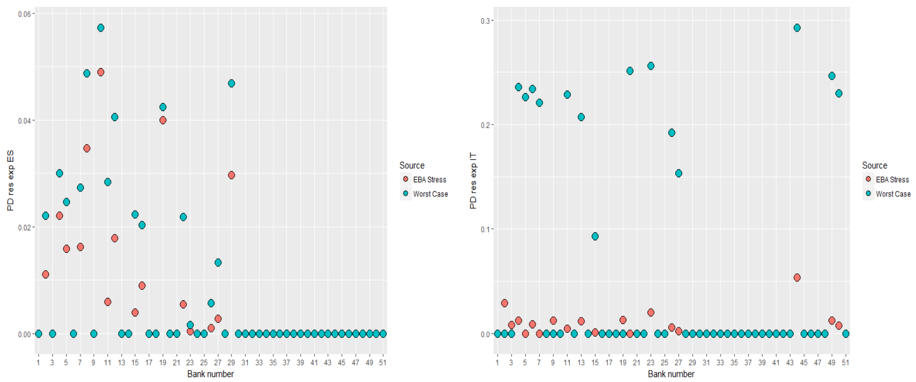


**Figure 4:** Probabilities of default for residential exposures in Spain (left) and residential exposures in Italy (right) for all 51 EBA banks. The PDs in the EBA stress scenario are shown as red dots, the PDs in the Worst Case scenario of equal plausibility are shown in blue dots. For all banks and for both sectors, PDs in the worst case scenario are higher than PDs in the EBA stress scenario.

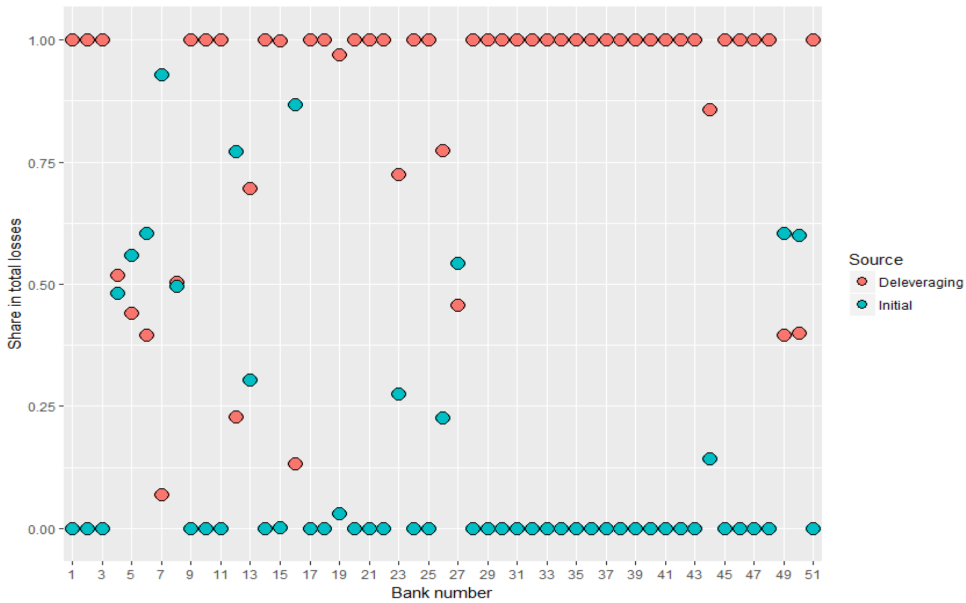


**Figure 5:** Shares of losses that come from the initial shock and from subsequent fire sales, for each of the 51 EBA banks. We see that fire sale losses are potentially important.

## 5.2 Fire sales

In the last part we ask whether the addition of fire sales would significantly amplify the losses in the EBA-scenario as well as in the worst case scenario. While the example is very stylised and not particularly realistic we hope that it illustrates how to work with generalized scenarios and to give an impression about the differences in quantitative effects we get from traditional versus worst case scenario search. A first impression of the potential importance of deleveraging is conveyed by Fig. 5.

**Initial Exposures** To get an impression of the data we look in Fig. 6 at the distribution of leverage among the 51 banks in the EBA dataset. Leverage is defined here as the ratio of the value of total (marketable and non-marketable assets over Tier 1 capital.)

The leverage distribution shows that on average banks in the sample are at around a value of Tier 1 capital of about 5% of total assets, with a few banks with worse and quite a number of banks with a significantly better capitalisation.

Figure 7 shows the distribution of the aggregate values over asset classes of marketable and non-marketable assets as percentage of total assets across the 51 banks. This shows that in terms of asset classes, lending to enterprises and households (the main non-marketable assets) is the most important

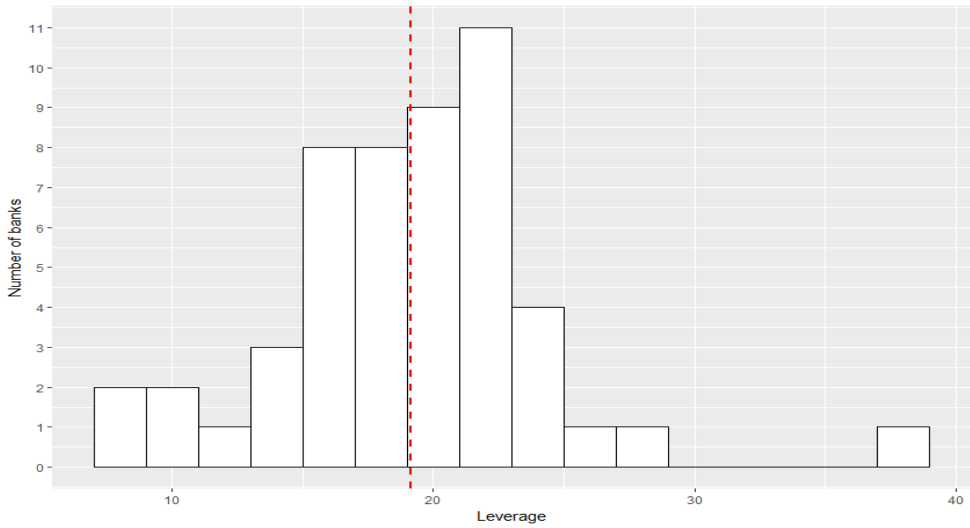


**Figure 6:** Distribution of Leverage across the 51 Banks in the EBA sample. The average leverage ratio is indicated by a dotted red line. It is near a value of 20 or 5 % Tier 1 capital.

business line. Holdings of marketable assets are for most banks substantially lower.

**Deleveraging and Loss Amplification** We finally ask, whether losses coming from expected defaults on residential exposures in Spain and Italy might induce deleveraging and price impacts. The answer of course depends on a number of assumptions about bank deleveraging behavior and the corresponding price impact. We use the Cont Schaanning model (Cont and Schaanning [2016]) and their data on average daily volumes of government and corporate bonds as well as the volatility of their respective price indices.

If the maximum leverage a bank is willing to tolerate before selling any of its marketable assets is high enough, there will be no deleveraging. In Cont and Schaanning [2016] the maximum leverage ratio is assumed to be 33 and a target ratio of 32, which corresponds to a Tier 1 capital of about 3%. At this critical level, there is one bank in the sample which would even without a shock sell a certain amount of its marketable assets to come back to target. Otherwise in this parametrisation, neither the EBA stress scenario nor the worst case scenario would kick off a deleveraging process that reaches beyond the bank which was initially over-leveraged.

When the maximum leverage is a bit tighter, say at 4% of Tier 1 capital, and the target leverage is set to 95% of the maximum leverage, the effects would look differently. At this tighter target, fire sales would occur for more than ten banks. This can be seen in Fig. 8.

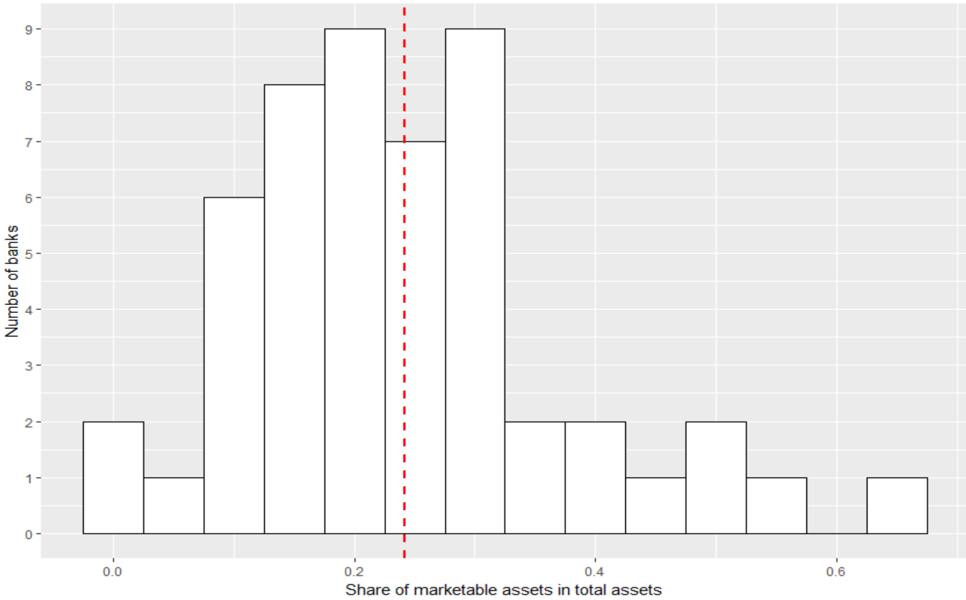


**Figure 7:** Share of marketable assets in total assets across the 51 EBA banks. For each bank the rest of the total assets are non-marketable.

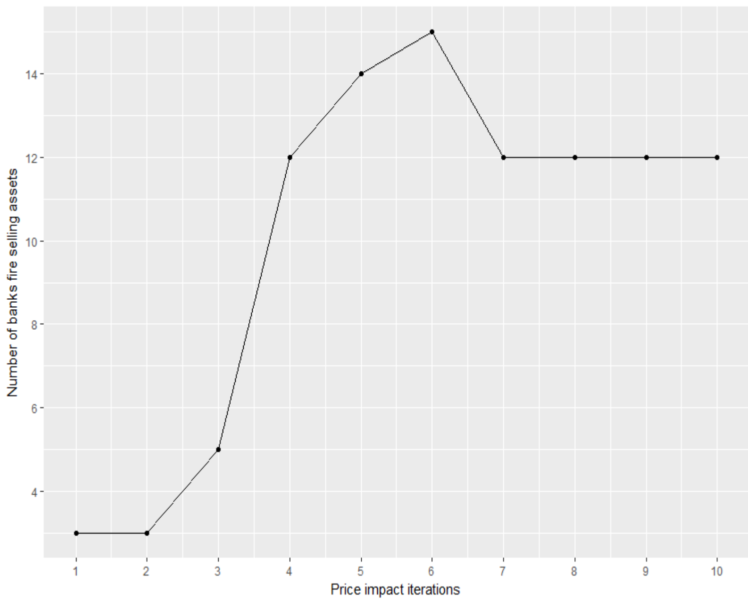


**Figure 8:** Looking at 10 rounds of deleveraging we see that initially there is a fire sale by 3 banks, which rises to 5 banks in round 3 and then even further to 12 and 15 banks. Then some banks manage to stabilise while 12 banks even after 10 rounds of deleveraging have not managed to restore their target.

When we look at losses it matters a great deal whether we look at potential deleveraging spirals or not. In Fig. 5 we compare the shares of total losses from the initial stress scenario plus the potential deleveraging process and compare the share these losses have in total losses.

We see that, if a deleveraging process occurs, the share in losses from deleveraging is for most banks higher than losses from the initial shock. Of course this is conditional on the fact that deleveraging occurs in the first place. We saw that with a leverage threshold of 25 we can get already significant deleveraging losses. Taking fire selling into account is potentially important for the evaluation of losses.

To get an impression of what deleveraging does to the price indices of the 122 marketable asset classes considered in our example we look at a histogram of changes (drops) in the Index value. This is shown in Fig. 9.

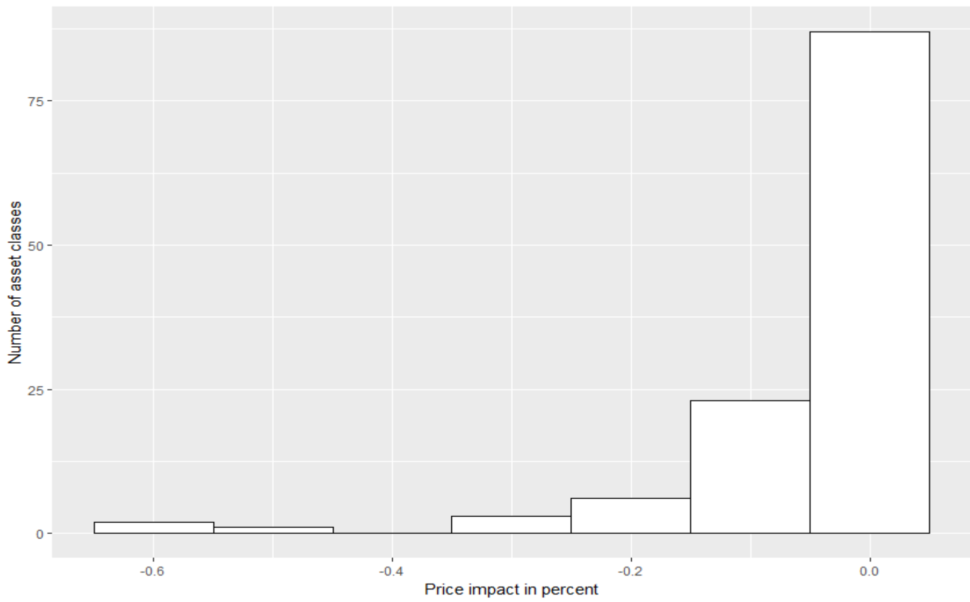


**Figure 9:** Price changes in marketable asset classes triggered by deleveraging, . Most asset classes show little price changes, while there are a few with extreme drops.

Overall our example suggests that the analysis of deleveraging or second round effects might be very important in the assessment of losses. The question of which is the most useful and most credible quantitative model of deleveraging is at the moment still very much open. The price impact model we use here draws on the market micro structure literature, where it was mainly used in empirical analysis of order book dynamics. Under which exact circumstances these methods fit very well to the context of a stress test needs further research.

## 6 Conclusions

The methodology of current stress testing is problematic both in the way stress scenarios are chosen and in the way bank losses are evaluated. While most research resources in stress testing over the last ten years have been invested in constructing very detailed models of portfolio loss functions, and in constructing statistical-econometric models that translate very broadly formulated macroeconomic scenarios to the actual risk parameters of portfolios, scenario selection and inclusion of systemic risk has been somewhat neglected.

Our paper calls for changing the focus in stress testing on systematic scenario selection and on the consideration of loss amplification by systemic risk. The reason is that otherwise, stress tests will be weak in answering the key questions: "Which scenarios lead to big losses?" and "How big are the worst losses?".

Specifically we propose to work with generalized scenarios. We argue that it is useful to think about scenarios as distributions rather than realisations. This allows for an integrated analysis of market and credit risk at a common time horizon. Systematic scenario selection is achieved by an appropriate form of worst case search over plausibility domains. This method finds among all equally plausible scenarios the scenario leading to the worst expected loss for any given portfolio.

For making stress tests more systemic, we propose an integration of recent results on the quantitative modelling of deleveraging processes with our approach to systematic scenario search. Rather than going for a fully fledged equilibrium analysis and sophisticated behavioural modelling, we advocate an approach that makes use of describing the very limited options a bank has in distress at a short time horizon plus tools from market micro structure literature to assess price impact.

We believe that our proposals can be implemented in a fairly straightforward way without drawing on exotic or new data sources. Our example gives ideas about how such an approach might work in a more or less traditional top down stress testing setup. We hope that our ideas provide foundations for future stress tests to become more systematic and systemic.

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# A How plausible is the EBA stress scenario relative to the EBA baseline scenario?

For banks $i = 1, \dots, I$ and sectors $n = 1, \dots N$ the exposures $f_{in}$ are given. Also for the EBA baseline scenario (call it $Q$) and the EBA stress scenario (call it $P$) we are given the PDs of the exposure of bank $i$ in sector $n$: $pd_{in}^Q, pd_{in}^P$. Also given are time series for the sector values.

1. For both scenarios $P, Q$, construct the marginals of $N$-dimensional distributions described by the differences $q_i^n, p_i^n$ of ordered PDs (so that $\sum_{i=0}^I p_i^n = 1$, $\sum_{i=0}^I q_i^n = 1$ for all $n$).

2. For the baseline scenario $Q$ calculate the $N$-dimensional distribution $Q^*$ from the given marginals $q^n$ by taking the Maximum Entropy distribution with respect to the given marginals, see Cover and Thomas [2006, Chap. 12].

3. For the stress scenario $P$, from the marginals $p^n$ calculate the $N$-dimensional distribution $P^*$ having minimal relative entropy with respect to $Q^*$.

4. We take the plausibility of the EBA stress scenario with respect to the EBA baseline scenario to be the relative entropy $D(P^*||Q^*)$.

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