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Dating and forecasting turning points by Bayesian clustering with dynamic structure: A suggestion with an application to Austrian data.

Sylvia Kaufmann

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Editorial

The information contained in a large panel data set is used to date historical turning points of the Austrian business cycle and to forecast future ones. The author estimates groups of series with similar time series dynamics and link the groups with a dynamic structure. The dynamic structure identifies a group of leading and a group of coincident series. Robust results across data vintages are obtained when series specific information is incorporated in the design of the prior group probability distribution. The results are consistent with common expectations, in particular the group of leading series includes Austrian confidence indicators and survey data, German survey indicators, some trade data, and, interestingly, the Austrian and the German stock market indices. The forecast evaluation confirms that the Markov switching panel with dynamic structure performs well when compared to other specifications.

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Dating and forecasting turning points by Bayesian clustering with dynamic structure: A suggestion with an application to Austrian data.

Sylvia Kaufmann*†

June 2008

Abstract

The information contained in a large panel data set is used to date historical turning points of the Austrian business cycle and to forecast future ones. We estimate groups of series with similar time series dynamics and link the groups with a dynamic structure. The dynamic structure identifies a group of leading and a group of coincident series. Robust results across data vintages are obtained when series specific information is incorporated in the design of the prior group probability distribution. The results are consistent with common expectations, in particular the group of leading series includes Austrian confidence indicators and survey data, German survey indicators, some trade data, and, interestingly, the Austrian and the German stock market indices. The forecast evaluation confirms that the Markov switching panel with dynamic structure performs well when compared to other specifications.

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*Oesterreichische Nationalbank, Economic Studies Division, P.O. Box 61, A-1010 Vienna, e-mail sylvia.kaufmann@oenb.co.at, phone +431 40420-7222, fax +431 40420-7299.

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1 Introduction

The model suggested in the present paper allows to identify business cycle turning points by using the information contained in a large set of economic, real and financial, variables. The information about the cyclical stance is extracted by estimating groups of series, i.e. by classifying those series together, that follow a similar time series process over time. Within each group, we allow for group-specific parameter heterogeneity. This means that the parameters are shrunk towards a group-specific mean rather than pooled, which is usually done in panel estimation. Two groups of series are additionally linked by a dynamic structure, whereby one group leads the other one in the business cycle. If the dynamic structure between the groups is not known a priori, that is if the leading and the coincident group of series are not known, then the model can be generalized to estimate the appropriate dynamic structure of the groups in the panel. Obviously, not all series can be classified into one of both the coincident and the leading group of variables. Therefore, the remaining group, which collects all the series not following the coincident or the leading group of series, is moving independently from the other two groups.

The methodological approach pursued in the paper is based on the idea of model-based clustering of multiple time series (Frühwirth-Schnatter and Kaufmann, 2008). An extension is introduced here that links two groups in the panel by a dynamic structure. How to form the groups and which groups are linked by the dynamic structure may be subject to estimation. The series are transformed to non-trending series, and demeaned and standardized before the analysis. Therefore, the estimation yields an inference on growth cycles. The growth cycle itself is modelled by a process which identifies periods of above-average and below-average growth. These periods usually cannot be identified a priori with certainty, therefore we introduce group-specific unobservable state indicators which follow a first-order Markov process.

Recently, research on the euro area business cycle has intensified. The areas which numerous papers deal with are dating business cycle turning points, assessing the current stance of the business cycle, forecasting the cycle itself and the probability of turning points as well. The issues also relate to defining an appropriate group of leading indicators which permits a timely assessment of the current business cycle state and an accurate forecast of turning points (see Marcellino, 2006, for an overview). The model of the present paper is related to Bengoechea and Pérez-Quirós (2004) who estimate a bivariate Markov switching model for the euro area industrial production index and the industrial confidence indicator. With the so-called filter probabilities of the state indicator, which reflect the state probability in period t given the information up to period t, they assess the current state of the euro area business cycle and form a forecast on the probability of a turning point. While they base the inference on a model for two aggregate variables, here the cyclical stance is extracted from the information contained in a large cross-section of economic series. Moreover, while they are modelling the state indicators of each series as switching either independently or jointly, here, the state indicator of the leading group switches before the state indicator of the coincident group.

A closely related paper to capture the dynamic structure of time series is Paap, Segers, and van Dijk (2007). The series-specific state indicators of a bivariate Markov switching

VAR model for US industrial production and the Conference Board's Composite Index of Leading Indicators (CLI) are linked together by allowing asynchronous cycles with different lead times for peaks and troughs in the CLI. The lead times are not restricted, allowing in principal different durations of business cycle phases in both series, such that cycle phases of even different cycles may overlap. We capture the different lead times for peaks and troughs directly in the transition probability matrix of an encompassing state indicator. The transition between states of the encompassing state indicator is restricted to ensure that the coincident group of series has first to switch into the actual state of the leading group of series before the latter can switch back to the other state.

With the multi-level panel STAR model proposed in Fok, van Dijk, and Franses (2005) one can also capture common non-linear dynamics across individual series. Although the parameters driving the regime switches are series-specific, they are determined by time-invariant properties of the individual series. Thus, the model is an alternative between estimating a STAR model for each individual series and between a fully pooled model, in which the parameters governing the regime changes are equal across series. The results for US industrial production sectors confirm that regime switches occur in different periods at different frequencies. The results also suggest that some series follow similar switching processes. In the present paper, we classify series into groups and estimate group-specific state-indicators, assuming the series to be in the same phase of the business cycle.

Another method of dimension reduction is factor analysis as pursued in Forni et al. (2000) and Stock and Watson (2002). Forni et al. (2001) suggest to use dynamic principal components to extract the coincident and the leading index of euro area economic activity. From a large cross-section, they choose a set of core variables usually considered to be the most relevant to describe the business cycle stance, and include additional variables that are most correlated with this core and have only minor idiosyncratic dynamics. The common component extracted from these series allows to compute a coincident indicator for the euro area as a whole and for each individual country as well. The Austrian series they include in the core are GDP, investment, consumption and industrial production. Austrian orders is the only series additionally taken into consideration in the final estimation. Generally, all financial and monetary variables are not sufficiently correlated to the core to be included in the final estimation, and neither are the price series and the share prices. Not surprisingly, orders turn out to be strongly correlated to the common component of the core series. Finally, the country-specific comparison of turning points with the euro area aggregate reveals that Germany, and also Austria, are not leading the euro area coincident indicator.

These results are of interest as the ones reported in the present paper using recent data depart from the previous evidence. Some financial variables like M1, interest rates and, interestingly, asset prices like the Austrian ATX and the German DAX stock market indices fall into the leading group of series. Orders, Austrian and German confidence indicators and survey data as well, fall also into the leading group of variables.

Another possibility to predict turning points is suggested in Canova and Ciccarelli (2004). Based on the estimation of a Bayesian panel VAR for the G7 countries, forecasts in the growth rates of GNP are used to predict turning points and the probability of turning points. In principle, one could use the approach for a single country and form several

VARs for related series in the panel, like business surveys, labor market series, trade series etc. Nevertheless, panel VARs appear most attractive to capture cross-country or country-specific inter-industry interdependencies. Our data set does not include many foreign variables, nor are the included series very disaggregate. Therefore, we use the "basic" panel approach described in the following section.

Finally, a very specific approach is described in Bruno and Lupi (2004). Using early released reliable indicators, specifically a business survey series on future production prospects and the quantity of goods transported by railways, the authors specify a parsimonious forecasting model to produce a forecast of actual industrial production which then is used in an unobserved components model to assess the actual stance of the business cycle. In the present paper, however, we want to exploit the information of many series, of which many are also timely released, to form an expectation about turning points. Missing data on actual industrial production or other national account series can be handled as missing values and replaced by an estimate given the information we have on other timely released series (see also the appendix in Kaufmann and Kugler, 2007).

The paper is organized as follows. Section 2 introduces the model, section 3 outlines the estimation procedure. The data and the results are summarized in section 4. Section 5 shows the effectiveness of using informative prior group probabilities in obtaining robust results across data vintages. Section 6 evaluates the in-sample and out-of-sample one-step-ahead forecast performance of the model and compares it to other related model specifications. Section 7 concludes.

2 The econometric model

2.1 The group-specific time series model

In business cycle studies analyzing large cross-section of time series with different trend and volatility levels (Forni et al., 2000, Stock and Watson, 2002) it is usual to work with detrended, demeaned and standardized series. Therefore, let y_{it} represent the mean-adjusted and standardized growth rate or change of time series i, $i = 1, \dots, N$ in period t, $t = 1, \dots, T$ in a panel of economic variables. Each time series is assumed to follow an autoregressive process with switching intercept:

$$y_{it} = \mu^i_{I_{it}} + \phi^i_1 y_{i,t-1} + \dots + \phi^i_p y_{i,t-p} + \varepsilon_{it}, \tag{1}$$

where $\varepsilon_{it} \sim$ i.i.d$N(0, \sigma^2/\lambda_i)$. The unobservable state indicator I_{it} takes on the value 1 or 2 and indicates the switches between periods of above- and below-average conditional growth rates:

$$\mu^i_{I_{it}} = \begin{cases} \mu^i_1 & \text{if} \quad I_{it} = 1 \\ \mu^i_2 & \text{if} \quad I_{it} = 2 \end{cases} . \tag{2}$$

As the purpose is to estimate business cycle turning points, we define state 1 as periods of below-average growth and state 2 as periods of above-average growth. Thus $\mu^i_1 \leq 0$ and $\mu^i_2 > 0$. The autoregressive process is assumed to lie in the stationarity area, such that it also follows $\mu^i_1/(1 - \sum_{j=1}^p \phi^i_p) \leq 0$ and $\mu^i_2/(1 - \sum_{j=1}^p \phi^i_p) > 0$

The superscript i is used to denote that each time series, in principle, can follow an independent process. However, efficiency gains in estimation might be exploited by grouping the series which follow a similar time series process (see e.g. Hoogstrate et al., 2000, and Frühwirth-Schnatter and Kaufmann, 2008). The difficulty is to form the appropriate groups of series. If we do not have a priori certain information, we may estimate the appropriate grouping of the series. To this aim, an additional latent indicator, a group indicator, S_i, $i = 1, \dots, N$, is defined which relates to group-specific parameters. If we assume to have K distinct groups of series in the panel, S_i takes one out of K different values, $S_i = k$, $k = 1, \dots, K$, and indicates into which group a specific time series is classified.

There are various possibilities to specify the group-specific model. In each group, we may pool the time series and restrict the parameters to be identical for all time series. To be less restrictive, we may assume that in each group, the series-specific parameters are shrunk towards a group-specific mean, allowing for group-specific parameter heterogeneity:

$$\begin{aligned} \mu^i_{I_{S_i,t}} &\sim \begin{cases} \mathcal{N}\left(\mu^k_1, q^k_1\right) & \text{if} \quad S_i = k \text{ and } I_{kt} = 1 \\ \mathcal{N}\left(\mu^k_2, q^k_2\right) & \text{if} \quad S_i = k \text{ and } I_{kt} = 2 \end{cases} \\ \left(\phi^i_1, \dots, \phi^i_p\right) &\sim \mathcal{N}\left(\phi^k, Q^k_\phi\right) \text{ if } S_i = k, \ k = 1, \dots K \end{aligned} \tag{3}$$

Generally, the autoregressive parameters may also be modelled as state-dependent. Larger autoregressive parameters during periods of below-average growth would reflect the fact that business cycle downturns are steeper than business cycle upturns. This pattern, however, is not found in the data used for the empirical investigation. Therefore, and also for expositional convenience, in model (3) the autoregressive parameters are modelled only group-specific.

2.2 The model for the state and the group indicators

Both latent indicators are discrete variables with different distributional assumptions. The model for each group-specific state indicator I_{kt} takes into account that the duration of above-average growth periods may differ from periods of below-average growth. We specify I_{kt} to follow a Markov switching process of order one, $P(I_{kt} = l | I_{k,t-1} = j) = \xi^k_{jl}$, $j, l = 1, 2$, with the restriction $\sum_{l=1}^2 \xi^k_{jl} = 1$, $j = 1, 2$. Thus, for a single time series, the model comes close to the one estimated in Hamilton (1989) for US GNP. The Markov switching specification is also appropriate to capture business cycle turning points in real time. Recent evidence in Chauvet and Piger (2008) shows that both the algorithm of Harding and Pagan (2006) and a small-scale dynamic Markov switching factor model are able to date NBER turning points of the US business cycle in real time.

For the group indicator we assume a multinomial logit model to include prior information on a particular series into the estimation of the group probability:

$$P(S_i = k | \gamma_1, \dots, \gamma_{K-1}, \gamma_{z1}, \dots, \gamma_{z,K-1}) = \frac{\exp(\gamma_k + Z_i \gamma_{zk})}{1 + \sum_{l=1}^{K-1} \exp(\gamma_l + Z_i \gamma_{zl})}, \tag{4}$$

where the last group K is the baseline group with $\gamma_K = \gamma_{zK} = 0$. The variable Z_i may be a vector of any series-specific features which are thought to determine the classification into a specific group, and the parameters $\gamma_{\cdot}$ are unknown but group-specific values. In the empirical application we will use correlation with GDP and with order books in the industrial sector as variables determining the prior group probability when specifying an informative prior for the model. Z_i may also be a set of dummy variables reflecting one's belief whether a variable like GDP, its components or industrial production pertain to the group of coincident series or whether a variable like a survey or a confidence indicator should fall into the group of leading series.

If no series-specific information is available, the (prior) group probabilities are constant across series and are assumed to be equal to the relative size of the groups:

$$P(S_i = k|\gamma_1, \ldots, \gamma_{K-1}) = \frac{\exp(\gamma_k)}{1 + \sum_{l=1}^{K-1} \exp(\gamma_l)} = \eta^k. \tag{5}$$

2.3 Modelling a leading group of time series

The last ingredient of the model is the dynamical structure between two groups of series. Assume that the second group of series, i.e. that all series for which $S_i = 2$, are leading the cycle of the coincident series, which, let us again assume, are all series for which $S_i = 1$. Then, we define the encompassing state variable I_t^* which captures all $J^* = 4$ possible constellations of both state indicators 1 and 2 in period t (see also Phillips, 1991):

$$\begin{aligned} I_t^* = 1 \quad &:= \quad (I_{1t} = 1,\ I_{2t} = 1) \\ I_t^* = 2 \quad &:= \quad (I_{1t} = 1,\ I_{2t} = 2) \\ I_t^* = 3 \quad &:= \quad (I_{1t} = 2,\ I_{2t} = 1) \\ I_t^* = 4 \quad &:= \quad (I_{1t} = 2,\ I_{2t} = 2). \end{aligned} \tag{6}$$

If the state indicator of group 2 is leading the state indicator of group 1,[1] eight of the 16 elements of the transition distribution of I_t^* will in fact be restricted to zero:

$$\xi^* = \begin{bmatrix} \xi_{11}^* & \xi_{12}^* & 0 & 0 \\ 0 & \xi_{22}^* & 0 & \xi_{24}^* \\ \xi_{31}^* & 0 & \xi_{33}^* & 0 \\ 0 & 0 & \xi_{43}^* & \xi_{44}^* \end{bmatrix} \tag{7}$$

If state 1 identifies periods of below-average growth, $1/(1 - \xi_{22}^*)$ will be the expected lead of group 2 out of a trough, and, correspondingly, $1/(1 - \xi_{33}^*)$ the expected lead in reaching a peak.

From (6) we can recover the group-specific indicators I_{1t} and I_{2t}. The group-specific transition probabilities implied by (7) are $\xi_{11}^1 = (1 + \xi_{22}^*)/2$ and $\xi_{22}^1 = (1 + \xi_{33}^*)/2$ for I_{1t}, and $\xi_{11}^2 = (1 + \xi_{11}^*)/2$ and $\xi_{22}^2 = (1 + \xi_{44}^*)/2$ for I_{2t}.

[1] The leading behavior of state 2 is modelled in a strict form in the sense that a switch in the state indicator of group 2 will be followed by a switch in the state indicator of group 1 before the state indicator of group 2 may switch back to the initial state.

2.4 Unknown dynamic structure

For expositional convenience we assumed so far that group 2 is leading group 1, while the remaining $K-2$ groups would behave independently over time. If there is uncertainty about which two groups may be linked by a dynamic structure, we can generalize the model by introducing a dynamic structure indicator ρ^*, which characterizes the dynamic structure between the groups. The indicator ρ^* takes on one realization ρ_l of the $L = K(K-1)$ possible permutations of $\{1, 2, 0_{K-2}\}$.[2] The element in ρ^* which takes the value 1 refers to the group of coincident series, the element which takes the value 2 refers to the leading group, and all other elements refer to the groups that behave independently. If we have no a priori knowledge on the dynamic structure between groups, each permutation is given a priori equal weight $\eta_\rho = 1/(K(K-1))$.

3 Bayesian estimation and forecasts

The following notation is adopted to describe the Bayesian estimation method in a convenient way. While y_{it} denotes observation t for time series i, y_i^t gathers all observations of time series i up to period t, $y_i^t = \{y_{it}, y_{i,t-1}, \ldots, y_{i1}\}$, $i = 1, \ldots, N$. The variables Y_t and Y^t denote the observations in and up to period t of all time series, respectively, $Y_t = \{y_{1t}, y_{2,t}, \ldots, y_{Nt}\}$ and $Y^t = \{Y_t, Y_{t-1}, \ldots, Y_1\}$. Likewise, the vectors $S^N = (S_1, \ldots, S_N)$ and $I^T = (I_1^T, \ldots, I_K^T)$, where $I_k^T = (I_{kT}, I_{k,T-1}, \ldots, I_{k1})$, $k = 1, \ldots, K$, and $\lambda^N = (\lambda_1, \ldots, \lambda_N)$ collect the group and the state indicators and the series-specific weights, respectively. Finally, all model parameters are gathered in θ.[3] By using Markov chain Monte Carlo simulation methods we obtain a posterior inference on the augmented parameter vector $\psi = (\theta, S^N, I^T, \lambda^N, \rho^*)$ which additionally includes the two latent indicators, the weights and, if also estimated, the dynamic structure variable ρ^*.

3.1 MCMC estimation

The posterior distribution $\pi(\psi|Y^T)$ is obtained by updating the prior distribution $\pi(\psi)$ with the information given in the data by the likelihood $L(Y^T|\psi)$:

$$\pi(\psi|Y^T) \propto L(Y^T|\psi)\pi(\psi). \tag{8}$$

[2] The vector 0_{K-2} denotes a vector of $K-2$ zeros.

[3] That is: $\theta = (\mu_1^1, \mu_2^1, \ldots, \mu_1^K, \mu_2^K, \phi^1, \ldots, \phi^K, Q^1, \ldots, Q^K, \sigma^2, \xi^*, \xi^{\rho^*(k)=0}, \gamma, \gamma_z)$, where Q^k includes all within-group heterogeneity,

$$Q^k = \begin{bmatrix} q_1^k & 0 & \mathbf{0} \\ 0 & q_2^k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Q_\phi^k \end{bmatrix}.$$

$\xi^{\rho^*(k)=0} = \{\xi^k\}$ has as elements the transition probabilities of the independent group-specific state indicators, $\xi^k = (\xi_{11}^k, \xi_{12}^k, \xi_{21}^k, \xi_{22}^k)$. The last two vectors include $\gamma = (\gamma_1, \ldots, \gamma_{K-1})$, $\gamma_z = (\gamma_{z1}, \ldots, \gamma_{z,K-1})$.

Conditional on S^N and I^T, the likelihood $L(Y^T|\psi)$ can be factorized as

$$L(Y^T|\psi) = \prod_{t=p+1}^{T} \prod_{i=1}^{N} f(y_{it}|y_i^{t-1}, \mu_{I_{S_i t}}^{S_i}, \phi^{S_i}, Q^{S_i}, \lambda_i, \sigma^2), \tag{9}$$

where $f(y_{it}|\cdot)$ denotes the density of the normal distribution:

$$f(y_{it}|y_i^{t-1}, \mu_{I_{S_i t}}^{S_i}, \phi^{S_i}, Q^{S_i}, \lambda_i, \sigma^2) = \\ \frac{1}{\sqrt{2\pi\nu_{it}^{S_i}}} \exp\left\{ -\frac{1}{2\nu_{it}^{S_i}} \left(y_{it} - \mu_{I_{S_i t}}^{S_i} - \sum_{j=1}^{p} \phi_j^{S_i} y_{i,t-j} \right)^2 \right\}. \tag{10}$$

The observation density in (10) is the density for the model marginalized with respect to the random effects:

$$\begin{aligned} y_{it} &= X_{it}^{S_i}\beta^{S_i} + \varepsilon_{it}^*, \qquad \varepsilon_{it}^* \sim N\left(0, \nu_{it}^{S_i}\right) \\ \nu_{it}^{S_i} &= X_{it}^{S_i} Q^{S_i} X_{it}^{S_i\prime} + \sigma^2/\lambda_i \end{aligned} \tag{11}$$

where $X_{it}^{S_i} = \left(D_{1t}^{I(S_i)}, D_{2t}^{I(S_i)}, y_{i,t-1}, \ldots, y_{i,t-p}\right)$ with $D_{jt}^{I(S_i)} = 1$ if $I_{S_i,t} = j$ and $D_{jt}^{I(S_i)} = 0$ otherwise, $j = 1, 2$. The vector of coefficients is $\beta^{S_i} = \left(\mu_1^{S_i}, \mu_2^{S_i}, \phi^{S_i}\right)$.

If the dynamic structure ρ^* is known, the prior distribution $\pi(\psi)$ is designed in way which assumes that the encompassing state indicator I^{*T}, the remaining $K-2$ state indicators I_k^T, the group indicator S^N and the weights λ^N are independent of each other and independent of the model parameters θ:

$$\pi(\psi) = \pi(I^{*T}|\rho^*, \xi^*) \prod_{\rho^*(k)=0} \pi(I_k^T|\rho^*, \xi^k)\pi(S^N|\gamma, \gamma_z, Z^N)\pi(\lambda^N)\pi(\theta), \tag{12}$$

with known densities for $\pi(I^{*T}|\rho^*, \xi^*)$, $\pi(I_k^T|\rho^*, \xi^k)$ and $\pi(S^N|\gamma, \gamma_z, Z^N)$, respectively. The weights are independent a priori and follow a Gamma distribution. To specify $\pi(\theta)$, the parameter vector θ is further broken down into parameter blocks, for all of which we assume standard prior distributions (see appendix A).

The sampling scheme to draw from the posterior $\pi(\psi|Y^T)$ follows Frühwirth-Schnatter and Kaufmann (2008) and involves the following steps (see appendix B for the derivation of the posterior distributions):

(i) $\pi(S^N|Y^T, I^T, \rho^*, \lambda^N, \theta)$,
(ii) $\pi(I^T|Y^T, S^N, \rho^*, \lambda^N, \theta)$,
(iii) $\pi(\lambda^N|Y^T, I^T, S^N, \theta)$,
(iv) $\pi(\theta|Y^T, S^N, I^T)$.

In step (i), the group indicator can be sampled individually for each time series. Given the dynamical structure ρ^*, we obtain a draw for the group-specific state indicators in step (ii) by multi-move sampling, using in particular the encompassing specification I^* for the state indicators of the coincident and the leading group. In step (iii) the series-specific weights are sampled independently from Gamma distributions. All posterior distributions in (iv) are conjugate to the priors, except for the posterior distribution of γ and γ_z, the parameters influencing the group probabilities. These posterior distributions are not of closed form and therefore, a Metropolis-Hastings step is used to sample them (see Albert and Chib, 1993 and Scott, 2006). In order to sample the covariance matrices Q, step (iv) additionally involves sampling of the series-specific random effects $\beta^i - \beta^{S_i}$ from a normal distribution or using the filter form derived in Frühwirth-Schnatter (2006, p. 266). Note however, that the extension to group-heterogeneity in parameters depending on the group-specific state indicator (the Markov switching mean intercept term) renders the predictor vector $X_{it}^{S_i}$ in the marginal model (11) group-specific, which is not the case for the random effects model without a latent state indicator. This has to be taken into account when deriving the moments of all posterior distributions and for the filter form. Finally, given that the prior of the transition distributions ξ^* and ξ^k is Dirichlet, the posterior simulation stems also from a Dirichlet (see also Sims, Waggoner, and Zha, 2008).

We sample from the constrained posterior which means that we restrict $\mu_1^k \leq 0$ and $\mu_2^k > 0$, $k = 1, \dots, K$ while sampling the model parameters. In step (iii), we therefore only accept draws that fulfill the latter state identifying restriction. If a priori other parameters in the time series model would be switching and we would not know with certainty with which one the states could uniquely be identified, we could apply the random permutation sampler (Frühwirth-Schnatter, 2001).

3.2 Estimating the dynamic structure

If the dynamic structure between the groups is not known a priori, we can estimate ρ^* from the data. Step (ii) of the sampling scheme described above is extended to:

(ii.a) $\pi(\rho^*|Y^T, S^N, \lambda^N, \theta)$,

(ii.b) $\pi(I^T|Y^T, S^N, \rho^*, \lambda^N, \theta)$.

If each permutation ρ_l, $l = 1, \dots, L$, out of the $L = K(K-1)$ possible ones from $\{1, 2, 0_{K-2}\}$ is given equal prior probability $\eta_\rho = 1/(K(K-1))$, the posterior distribution of the dynamic structure ρ^* is discrete:

$$\pi(\rho^* = \rho_l|Y^T, S^N, \lambda^N, \theta) \propto L(Y^T|S^N, \lambda^N, \theta, \rho_l) \cdot \eta_\rho, \tag{13}$$

for $l = 1, \dots, K(K-1)$. The marginal likelihood associated to the dynamic structure ρ_l is derived in detail in appendix C.

The issue of label switching traditionally encountered in mixture models also arises when the dynamic structure is unknown a priori. The likelihood $L(Y^T|S^N, \lambda^N, \theta, \rho^*)$ is invariant to any permutation ϕ of the groups $\{1, \dots, K\}$, associated with a dynamic structure ρ_l:

$$L(Y^T|S^N, \lambda^N, \theta, \rho^*) = L(Y^T|\phi(S^N), \lambda^N, \phi(\theta_{-\sigma^2}), \phi(\rho^*)), \tag{14}$$

where $\phi(\theta_{-\sigma^2})$ means that all group-specific parameters are re-ordered according to permutation ϕ, except σ^2 which is the only parameter being group-independent. We obtain an estimate of the unconstrained model by concluding each sweep of the sampler by a permutation step:

(v) $S^N := \phi(S^N), \theta := \phi(\theta_{-\sigma^2})$,
$I^T := \phi(I^T), \rho^* := \phi(\rho^*)$

Drawing randomly ϕ ensures that we visit all modes of the posterior distribution. To identify a unique specification, the simulations from the unconstrained distribution are then re-ordered according to a group-identifying restriction. Such restrictions can usually easily be find by means of graphical tools. A detailed description of the approach can be found in Frühwirth-Schnatter and Kaufmann (2006).

3.3 Forecasting

Given the model estimate, we may simulate recursively future paths of the encompassing state indicator between $T+1$ and $T+H$ from the posterior predictive density $\pi(I^*_{T+h}|Y^T)$, for $h = 1, \ldots, H$. We simulate the distribution recursively from

$$\pi(I^{*(m)}_{T+h}|Y^T, I^{*(m)}_{T+h-1}, \xi^{*(m)}), \tag{15}$$

which is equal to the jth column of $\xi^{*(m)\prime}$ if $I^{*(m)}_{T+h-1} = j$ at $T+h-1$. The superscript (m) indicates the mth parameter draw of the MCMC output and $I^{*(m)}_{T|T} = I^{*(m)}_T$.

We obtain a conditional probabilistic forecast $I^*_{T+h}|Y^T, I^*_T$ by averaging over the sampled values:

$$I^*_{T+h}|Y^T, I^*_T = \frac{1}{M}\sum_{m=1}^{M} I^{*(m)}_{T+h}, \quad h = 1, \ldots, H. \tag{16}$$

By relating the states of the encompassing state indicator $I^*_{T+h|T}$ to the group-specific state indicators $I_{k,T+h|T}$ according to (6), we can make a probabilistic forecast of reaching a turning point within the next h periods ahead, conditional on the state prevailing at T.

The simulated future paths for the encompassing state indicator can be used to obtain forecasts of a specific time series in the panel from the joint predictive density $\pi(y_{i,T+1}, \ldots, y_{i,T+H}|Y^T)$, with $k = S^{(m)}_i$ in the following:

$$y^{(m)}_{i,T+h|T} = \mu^{k,(m)}_{I^{(m)}_{k,T+h|T}} + \phi^{k,(m)}_1 y^{(m)}_{i,T+h-1|T} + \cdots + \phi^{k,(m)}_p y^{(m)}_{i,T+h-p|T} + \varepsilon^{*(m)}_{i,T+h|T}. \tag{17}$$

If $T+h-j \leq T$ we insert observed values $y^{(m)}_{i,T+h-j|T} = y_{i,T+h-j}$, and $\varepsilon^{*(m)}_{i,T+h|T}$ corresponds to a draw from the error distribution $N\left(0, \nu^{(m)}_{i,T+h|T}\right)$,

$$\begin{aligned}
\nu^{(m)}_{i,T+h|T} &= X^k_{i,T+h|T} Q^k X^k_{i,T+h|T}{}' + \sigma^{2(m)}/\lambda^{(m)}_i \quad \text{with} \\
X^k_{i,T+h|T} &= \left(D^{I^{(m)}(k)}_{1,T+h|T}, D^{I^{(m)}(k)}_{2,T+h|T}, y^{(m)}_{i,T+h-1|T}, \ldots, y^{(m)}_{i,T+h-p|T}\right),
\end{aligned}$$

and $D^{I^{(m)}(k)}_{j,T+h|T} = 1$ if $I^{(m)}_{k,T+h|T} = j$ and 0 otherwise.

4 Results using prior information on classification

4.1 Data

The analysis is done with a large cross-section of Austrian quarterly time series covering the period of the first quarter of 1988 through the fourth quarter of 2006. To assess the robustness of the method, we will compare the results with those obtained with a dataset ending in 2003 as it was available at the beginning of 2004 (see section 5.1 below). The data include GDP, its components and industrial production, economic confidence indicators and survey data for Austria, Germany and the US, the consumer price index, the harmonized consumer price index as well as its components, wholesale prices, wages and labor market series, trade series and exchange rates; finally, monetary and credit aggregates, and financial variables also containing the ATX, the DAX and the Dow Jones index. The complete set is found in table 1 in appendix D. Before the estimation, the data are transformed to non-trending series by taking first differences or first differences of the logarithmic level multiplied by 100 (see also table 1).[4] All series are mean-adjusted to remove long-run trends and standardized to account for the different volatility levels. Finally, those series that have a significant negative correlation with GDP or with order books total in the industrial sector (KTAUF), a series that is commonly seen as leading the cycle, are multiplied by -1 (see figure 1).

Some basic data properties are displayed in figure 1. For expositional convenience, only those series are plotted for which the contemporaneous correlation with GDP or with orders in the industrial sector is significant. In panel (a), we see that all series have distinct above-average and below-average mean growth rates, which justifies the two-state specification (this is also the case for the series not reported). Panel (b) plots the mean against the standard deviation of each series. The different volatility level of the series justifies the normalization and the specification of series-specific error variances. The correlations in panel (c) give a first hint about the series that might be coincident or leading the business cycle. Obviously, the components of GDP and industrial production are correlated with GDP. Some confidence and economic sentiment indicators correlated with order books, in particular the German IFO indices, some trade series and labor market series are also significantly correlated with GDP. The correlation with GDP is negative for various unemployment rates. In many cases, however, the correlation is higher with order books in the industrial sector. We do not find significant correlation with GDP for the price series except for the wholesale price of intermediate goods. Among the financial variables, we find the 3-month interest rate and the government bond yield that are positively correlated with GDP.

[4]The survey and confidence indicators are also taken in differences or growth rates because increases or decreases indicate whether periods of recovery or downturn are prevailing, respectively.

4.2 Model specification

The model is estimated for three groups, $K = 3$, and the lag length is set to $p = 2$, after having checked that higher order autocorrelation is insignificant. We link group 1 and group 2 by the dynamic structure described in section 2.3, which means that we fix ρ^* in a first round. We put additional information onto the model by pre-classifying GDP and its components, except for government consumption, into the first group. Some series, which are traditionally acknowledged to lead the business cycle, namely production in recent months (KTPROL), the expectations about orders (QTAUF), order books (KTAUF) and the confidence survey in the industrial sector (EINDSE), are pre-classified into group 2. These series are thought to determine the processes of the coincident and the leading group, respectively. For all other series, we will assume that the prior group probability depends on the correlation with GDP and with order books in the industrial sector. Given that in this setting GDP and order books are exogenously pre-classified into group 1 and 2, respectively, the posterior distribution of the group indicator for each series is discrete and remains independent of all other series. Thus, the sampler discussed in section 3.1 remains unchanged (see also (22) in appendix B).

It proved necessary to impose these three pieces of information on the model specification to obtain robust results across the two data vintages for the Austrian economy. In section 5.2 we will show the results for the model estimation without a priori information on the group structure, the group processes and the prior group probability. In section 5.3 we will additionally assess which of all three elements of the prior information is necessary to obtain robust results across vintages.

To estimate the model, we iterate 13,000 times over the sampling steps (i)-(iv) described in section 3. The first 8,000 iterations are discarded to remove dependence on starting conditions.

One characteristic feature of the model specification is its explicit modelling of series-specific heterogeneity. To illustrate the importance of it, figure 2 displays the marginal posterior distributions of the error variance σ^2, of the conditional rate in the above-average growth state, μ_2^k, and the first autoregressive coefficient ϕ_1^k of group 1 and 2, respectively. In panel (a), the dots plotted at the height of $10 \cdot \sigma^2$ show the mean of the weighted series-specific variance σ^2/λ_i. Despite that the series have been standardized before estimating the model, the dispersion of the error variances across series is significant. In panel (b) and (c) the dots , plotted at the height of 3 times the respective mean of the group-specific parameter, $3 \cdot \mu_2^k$ and $3 \cdot \phi_1^k$, represent the series-specific mean parameter estimate μ_2^i and ϕ_1^i, respectively. The panels represent two situations. Panel (b), in which within-group heterogeneity may not be that significant, given that most of the series-specific mean estimate μ_2^i lie within the region with highest posterior density of the marginal posterior distribution of the group-specific mean. In panel (c), we see that within-group heterogeneity is significant.

The significance of the informative prior on classification probability is shown in figure 3. For the prior probability of classification into group 1 (solid line) the posterior distribution of the effects of both GDP and order books correlation is clearly shifted away from zero while for the probability of classification into group 2 (solid dotted) only the correlation with order books is significant. Table 3 panel (a) contains the moment estimates of the posterior distributions.

Figure 4 visualizes the effect of GDP and order books correlation on the prior probability of classification into group 1 (left panel) and 2 (right panel). The mesh represents $P(S_i = j|Z_i)$ obtained by marginalizing over (γ, γ_z) using the simulations from the posterior distribution. The prior probability of classification into group 1 is low but nevertheless positive for series perfectly correlated with GDP and negatively correlated with order books. The prior probability of classification increases as the negative correlation with order books decreases, reaching the highest level at zero correlation with order books. Interestingly, the prior probability decreases again when correlation with order books turns positive, but it does not fall below 0.5. The prior probability of classification into group 2 is positive for series perfectly correlated with order books. In contrast to before, it further monotonically increases as correlation with GDP decreases.

The height of the lines in figure 4 represent $P(S_i = j|Z_i, Y^T)$, the posterior probability of series i to pertain to group 1 or 2. We observe that, for most series, the probabilities of classification are clearly updated towards 1 or 0 with the additional information contained in the data.

Overall, the results obtained for the series-specific error variance, for the within-group heterogeneity of model-parameters and the significance of the effect of GDP and order books correlation for the prior probability on classification, convey the model's ability to capture data specific features.

4.3 The classification of series

Table 2 panel (a) lists the variables falling into the coincident and the leading group of series. Comparing with table 1, we observe that with some exceptions, variables of the same kind fall into the same group. As already mentioned, GDP and its components are a priori classified into the coincident group of variables. Besides some trade data, minimum wages and the index of wholesale prices are classified into group 1.

The series which are traditionally seen as leading the business cycle fall into the group of leading variables. The actual situation and the expectations in industrial production and the construction sector fall into this group, confidence indicators and survey data of the industry and the construction sector as well. As the Austrian economy heavily relies on exports, it does not surprise that also the German IFO economic indicators are in the leading group of series. For this data vintage, some HICP components, like processed food, energy and services prices, and some wholesale price components, are also classified into group 2. The classification of some labor market data, like vacancies and number of employees into the leading group is intuitive, whereas for unemployment it is less intuitive. However, the classification of price and labor market series turns out to vary across data vintages. The explanation of this fact is left for future research. Finally, it is interesting to note that the ATX, the DAX and other financial market data like M1, the 3-month money market rate and direct credits to government are also classified as leading the business cycle.

4.4 Cycle duration and turning points

Figure 5 depicts the posterior state probabilities $P(I_k^T = 1|Y^T)$ of the coincident ($S_i = 1$) and of the leading group ($S_i = 2$) of series. They are obtained by averaging over the M simulated values $I^{T,(m)}$, $m = 1, \dots, M$. The inference is quite clear as nearly all posterior probabilities are either one or zero. For both groups, the switches into and out of state 1 clearly identify turning points in the business cycle (see also table 4). For this sample period, we observe that the lead into periods of below-average growth rates is nearly equal to the lead into recovery, except for the most recent upturn in 2001/2002. This is reflected in the mean posterior estimate of the transition probability matrix ξ^*, obtained by averaging over the MCMC draws:

$$E(\xi^*|Y^T) \begin{bmatrix} 0.78 & 0.22 & 0 & 0 \\ 0 & 0.27 & 0 & 0.73 \\ 0.76 & 0 & 0.24 & 0 \\ 0 & 0 & 0.16 & 0.84 \end{bmatrix}. \tag{18}$$

The expected lead of the leading group into a downturn, $1/(1 - \hat{\xi}^*_{33}) = 1.32$, is somewhat less than 4 months while the lead out of a trough, $1/(1 - \hat{\xi}^*_{22}) = 1.37$, is somewhat more than 4 months.

Based on figure 5 which plots the posterior probabilities of being in the below-average growth state, we may date turning points on the basis of the posterior state probabilities of the coincident group of series ($S_i = 1$). Period t will be identified as a peak if $P(I_{k,t-1} = 1, I_{k,t} = 1|Y^T) < 0.5$ and $P(I_{k,t+1} = 1, I_{k,t+2} = 1|Y^T) > 0.5$; likewise, period t will be identified as a trough, if $P(I_{k,t-1} = 1, I_{k,t} = 1|Y^T) > 0.5$ and $P(I_{k,t+1} = 1, I_{k,t+2} = 1|Y^T) < 0.5$, where k refers to the group of the coincident variables, in our case group 1.

The turning points identified with this rule are found in table 4, on the line labelled "PDS 88-06". As no official dates are available for Austria, we compare the dates with those reported by the Economic Cycle Research Institute (ECRI, www.businesscycle.com). The last ECRI release of a turning point dates back to 2004. Therefore, we will compare the estimated chronologies up to that period.

Up to 2001, the two chronologies are similar, which can also be seen in the first two panels of figure 6. However, PDS 88-06 usually identifies turning points one or two quarters later than ECRI. ECRI dates an additional cycle in 2003/2004, which is not identified by PDS 88-06 (nor by PDS 88-03). In table 5, first line, the concordance index between both state indicators of 0.7 is nevertheless quite considerable. The overall net lag of 7 quarters in identifying the four peaks is mainly due to the later identification of the fourth peak in the last quarter of 2000.

We also compare the dates to those obtained with other model specifications. We estimate (i) a Markov switching panel with shrinkage only for the series of the coincident group (PP), (ii) a Markov switching bivariate model with a dynamic structure (BDS) for GDP and order books total, and (iii) a Markov switching univariate model (UNI) for GDP. The business cycle phases are depicted in figure 6 in the last three panels. These other three models identify chronologies which are consistent with those identified by the PDS model. The periods of below-average growth identified by PP and BDS, with the exception of the downturn in 1991/1992 for PP, are longer than those identified by PDS. Also PP and BDS identify additional turning points in 2003/2004. The univariate specification only identifies two periods of below-average growth which relate to the recession in 1992/93 and the latest slowdown in 2000/01.

In the last three lines of table 5 we see that the concordance index between the state indicators estimated by the BDS and UNI models and the one estimated by PDS is higher than between the former two and the one identified with ECRI. The concordance between BDS and PDS is highest with 0.80. Relative to ECRI, all models identify the peaks and troughs later, except for the troughs identified by PP. Worth mentioning is that, except for PP, the dates of troughs are more consistent across the various model specifications than the peaks.

4.5 Probabilistic forecasts of turning points

At the end of 2006, according to PDS, both groups are in the above-average growth state with probability 1. Thus, we are in state 4 of I_T^*. Given the mean estimate of the transition probability matrix in (18), the mean conditional forecast $E(I_{T+h|T}^*|Y^T, I_T^*, \xi^*)$ two quarters ahead can be estimated as:

$$E(I_{T+2}^*|Y^T, I_T^*, \xi^*) = \left(E(\xi^{*\prime})\right)^2 \cdot \pi(I_T^*|Y^T), \tag{19}$$

which would yield a 71% probability of staying in the above-average growth state over the next half year and a 12% probability of reaching a below-average growth state in both groups within the same time span.

Another formulation would be that the expected duration of the above-average growth state at the end of 2006 is $1/(1-\hat{\xi}_{44}^*) = 3.45$ periods, i.e. between 10 and 11 months. If the leading group of series would switch into the below-average growth state, which can happen with a 16% probability within the next quarter, we expect the coincident group of series to follow after further $1/(1-\hat{\xi}_{33}^*) = 1.32$ periods, hence after further 4 months.

5 Effectiveness of informative prior classification

In this section we assess the effectiveness of informative prior classification by comparing the results to those obtained with a data vintage ending in 2003 in two ways. We first show that turning points are consistently estimated across data vintages when we impose the three pieces of information on the model, namely the fixed dynamic structure $\rho^* = (1, 2, 0)$; the fixed classification of GDP and its main components into the coincident group of series, and the fixed classification of expectations about orders, recent production, order books and the confidence survey in the industrial sector into the group of leading series; informative prior group probability depending on the correlation with GDP and with orders in the industrial sector.

In a second step we remove all prior information and estimate the model for both data vintages. We show that the results obtained for the turning point chronologies, the dynamic structure and the classification of series are not consistent across the vintages without prior information.

Finally, we assess which element forming the informative prior is relevant to obtain consistent estimates across data vintages by comparing results for estimations with different levels of prior information.

5.1 Estimation for data vintage ending in 2003

The results obtained with the data vintage ending in 2003 (vintage 2003 henceforth) are broadly consistent with those obtained with the data vintage ending in 2006 (vintage 2006 henceforth). Table 2 panel (b) shows that there are fewer series falling into the leading group. In particular, all HICP components, most of the wholesale price series and the financial variables, with the exception of the ATX and the DAX, are not classified into the leading group. Most trade data remain consistently in the coincident group across vintages while the classification of price and labor market series changes across vintages.

Table 3, panel (b) contains the estimates of the parameters for the prior group probability. The effect of the correlation with GDP is significant for the prior probability for classification into the coincident group while it is insignificant for the prior classification probability into the leading group. The effect of the correlation with order books in the industrial sector is marginally significant and significant for the prior classification into the coincident and the leading group, respectively.

We essentially obtained the same results for the vintage 2006, although the effect of the correlation with order books in the industrial sector on the prior classification probability into the coincident group becomes significant, but remains smaller than the estimated effect on the prior classification probability into the leading group.

The posterior state probabilities of being in below-average growth are depicted in figure 7. We see that the downturn period at the beginning of the 1990s is estimated to begin a year later than estimated from the vintage 2006. Otherwise, when we compare the turning point estimates across the vintages (see table 4, the lines labelled PDS 88-06 and PDS 88-03) the chronologies are quite consistent.

5.2 Estimation without prior information

To illustrate the usefulness of a specification with informative prior classification, we estimate the model without any prior information on classification. These results confirm that for Austrian data prior information is necessary to obtain robust results across data vintages. Removing prior classification information means that no series are classified a priori into one of the groups, the dynamical structure is not fixed a priori but also estimated from the data, and finally, that we use the non-informative prior distribution on group probabilities (see specification (5)).

Table 6 briefly characterizes the classification and the dynamical structure that is estimated from the data. We see that the classification of GDP and its components across data vintages is not robust. For the data vintage 2006, GDP and the components are classified into the leading group of series, while the leading indicators fall into the group interpreted as coincident. For the data vintage 2003, GDP, consumption and exports fall together with the leading indicators into the coincident group. Government consumption together with some price series and financial variables are classified as the leading group. It is obvious that these estimated dynamic structures cannot be interpreted in a business cycle context and that turning points will not be robust across data vintages. The changing pattern of the estimated dynamic structure also reflects the fact that the frequency of turning points is quite high for the relatively short sample period. Moreover, given that the coincident group usually closely follows the leading group of series (see table 4), it is difficult to estimate a robust dynamic structure across data vintages without setting prior classification information.

5.3 Robustness across vintages with different levels of prior information

Table 9 complements the differences between the estimations obtained with different levels of prior information on classification reported so far. In the first two lines we find the setting with informative priors and non-informative priors in which ρ^* is additionally estimated, respectively. We report three concordance indices. The concordance across the vintages of the estimates for the state indicator of the coincident group of series, and the concordance of series classification into the coincident and the leading group of series. While the estimates of the state indicator do not differ that much, the low concordance indices of group classification reflect the results presented so far for the non-informative prior specification.

The next two lines report the results obtained for different levels of a priori information. First, the logit prior on classification is substituted for the non-informative prior (5) and then, pre-classification of GDP and its components and of some leading indicators into, respectively, the coincident and leading group is dropped. Pre-classification of series is important to obtain consistent estimates of the state indicators across vintages, given that the concordance decreases from 0.81 to 0.63 when pre-classification is dropped. The informative logit prior turns out to be important for consistent estimation of the classification of series across vintages. Dropping the logit prior leads to a deterioration of the concordance index from 0.85 and 0.86 to 0.48 and 0.55 for the coincident and the leading group, respectively. Finally, note that the concordance for the classification of the leading group is higher under each prior information level than the one for the classification of the coincident group.

The last two lines report the results obtained when only GDP and order books in the industrial sector are pre-classified into group 1 and 2 (partial pre-classification). The conclusions drawn so far are corroborated. Pre-classification of series is important to obtain consistent estimates of the state indicators across vintages. We observe that the concordance decreases from 0.8 with full pre-classification to around 0.7 with partial pre-classification. The logit specification for the prior classification probability is important for the grouping of the series, in particular for the consistent classification of the coincident group. As in the setting with full pre-classification, the concordance for the coincident group decreases when the logit prior is dropped.

6 Forecasting turning points

We evaluate the ability of the model to forecast turning points from an in-sample and an out-of-sample perspective. Given that the usefulness of a model lies in its timeliness to recognize at the end of the sample a turning point, we focus on one-step-ahead forecasts of the state indicator by using equation (15). The forecast period in both the in-sample and out-of-sample exercises runs from 2001 to the end of 2006. We produce forecasts based on all models estimated in section 4.4 to date turning points, that is for the Markov switching PDS, PP and UNI models. The state indicator forecasts are evaluated against the own model-based state indicator and against the state indicators identified according to ECRI dates and according to PDS by assessing how many states out of the 24 forecasts and how many turning points are correctly predicted.

6.1 In-sample evaluation

For the in-sample evaluation, we estimate the parameters and the model-based state indicators using all information available up to the end of 2006. Then, for each quarter t from the first quarter of 2001 to the fourth quarter of 2006, we forecast the one-step-ahead state indicator based on the posterior state distribution $\pi(I_{t-1}^{*}|Y^{T})$.

Table 7 presents the forecast evaluation. Overall, the in-sample performance is best for PDS and UNI, for which the states are correctly forecasted in 24 and 23 out of 24 periods, respectively. BDS follows with 22 and PP ranks last with 20 out of 24 forecasted states. PDS identifies two turning points over the forecast period, both of which are correctly forecasted. PP and BDS have a similar performance, both identify two out of two and three turning points. BDS even identifies the trough in the third quarter of 2005 a quarter earlier.

When compared against ECRI identified states, we observe that all models have a low overall performance. In only half or even less of the quarters, the forecasts coincide with ECRI states, the exception being PP with 17 out of 24 states coinciding with ECRI states. Likewise, all models miss all turning points identified by ECRI. Of course, this is due to the low concordance between the model-based estimated state-indicators and the ECRI states. This is nevertheless worth mentioning, given that he concordance between the PDS and the ECRI state indicators is 0.7. Against the state identified by PDS, the performance is best for BDS with 20 out of 24 quarters being forecasted correctly. The overall forecast is still considerable for UNI with 17 equally identified states while PP forecasted states only coincide in 10 out of 24 periods. None of both models can forecast turning points identified by PDS.

6.2 Out-of-sample evaluation

The out-of-sample forecast evaluation is based on rolling estimations respectively extending the sample window up to the last quarter before the one to be forecasted. We thus forecast the one-step-ahead state indicator I_t^* based on the posterior state distribution $\pi(I_{t-1}^*|Y^{t-1})$. The evaluation is then performed against the model-based state indicator estimated for the whole sample and against ECRI and PDS states, respectively.

The evaluation in table 8 conveys the same picture as for the in-sample forecast evaluation. The overall performance is best for PDS and BDS. PDS forecasts one out of the two estimated turning points correctly. The peak at the end of 2000 is missed by one quarter. BDS forecasts two out of three turning points correctly. The performance against ECRI is low, both in terms of overall performance and forecasting turning points. The overall out-of-sample performance against the PDS state indicator is best for BDS, which forecasts 18 out of 24 states correctly. None of the three models forecasts the trough in 2002 identified by PDS. This is due to the fact that all three models identify the downturn to last until 2003 rather than being short-lived as estimated by PDS.

It is obvious that the number of turning points identified by PDS available for evaluating the models' forecasting performance is very small. However, these are restricted by degrees of freedom considerations. The forecast performance may be reassessed in the future as time series become longer.

7 Conclusion

In the present paper the information contained in a large panel of quarterly economic and financial variables is used to estimate business cycle turning points for Austria. The econometric model is based on the idea of model-based clustering of multiple time series, which suggests to group those series together which display similar time series and business cycle dynamics, whereby the appropriate classification of series is also part of the estimation method. Within a group, we allow for parameter heterogeneity by shrinking the series-specific parameters towards a group-specific mean rather than pooling all series within a group and assuming the same parameters for all of them. To account for the fact that some series are leading the business cycle, two groups are explicitly linked by a dynamic structure by defining one of them as leading the other one. To obtain robust results across data vintages, it is useful to design an informative prior for the group probability. In particular, GDP and its components except government consumption are classified a priori into the group of coincident series, while production in recent months, the expectations about orders and order books are pre-classified into the group of leading series. Series that are negatively correlated with either GDP or with order books in the industrial sector are multiplied by -1. Finally, for all series not pre-classified into a group, the prior group probability depends on the correlation with GDP and with order books in the industrial sector.

The results are broadly consistent with expectations. GDP and its components, some trade series, the wholesale price index (excluding seasonal goods) and agreed minimum wages fall into the coincident group. The group of leading series consists of Austrian confidence indicators and survey data in the industrial and the construction sectors, labor market data like unemployment rates, job vacancies and the number of employees, and some components of the HICP and the wholesale price index. Given that Germany is a major trading partner of Austria, it is intuitive that German survey indicators (IFO business cycle indicator) also fall into the leading group. Finally, it is interesting that some financial market series like M1, the Austrian and the German stock market indices, interest rates and direct credits to government fall into the leading group.

Based on the posterior state probabilities we date growth cycle turning points which closely correspond to those identified by the Economic Cycle Research Institute. Based on the posterior estimate of the transition probabilities and of the state indicator we can make a probabilistic forecast of reaching a turning point in the future. The ability of the model to predict turning points is evaluated in an in-sample and an out-of-sample forecast exercise. Out-of-sample, the model is able to predict one out of two turning points, whereby one is missed by one quarter. The performance is compared to other models. The Markov switching bivariate model with dynamic structure for GDP and order books in the industrial sector and the Markov switching panel for the group of coincident series predicts 2 out of 3 and 1 out of tow turning points, respectively. The univariate Markov switching model for GDP does not correctly predict out-of-sample turning points.

One feature not addressed in the paper and left for future research is to capture the changing pattern of the business cycle. The results suggest that recently the periods of below-average growth have become shorter. This may be modelled in various ways. One possibility would be to allow for time-varying transition probabilities (Diebold, Lee, and Weinbach, 1994, Filardo and Gordon, 1998). Another issue left for future research is the evaluation of the appropriate sample length to produce the forecasts of turning points (Pesaran and Timmermann, 2007). Model estimates of different sample length can additionally take into account the changing composition of the groups of series. Averaging over forecasts produced with these models would take into account changing business cycle features and group composition uncertainty.

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A The prior distribution of $(\lambda^N, \theta, \rho^*)$

To specify the prior on θ in a compact way, write model (1) as:

$$\begin{aligned} y_{it} &= X_{it}^{S_i}\beta^i + \varepsilon_{it}, \qquad \varepsilon_{it} \sim N(0, \sigma^2/\lambda_i) \\ \beta^i &\sim N\left(\beta^{S_i}, Q^{S_i}\right), \text{ if } S_i = k, \ k = 1, \ldots, K \end{aligned} \tag{20}$$

where $\beta^i = (\mu_1^i, \mu_2^i, \phi_1^i, \ldots, \phi_p^i)'$ and $X_{it}^{S_i} = \left(D_{1t}^{I(S_i)}, D_{2t}^{I(S_i)}, y_{i,t-1}, \ldots, y_{i,t-p}\right)$ with $D_{jt}^{I(S_i)} = 1$ if $I_{S_i,t} = j$ and $D_{jt}^{I(S_i)} = 0$ otherwise, $j = 1, 2$.

The prior distribution of the group-specific parameter vectors β^k, $k = 1, \ldots, K$ consists of two parts:

- $\left(\mu_1^k, \mu_2^k\right) \sim N(m_0, M_0) \cdot I_{(\mu_1^k \leq 0, \mu_2^k > 0)}$, , where $I_{(\cdot)}$ is the indicator function.
- $\left(\phi_1^k, \ldots, \phi_p^k\right) \sim N(0, \kappa I_p) \cdot I_{\left(\sum_{j=1}^p \phi_j^k < 1\right)}$, where I_p is the identity matrix of dimension p,

which leads to the prior

$$\beta^k \sim N\left(\left(\begin{array}{c} m_0 \\ 0 \end{array}\right), \left[\begin{array}{cc} M_0 & 0 \\ 0 & \kappa I_p \end{array}\right]\right) \cdot I_{\left(\mu_1^k \leq 0, \mu_2^k > 0, \sum_{j=1}^p \phi_j^k < 1\right)}.$$

In general, $\beta^k \sim TN(b_0, B_0)$, TN meaning truncated normal.

Group-specific parameter heterogeneity Q^k a priori follows an inverse Wishart distribution

$$Q^k \sim \mathcal{W}^{-1}(c_0, C_0)$$

The variance of the error terms and the series-specific weights, σ^2 and λ_i, a priori follow an inverse Gamma and a Gamma distribution, respectively:

$$\sigma^2 \sim IG(g_0, G_0) \quad \lambda_i \sim G\left(\frac{\nu}{2}, \frac{\nu}{2}\right), \ i = 1, \ldots, N$$

The parameters governing the prior group probabilities, $\gamma = (\gamma, \gamma_z)$ are assumed to have a normal prior $N(0, \tau I_g)$, where g is the dimension of the vector γ.

The transition distribution ξ^* of the encompassing state indicator is specified by independent Dirichlet distributions

$$\xi^* \sim D(e_{11,0}^*, e_{12,0}^*) D(e_{22,0}^*, e_{24,0}^*) D(e_{31,0}^*, e_{33,0}^*) D(e_{43,0}^*, e_{44,0}^*).$$

The transition distribution of the independent groups is also Dirichlet, $\xi^{\rho^*(k)=0} \sim \Pi_{j=1}^2 D(e_{j1,0}, e_{j2,0})$, $k = 1, \ldots, K$.

The hyperparameters we choose are: $m_0 = (-0.25, 0.25)$ and $M_0 = 2.22 \cdot I_2$; $\kappa = 0.25$; $c_0 = 12$ and $C_0 = 0.1 \cdot I_{p+2}$; $g_0 = 1$, $G_0 = 1$ and $\nu = 8$; $\tau = 20$; $(e_{11,0}^*, e_{12,0}^*) =$

$(5,2)$, $(e^*_{22,0}, e^*_{24,0}) = (3,7)$, $(e^*_{31,0}, e^*_{33,0}) = (7,3)$, $(e^*_{43,0}, e^*_{44,0}) = (3,7)$, $(e_{11,0}, e_{12,0}) = (2,1)$, $(e_{21,0}, e_{22,0}) = (1,2)$. The hyperparameters governing the prior transition distribution of the encompassing state yield a mean persistence of the coincident group to remain in the below-average and in the above-average state of 0.59 and 0.65, respectively. The lead out of a trough and into the below-average growth state of the leading group is 1.4 periods, i.e. about four and a half months.

If ρ^* is estimated, the prior distribution is discrete, and each permutation ρ_l, $l = 1, \ldots, L$, out of the $L = K(K-1)$ possible ones from $\{1, 2, 0_{K-2}\}$ is given a prior probability of $\eta_\rho = 1/(K(K-1))$.

B Posterior distributions

In the following, we assume that the dynamic structure ρ^* is known. In case ρ^* is not known, the sampler involves an additional step. This extension is described in the next appendix.

Some posterior distributions will be derived using the marginal model in which the series-specific random effects are integrated out (see also equation (11)):

$$\begin{aligned} y_{it} &= X^{S_i}_{it} \beta^{S_i} + \varepsilon^*_{it}, \qquad \varepsilon^*_{it} \sim N\left(0, \nu^{S_i}_{it}\right) \\ \nu^{S_i}_{it} &= X^{S_i}_{it} Q^{S_i} X^{S_i\prime}_{it} + \sigma^2/\lambda_i \end{aligned} \tag{21}$$

where $X^{S_i}_{it}$ is defined as in (21). The difference to the model with group-specific parameter heterogeneity discussed in Frühwirth-Schnatter (2006, pp. 260-269) appears in the predictor matrix $X^{S_i}_{it}$, which is group-specific due to the group-specific state indicator determining the time-varying intercept term $\mu^i_{I_{S_i,t}}$.

We estimate model (20) by sampling from the following posterior distributions:
(i) $\pi(S^N|Y^T, I^T, \rho^*, \lambda^N, \theta)$,
(ii) $\pi(I^T|Y^T, S^N, \rho^*, \lambda^N, \theta)$,
(iii) $\pi(\lambda^N|Y^T, S^N, I^T, \theta)$,
(iv) $\pi(\theta|Y^T, S^N, I^T, \lambda^N)$.

(i) Simulating S^N out of $\pi(S^N|Y^T, I^T, \rho^*, \lambda^N, \theta)$ using the marginal model (21). The group indicator is simulated for each series independently, given that the posterior distribution can be factorized:

$$\pi(S^N|Y^T, I^T, \rho^*, \lambda^N, \theta) \;\propto\; \prod_{i=1}^{N} \pi(S_i|Y^T, I^T, \rho^*, \lambda^N, \theta).$$

For each series, S_i is sampled from the discrete distribution

$$P(S_i = k|\,\cdot) \propto \prod_{t=p+1}^{T} f(y_{it}|X^k_{it}, \beta^k, Q^k, \sigma^2, \lambda_i) P(S_i = k|\gamma_k, \gamma_{zk}), \tag{22}$$

where the observation density $f(y_{it}|\,\cdot^k)$ is given in (10).

(ii) Simulating I^T out of $\pi(I^T|Y^T, S^N, \rho^*, \lambda^N, \theta)$. For a given dynamic structure ρ^*, the posterior distribution of I^T can be factorized as

$$\pi(I^T|Y^T, S^N, \lambda^N, \rho^*, \theta) \quad = \quad \pi(I^{*T}|Y^T, S^N, \lambda^N, \rho^*, \theta) \prod_{\rho^*(k)=0} \pi(I_k^T|Y^T, S^N, \lambda^N, \theta),$$

$k = 1, \ldots, K$, and where I^{*T} and I_k^T are the four-state encompassing and the two-state independent state-indicators, respectively. The terms can typically be factorized as

$$\begin{aligned}\pi(I^T|Y^T, S^N, \lambda^N, \rho^*, \theta) \quad &= \quad \pi(I_T|Y^T, S^N, \lambda^N, \rho^*, \theta) \times \\ &\prod_{t=0}^{T-1} \pi(I_t|Y^t, I_{t+1}, S^N, \lambda^N, \rho^*, \theta).\end{aligned}$$

The encompassing state indicator I^{*T} and each of the $K-2$ remaining independent state indicators I_k^T can thus be simulated independently of each other. For all indicators, the sampling densities can be derived from the multi-move sampler described in Chib (1996). Note that the typical element $\pi(I_t|Y^t, I_{t+1}, S^N, \rho^*, \theta)$, is proportional to

$$\pi(I_t|Y^t, I_{t+1}, S^N, \lambda^N, \rho^*, \theta) \propto \pi(I_t|Y^t, S^N, \lambda^N, \rho^*, \theta) \cdot \xi_{I_t, I_{t+1}}. \tag{23}$$

The first factor $\pi(I_t|Y^t, S^N, \lambda^N, \rho^*, \theta)$ corresponds to the filter density

$$\pi(I_t|Y^t, S^N, \lambda^N, \rho^*, \theta) \propto \prod_{S_i = \ \cdot} f\left(y_{it}|X_{it}^{S_i}\beta^{S_i}, \lambda_i, \sigma^2, Q^{S_i}, I_{S_i,t}\right) \cdot \pi\left(I_{S_i,t|t-1}\right) \tag{24}$$

where the product is build over $S_i = k$ if $\rho^*(k) = 0$, or jointly for the coincident and the leading group of series, $S_i = \{k, \tilde{k}\}$, which are indicated by $\rho^*(k) = 1$ and $\rho^*(\tilde{k}) = 2$, respectively, $k = 1, \ldots, K$. Conditional on state j, the observation density $f\left(y_{it}|\ \cdot, I_{S_i,t} = j\right)$ is normal

$$f\left(y_{it}|\ \cdot, I_{S_i,t} = j\right) = \frac{1}{\sqrt{2\pi\nu_{it}^{S_i(j)}}} \exp\left\{-\frac{1}{2\nu_{it}^{S_i(j)}}\left(y_{it} - X_{it}^{S_i(j)}\beta^{S_i}\right)^2\right\}$$

where $X_{it}^{S_i(j)} = \left(D_{1t}^{I(S_i)}, D_{2t}^{I(S_i)}, y_{i,t-1}, \ldots, y_{i,t-p}\right)$, with $D_{jt}^{I(S_i)} = 1$ if $I_{S_i,t} = j$ and 0 otherwise, $j = 1, 2$. In analogy to (21), $\nu_{it}^{S_i(j)} = X_{it}^{S_i(j)} Q^{S_i} X_{it}^{S_i(j)\prime} + \sigma^2/\lambda_i$.

The term $\pi\left(I_{S_i,t|t-1}\right)$ is obtained by extrapolation

$$\pi\left(I_{S_i,t|t-1}\right) = \sum_{I_{t-1}} \pi(I_{t-1}|Y^{t-1}, S^N, \lambda^N, \rho^*, \theta) \cdot \xi_{I_{t-1}, I_t}^{S_i}. \tag{25}$$

Given the filter densities $\pi\left(I_t|Y^t, S^N, \lambda^N, \rho^*, \theta\right)$, $t = 1, \ldots, T$, beginning in T, we sample I_T from $\pi(I_T|Y^T, S^N, \lambda^N, \rho^*, \theta)$. Then, the recursion in (23) is used to simulate I_t for $t = T-1, \ldots, 0$.

(iii) The weights λ^N are simulated from $\pi(\lambda^N|Y^T, S^N, I^T, \theta)$ using the model (20). Details can be found in Frühwirth-Schnatter and Kaufmann (2008).

(iv) Given S^N and I^T, the vector θ is blocked appropriately to simulate the model parameters out of their posterior conditional distributions.

Simulating the group-specific parameter vectors from $\pi\left(\beta^1,\ldots,\beta^K|Y^T,S^N,I^T,\lambda^N,Q,\sigma^2\right)$. We use the marginal model (21) and write it

$$y_{it}=X_{it}\beta+\varepsilon_{it}^* \quad \varepsilon_{it}^*\sim N\left(0,\nu_{it}^{S_i}\right)$$

with $\beta = vec\left(\beta^1,\ldots,\beta^K\right)$ and, this time, $X_{it}^{S_i} = \left(D_i^{G(1)}X_{it}^1,\ldots,D_i^{G(K)}X_{it}^K\right)$, where $D_i^{G(k)} = 1$ if $S_i = k$ and 0 otherwise. X_{it}^k and $\nu_{it}^{S_i}$ are defined as in (21). The posterior distribution can be derived as

$$\pi\left(\beta|\,\cdot\right)\sim N\left(b,B\right)\cdot I_{\left(\mu_1^k\leq 0,\mu_2^k>0,\sum_{j=1}^p\phi_j^k<1,\forall k\right)}$$

with

$$\begin{aligned} B &= \left(\sum_i\sum_t X_{it}{}'X_{it}/\nu_{it}^{S_i}+B_0^{-1}\right)^{-1} \\ b &= B\left(\sum_i\sum_t X_{it}{}'y_{it}/\nu_{it}^{S_i}+B_0^{-1}b_0\right) \end{aligned}$$

and appropriately inflated vector and matrix b_0 and B_0, respectively.

Simulating group-specific parameter heterogeneity Q. To derive the posterior distribution we first have to simulate the series-specific random effects $\beta_i-\beta^{S_i}$. Re-write model (20) as

$$y_{it}=X_{it}^{S_i}\left(\beta^{S_i}+\left(\beta^i-\beta^{S_i}\right)\right)+\varepsilon_{it} \qquad \varepsilon_{it}\sim N\left(0,\sigma^2/\lambda_i\right).$$

Given the normal prior for the random effects, $\beta^i-\beta^{S_i}\sim N\left(0,Q^{S_i}\right)$, the posterior is normal $N\left(b_i,B_i\right)$ with

$$\begin{aligned} B_i &= \left(\frac{\lambda_i}{\sigma^2}\sum_t X_{it}^{S_i\prime}X_{it}^{S_i}+Q^{S_i-1}\right)^{-1} \\ b_i &= B_i^{-1}\left(\frac{\lambda_i}{\sigma^2}\sum_t X_{it}^{S_i\prime}\left(y_{it}-X_{it}^{S_i}\beta^{S_i}\right)\right) \end{aligned}$$

Alternatively, the filter form proposed in Frühwirth-Schnatter (2006, p. 266) can be adjusted appropriately.

The posterior distributions of group-specific parameter heterogeneity Q^k are then independent of each other. We can sample from $\pi\left(Q^k|Y^T,S^N,I^T,\lambda^N,\beta,\beta^i,\sigma^2\right)$ which is inverse Wishart $\mathcal{W}^{-1}\left(c^k,C^k\right)$ with

$$\begin{aligned} c^k &= c_0+N_k/2 \qquad N_k=\#\left\{S_i=k\right\} \\ C^k &= C_0+1/2\sum_{S_i=k}\left(\beta^i-\beta^k\right)'\left(\beta^i-\beta^k\right) \end{aligned}$$

Simulating the transition probabilities ξ^*,ξ^k, the variance σ^2 and the parameters governing the prior group probability, (γ,γ_z), is standard (including a Metropolis-Hastings step for the latter) and is discussed in Frühwirth-Schnatter and Kaufmann (2008).

C Sampling the dynamic structure ρ^*

If the dynamic structure is not known, the sampler involves an additional step in (ii):

(i) $\pi(S^N|Y^T, I^T, \lambda^N, \theta)$,
(ii.a) $\pi(\rho^*|Y^T, S^N, \lambda^N, \theta)$,
(ii.b) $\pi(I^T|Y^T, S^N, \lambda^N, \rho^*, \theta)$,
(iii) $\pi(\lambda^N|Y^T, S^N, I^T, \theta)$,
(iv) $\pi(\theta|Y^T, S^N, I^T, \lambda^N,)$.

(ii.a) Simulating ρ^* out of $\pi(\rho^*|Y^T, S^N, \lambda^N, \theta)$. The posterior of the dynamic structure is discrete:

$$\pi(\rho^* = \rho_l|Y^T, S^N, \lambda^N, \theta) \propto L(Y^T|S^N, \lambda^N, \theta, \rho_l) \cdot \eta_\rho,$$

for $l = 1, \ldots, K(K-1)$. The marginal likelihood associated with the dynamic structure ρ_l, $L(Y^T|S^N, \lambda^N, \theta, \rho_l)$ is given by:

$$L(Y^T|S^N, \lambda^N, \theta, \rho_l) = L_{\rho_l(k)\neq 0}(Y^T|S^N, \lambda^N, \theta) \cdot L_{\rho_l(k)=0}(Y^T|S^N, \lambda^N, \theta), \tag{26}$$

whereby the first factor is the likelihood part of the coincident and the leading group of series marginalized over the state indicator, and the second factor is the part contributed by the remaining groups of series. The second factor can be written

$$\begin{aligned} L_{\rho_l(k)=0}(Y^T|S^N, \lambda^N, \theta) &= \prod_{t=p+1}^{T} L_{\rho_l(k)=0}(Y_t|Y^{t-1}, S^N, \lambda^N, \theta) \\ &= \prod_{t=p+1}^{T} \prod_{\rho_l(k)=0} \prod_{S_i=k} f(y_{it}|y^{i,t-1}, S_i, \lambda_i, \theta), \quad k = 1, \ldots, K, \end{aligned}$$

where the term $f(y_{it}|y^{i,t-1}, S_i, \theta)$ turns out to be the normalizing constant of the filter distribution :

$$\begin{aligned} &\prod_{\rho_l(k)=0} \prod_{S_i=k} f(y_{it}|y^{i,t-1}, S_i, \lambda_i, \theta) = \\ &\qquad \prod_{\rho_l(k)=0} \prod_{S_i=k} \sum_{j=1}^{2} f(y_{it}|y_i^{t-1}, S_i, \lambda_i, I_{kt} = j, \theta) \cdot \pi(I_{kt} = j|Y^{t-1}, S^N, \lambda^N, \theta), \\ &\qquad k = 1, \ldots, K. \end{aligned}$$

The first factor of the marginal likelihood in (26) can analogously be derived using the filter distribution for the encompassing state I^{*T} driving the coincident group k, $\rho_l(k) = 1$, and the leading group $\tilde{k}$, $\rho_l(\tilde{k}) = 2$, respectively:

$$\begin{aligned} L_{\rho_l(k)\neq 0}(Y^T|S^N, \lambda^N, \theta) &= \prod_{t=p+1}^{T} \prod_{\substack{\rho_l(k)\neq 0 \\ S_i=k}} f(y_{it}|y^{i,t-1}, S_i, \lambda_i, \theta), \quad k = 1, \ldots, K, \\ &= \prod_{t=1}^{T} \prod_{\substack{\rho_l(k)\neq 0 \\ S_i=k}} \sum_{j=1}^{4} f(y_{it}|y_i^{t-1}, S_i, \lambda_i, I_t^* = j, \theta) \cdot \\ &\qquad \pi(I_t^* = j|Y^{t-1}, S^N, \lambda^N, \theta), \quad k = 1, \ldots, K. \end{aligned}$$

(ii.b) Simulating I^T out of $\pi(I^T|Y^T, S^N, \lambda^N, \rho^*, \theta)$. Given the dynamic structure parameter ρ^* and the group indicator S^N, the group-specific state indicators can be simulated as described in appendix B.

D Tables

Table 1: Time series. L means the logarithmic level, D first differenced and 100 multiplied by 100.

Acronym		Class	Name
YER	LD100	National account	GDP real
PCR	LD100	data	Private consumption, real
ITR	LD100		Total investment, real
GCR	LD100		Government consumption, real
MTR	LD100		Imports, real
XTR	LD100		Exports, real
TOT	LD100		Terms of trade
EEN	LD100		Nominal effective exchange rate on the export side
USD	LD100		Euro/US-Dollar exchange rate
QTAUF	D	WIFO Quarterly	Assessment of order books, industry,total, balance in %
QTEXPA	D	Survey	Assessment of export order books, industry, total, balance in %
QTLAG	D		Assessment of stocks, industry, total, balance in %
QTPR	D		Selling-price expectations, industry, total, balance in %
QTPRO	D		Production expectations, industry, total, balance in %
QTBAUF	D		Assessment of order books, main construction trade, total, balance in %
QTBPR	D		Assessment of selling price expectations, main construction trade, total, balance in %
QTBBGL	D		Assessment of business situation, main construction trade, total, balance in %
QTBAGL	D		Evolution of business situation, main construction trade, total, balance in %
EECOS	LD100	Monthly survey	Economic sentiment indicator
EINDSE	D	data	Industry - Confidence indicator
KTPROL	D		Industry - Production in recent months
KTAUF	D		Industry - Order books total
KTAUSL	D		Industry - Exports order books
KTLAG	D		Industry - Stocks of finished goods
KTPRON	D		Industry - Production over next months

Table 1: Time series, continued. L means the logarithmic level, D first differenced and 100 multiplied by 100.

Acronym		Class	Name
KTVPN	D		Industry - Selling prices over next months
BAUVPN	D		Construction - Selling prices over next months
EBAUSE	D		Construction - Confidence indicator
EHANSE	D		Retail trade - Confidence indicator
EKONSE	D		Consumer confidence indicator
IFOERW	LD100	Foreign confidence	IFO - Business expectations in Western Germany, Index 1991 = 100
IFOKL	LD100	indicator	IFO - Business climate index for Western Germany, 1991=100
IFOGL	LD100		IFO - Business situation in Western Germany, Index 1991 = 100
PMI	LD100		Purchasing Manager Index USA
HICP	LD100	HICP	HICP - Overall index
HICP-FO	LD100		HICP - Food incl. alcohol and tobacco
HICP-PF	LD100		HICP - Processed food incl. alcohol and tobacco
HICP-UF	LD100		HICP - Unprocessed food
HICP-G	LD100		HICP - Goods
HICP-IG	LD100		HICP - Industrial goods
HICP-GX	LD100		HICP - Industrial goods excluding energy
HICP-E	LD100		HICP - Energy
HICP-S	LD100		HICP - Services
HICP-XA	LD100		HICP - All-items excluding alcoholic beverages, tobacco
HICP-XE	LD100		HICP - All-items excluding energy
HICP-XF	LD100		HICP - All-items excluding energy and food
HICP-XG	LD100		HICP - All-items excluding energy and unprocessed food
VPIG86	LD100	CPI	Consumer price index, all items, 1986=100
VPI-WOH86	LD100		CPI 86, housing, 1986=100
TLIG86	LD100	Wages	Agreed minimum wages, overall index, 1986=100
TLIARG86	LD100		Agreed minimum wages, workers, index 1986=100
TLIANG86	LD100		Agreed minimum wages, salary earners, index 1986=100

Table 1: Time series, continued. L means the logarithmic level, D first differenced and 100 multiplied by 100.

Acronym		Class	Name
GHPIG	LD100	Wholesale prices	Wholesale prices - overall index, 1986=100
GHPIOS	LD100		Wholesale prices - excl. seasonal goods, 1986=100
GHPIGG	LD100		Wholesale prices - consumer goods, 1986=100
GHPIGL	LD100		Wholesale prices - durable commodities, 1986=100
GHPIGK	LD100		Wholesale prices - non-durable commodities, 1986=100
GHPIVBG	LD100		Wholesale prices - non-durables, 1986=100
GHPIKONG	LD100		Wholesale prices - consumer goods, 1986=100
GHPIINVG	LD100		Wholesale prices - investment goods, 1986=100
GHPIINTG	LD100		Wholesale prices - intermediate goods, 1986=100
OEL	LD		Oil price, USD per Barrel - IMF
EXPG	LD100	Foreign trade	Exports - FOB total
EXP6	LD100		Exports - FOB basic manufactures, SITC 6
EXP7	LD100		Exports - FOB machines and transport equipment, SITC 7
EXP8	LD100		Exports - FOB misc. manufactured goods, SITC 8
IMPG	LD100		Imports - CIF total
IMP6	LD100		Imports - CIF basic manufactures, SITC 6
IMP7	LD100		Imports - CIF machines and transport equipment, SITC 7
IMP8	LD100		Imports - CIF misc. manufactured goods, SITC 8
EXP-US	LD100		Exports of commodities to US
EXP-EU	LD100		Exports of commodities to EU
EXP-DE	LD100		Exports of commodities to Germany
IMP-US	LD100		Imports of commodities from US
IMP-EU	LD100		Imports of commodities from EU
IMP-DE	LD100		Imports of commodities from Germany
ALQNSA	D		Unemployment rate, national definition, total SA
ALOSM	LD100		Unemployment, male
ALOSW	LD100		Unemployment, female

Table 1: Time series, continued. L means the logarithmic level, D first differenced and 100 multiplied by 100.

Acronym		Class	Name
OFST	LD100		Vacancies
STANDR	LD100		Employees
INDPROD	LD100	IP	Industrial production, overall index (excl. construction and energy), 1995=100
ATX	LD100	Financial variables	ATX (Austrian Trading Index)
M1	LD100		M1
M2	LD100		M2
M3	LD100		M3
DAX	LD100		DAX
DowJones	LD100		Dow Jones Index
STI	D		3-month money market rate
SEKMRE	D		Secondary market yield on government bonds (9 to 10 years)
YIELD	D		Yield spread
DCR-H	LD100		Direct credits to private households
DCR-F	LD100		Direct credits to private firms
DCR-G	LD100		Direct credits to government
DEBT	LD100		Outstanding debt
DCR	LD100		Direct credits, total

Table 2: Series classification, informative prior

(a) data vintage 1988-2006	
coincident group	leading group
YER PCR ITR GCR MTR XTR TLIARG86 TLIANG86 GHPIOS EXPG EXP7 EXP8 IMPG IMP6 IMP7 EXP-US EXP-EU EXP-DE IMP-US IMP-EU IMP-DE	QTAUF QTEXPA QTLAG QTPR QTPRO QTBAUF QTBPR QTBBGL QTBAGL KTPROL KTAUF KTAUSL KTLAG KTPRON KTVPN BAUVPN EECOS EINDSE EBAUSE EHANSE EKONSE IFOERW IFOKL IFOGL PMI HICP-PF HICP-E HICP-S HICP-XF GHPIG GHPIGK GHPIVBG GHPIINT OEL EXP6 ALQNSA ALOSM ALOSW OFST STANDR INDPROD ATX M1 DAX STI SEKMRE DCR-G
(b) data vintage 1988-2003	
coincident group	leading group
YER PCR ITR MTR XTR TOT EEN HICP-E GHPIG GHPIOS GHPIVBG GHPIKONG EXP7 IMPG IMP6 IMP7 EXP-EU EXP-DE IMP-US IMP-EU IMP-DE INDPROD SEKMRE	QTAUF QTEXPA QTLAG QTPR QTPRO QTBAUF QTBPR QTBBGL QTBAGL KTPROL KTAUF KTAUSL KTLAG KTPRON KTVPN BAUVPN EECOS EINDSE EBAUSE EHANSE EKONSE IFOERW IFOKL IFOGL PMI GHPIGK GHPIINTG OEL EXPG EXP6 EXP-US ALQNSA ALOSM OFST STANDR ATX DAX

Table 3: Posterior distribution of the parameters influencing the group probabilities, $\pi(\gamma, \gamma_z | Z^N, Y^T)$, γ_z includes first the effect of GDP correlation and second the effect of the correlation with orders in the industrial sector. Mean estimate and standard deviation.

	(a) data vintage 1988-2006	
	coincident group	leading group
$E(\gamma, \gamma_z)$	(-2.66, 5.86, 7.56)	(-1.95, 1.20, 11.59)
$SD(\gamma, \gamma_z)$	(0.74, 2.23, 2.10)	(0.67, 1.92, 2.22)
	(b) data vintage 1988-2003	
	coincident group	leading group
$E(\gamma, \gamma_z)$	(-2.01, 7.47, 4.44)	(-3.44, 1.68, 14.09)
$SD(\gamma, \gamma_z)$	(0.60, 2.12, 2.00)	(0.92, 2.34, 2.69)

Table 4: Growth cycle peak (P) and trough (T) dates (YY:Q, M/YY) identified by various models and by the Economic Cycle Research Institute (ECRI).

	P	T	P	T	P	T	P	T	P	T	P	T	P	T
PDS 88-06	90:2	93:3	95:1	96:2			98:3	99:1	00:4	02:1				
PDS 88-03	91:2	93:4	95:1	96:2			98:1	99:1	00:4			03:2		
ECRI[b)]														
quarterly	90:1	93:1	94:4	96:1			98:2	99:1	99:3	01:4	03:1	03:4	04:3	
monthly	2/90	3/93	11/94	3/96			5/98	2/99	7/99	12/01	1/03	12/03	8/04	
PP	91:2	92:3	94:4	95:3			97:3	98:4	00:2			03:2	04:2	
BDS	90:2	93:4	95:1	96:3			98:1	99:1	00:3			03:3	04:4	05:3
UNI	92:1	93:4							00:2			03:3		

[b)]The ECRI dates growth cycles on a monthly basis. The quarterly dates are derived from the monthly ones (www.businesscycle.com).

PDS: Markov switching panel with dynamic structure; PP: Markov switching panel estimation (with shrinkage) of coincident group; BDS: Bivariate Markov switching model for GDP and Order books total with dynamic structure; UNI: Univariate Markov switching model for GDP.

Table 5: Properties of turning points 1989:3-2005:4. Comparison of various models relative to ECRI turning points and to turning points identified by the Markov switching panel with dynamic structure (PDS).

Model	Relative to ECRI concordance index	Peaks	Troughs	Relative to PDS concordance	Peaks	Troughs
PDS	0.71	+7/4*	+4/4	-	-	-
PP	0.59	+4/5	-6/4	0.55	-3/4	-8/3
BDS	0.73	+6/5	+4/4	0.80	-3/4	+2/3
UNI	0.52	+11/2	+2/2	0.65	+5/2	+1/1

* Net lag (+) or lead (-) of all identified peaks or troughs relative to the ECRI or PDS cycle.
PP: Markov switching panel estimation (with shrinkage) of coincident group; BDS: Bivariate Markov switching model for GDP and Order books total with dynamic structure; UNI: Univariate Markov switching model for GDP.

Table 6: Series classification, non-informative prior

(a) data vintage 1988-2006		
$E(\rho^*(1)) = 1.93$	$E(\rho^*(2)) = 0.93$	$E(\rho^*(3)) = 0.14$
$STD(\rho^*(1)) = 0.33$	$STD(\rho^*(2)) = 0.36$	$STD(\rho^*(3)) = 0.37$
leading group	coincident group	independent group
GDP and components	leading series	other series
trade with US and Germany	financial variables	

(b) data vintage 1988-2003		
$E(\rho^*(1)) = 1.49$	$E(\rho^*(2)) = 0.32$	$E(\rho^*(3)) = 1.18$
$STD(\rho^*(1)) = 0.50$	$STD(\rho^*(2)) = 0.63$	$STD(\rho^*(3)) = 0.79$
coincident group	independent group	leading group
government cons.,	GDP and components	leading indicators
price series	trade series	exports
financial variables		stock prices

Table 7: In-sample one-step ahead forecasts, 2001:1-2006:4. Model-based evaluation and relative to the state indicators identified by ECRI and by the Markov switching panel with dynamic structure (PDS).

Model	Model-based estimate		Relative to ECRI		Relative to PDS	
	overall	turning points	overall	turning points	overall	turning points
PDS	24/24	2/2	11/24	0/4	-	-
PP	20/24	2/2	17/24	0/4	10/24	0/1
BDS	22/24	2/3	9/24	0/4	20/24	0/1
		identified T in 05:02				
UNI	23/24	0/1	10/24	0/4	17/24	0/1

PP: Markov switching panel estimation (with shrinkage) of coincident group; BDS: Bivariate Markov switching model for GDP and Order books total with dynamic structure; UNI: Univariate Markov switching model for GDP.

Table 8: Out-of-sample one-step ahead forecasts, 2001:1-2006:4. Model-based evaluation and relative to the state indicators identified by ECRI and by the Markov switching panel with dynamic structure (PDS).

Model	Model-based estimate		Relative to ECRI		Relative to PDS	
	overall	turning points	overall	turning points	overall	turning points
PDS	20/24	1/2	9/24	0/4		
		identified P in 01:1				
PP	19/24	1/2	16/24	0/2	11/24	0/1
BDS	20/24	2/3	13/24	1/4	18/24	0/1
		missed T in 03:3		identified T in 03:4		
UNI	19/24	0/1	10/24	0/4	13/24	0/1

PP: Markov switching panel estimation (with shrinkage) of coincident group; BDS: Bivariate Markov switching model for GDP and Order books total with dynamic structure; UNI: Univariate Markov switching model for GDP.

Table 9: Evaluation of prior information

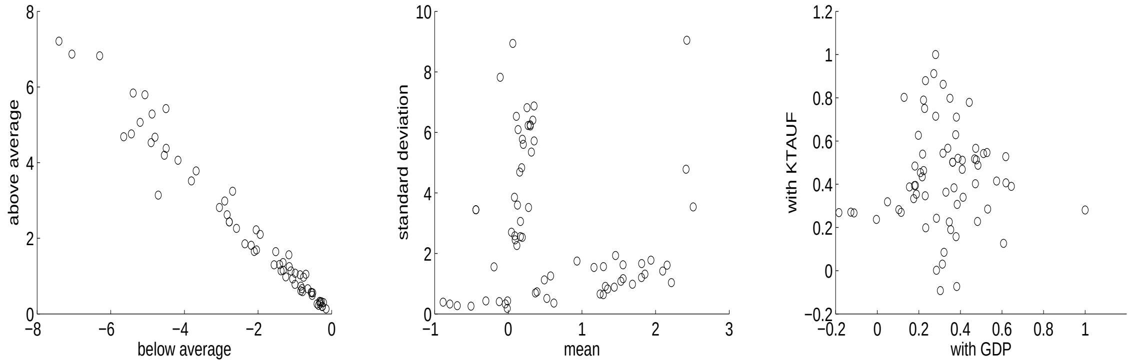
Prior information setting	Concordance index between vintages		
		Identification of	
	state indicator	coincident group	leading group
ρ^* fixed, logit prior full pre-classification[a])	0.80	0.85	0.86
ρ^* estimated no pre-classification	0.86	0.30	0.49
ρ^* fixed full pre-classification	0.81	0.48	0.55
ρ^* fixed no pre-classification	0.63	0.27	0.89
ρ^* fixed, logit prior partial pre-classification[b])	0.68	0.57	0.35
ρ^* fixed partial pre-classification	0.71	0.45	0.70

[a]) GDP and its components, except government consumption, are in the coincident group, production in recent months, expectations about orders, order books and the confidence survey in the industrial sector are in the leading group

[b]) GDP is in the coincident group, order books in the industrial sector is in the leading group

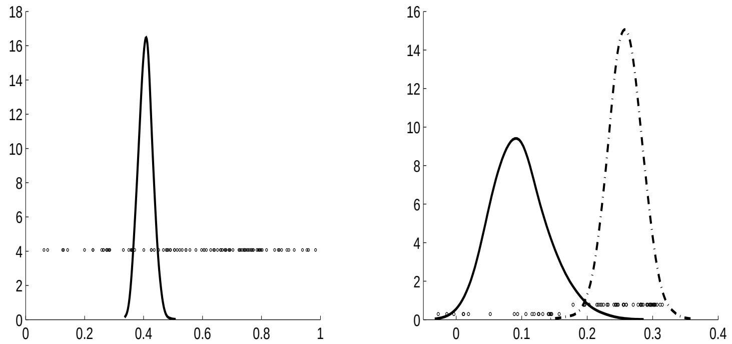
E Figures

Figure 1: Data properties



(a) mean (b) (c) correlation

Figure 2: Series-specific heterogeneity. Group 1 (solid line), group 2 (dash-dotted).



(a) $\sigma^2,\ \sigma^2/\lambda_i$ (b) $\mu_2^k,\ \mu_2^i$ (c) $\phi_1^k,\ \phi_1^i$

Figure 3: $\pi(\gamma, \gamma_z|Y^T)$, group 1 (solid line), group 2 (dash-dotted).

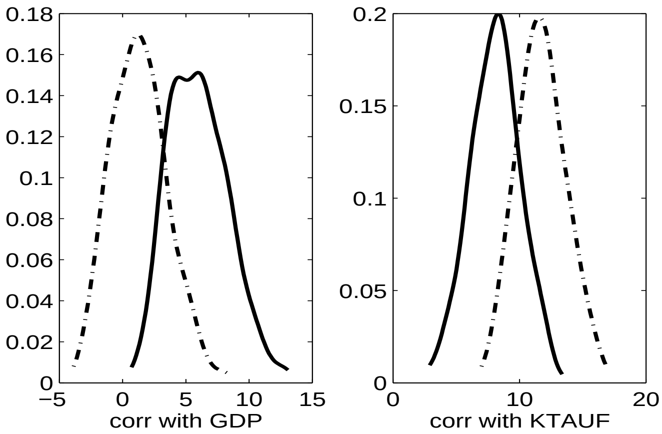


Figure 4: Prior and posterior group probabilities, $P(S_i = j|Z_i)$ and $P(S_i = j|Z_i, Y^T)$

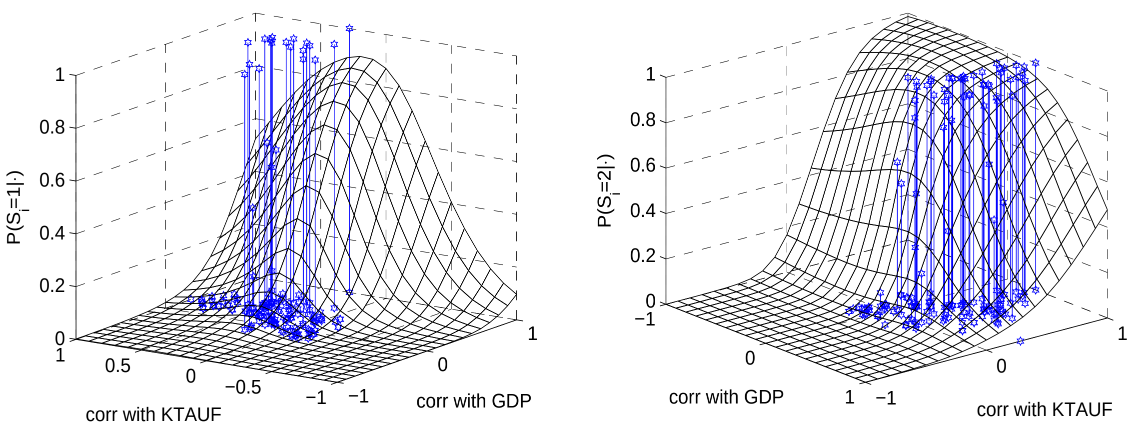


Figure 5: Posterior probabilities, $P(I_{kt} = 1|Y^T)$, of the coincident ($S_i = 1$) and the leading group ($S_i = 2$), 1988-2006, informative prior classification.

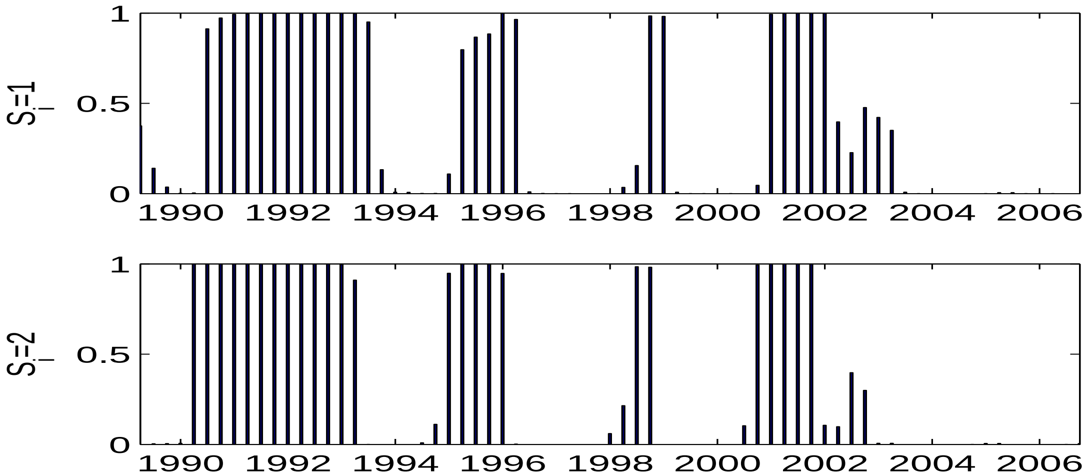
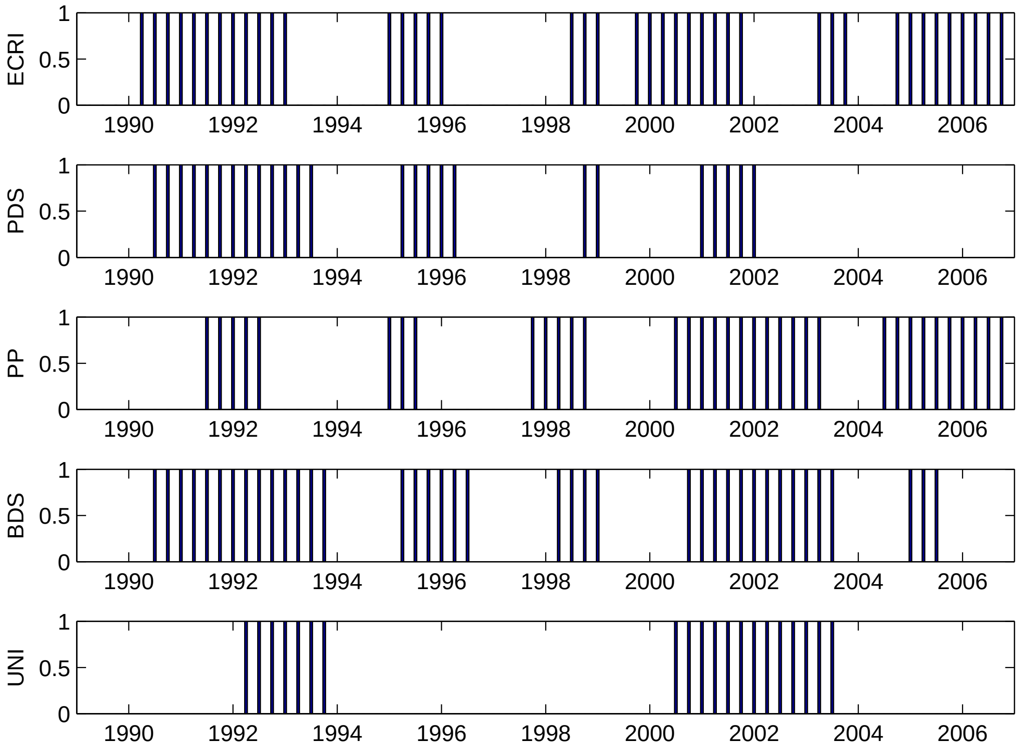


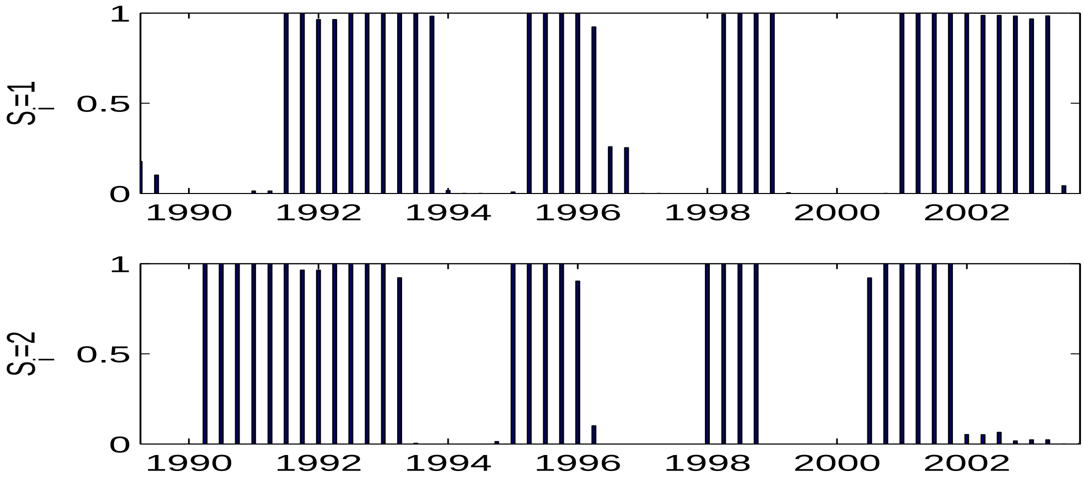
Figure 6: Business cycle phases identified by various models. $I_{kt} = 1$ identifies periods of below-average growth, 1988-2006.



PDS: Markov switching panel with dynamic structure; PP: Markov switching panel estimation (with shrinkage) of coincident group; BDS: Bivariate Markov switching model for GDP and Order books total with dynamic structure; UNI: Univariate Markov switching model for GDP.

Turning points are identified using the dates of ECRI and applying the rule: $P(I_{k,t-1} = 1, I_{k,t} = 1|Y^T) < 0.5$ and $P(I_{k,t+1} = 1, I_{k,t+2} = 1|Y^T) > 0.5$ for a peak and vice versa for a trough.

Figure 7: Posterior probabilities, $P(I_{kt} = 1|Y^T)$, of the coincident ($S_i = 1$) and the leading group ($S_i = 2$), 1988-2003, informative prior classification.



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